ELECTRICAL ENGINEERING TEXTS

A COURSE IN
ELECTRICAL ENGINEERING

VOLUME I
DIRECT CURRENTS

ELECTRICAL ENGINEERING TEXTS

A SERIES OF TEXTBOOKS OUTLINED BY THE

Following Committee.

ELECTRICAL ENGINEERING TEXTS

A COURSE IN ELECTRICAL ENGINEERING

VOLUME I

DIRECT CURRENTS

BY

CHESTER L. DAWES, S. B.
ASSISTANT PROFESSOR OF ELECTRICAL ENGINEERING, THE HARVARD ENGINEERING SCHOOL; MEMBER, AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS, ETC.

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PREFACE

For some time past the editors of the McGraw-Hill Electrical Engineering Texts have experienced a demand for a comprehensive text covering in a simple manner the general field of Electrical Engineering. Accordingly, these two volumes were written at their request, after the scope and general character of the two volumes had been carefully considered.

As the title implies, the books begin with the most elementary conceptions of magnetism and current-flow and gradually advance to a more or less thorough discussion of the many types of direct and alternating current machinery, transmission devices, etc., which are met in practice. These two books are intended for Electrical Engineering students as a stepping stone to the more advanced Electrical Engineering Texts which are already a part of the series.

These two volumes should be useful also to students not planning to specialize in the electrical engineering field, who are taking courses in Electrical Engineering as a part of their general training. Such men often find difficulty in obtaining detailed and straightforward discussions of the subject in any one text and the brevity of their course does not give them time to assimilate fragmentary information obtainable only by consulting a number of references. Men taking foremen's and industrial courses in Electrical Engineering, which as a rule are carried on only in the evening, require text books sufficiently comprehensive, but at the same time not involving much mathematical analysis. Ordinarily, this type of student does not have ready access to reference libraries and is usually out of contact with his instructors except during the short time available for class-room work. In preparing this work the needs of the foregoing types of students have been carefully kept in mind and as a result, a liberal use of figures and illustrative problems has been made.

Also frequent discussions of the methods of making measurements and laboratory tests are included.

In any course in Electrical Engineering, even though it be intended for non-electrical engineers, the author feels that the student gains little from a hurried and superficial treatment of the subject, as such treatment tends only to develop the memorizing of certain formulæ which are soon forgotten. Accordingly the attempt has been made in this text to develop and explain each phenomenon from a few fundamental and well-understood laws rather than to give mere statements of facts. Such treatment will develop the student's reasoning powers and give him training that will be useful in the solution of the more involved engineering problems that may arise later in his career.

Throughout the text, especially in the treatment of the more abstract portions, attempt has been made to show the ultimate bearing upon general engineering practice. The student takes more interest in the theory when he sees that it can be applied to the solving of practical problems. Because this work is not intended for advanced students in Electrical Engineering, little or no calculus is used and the mathematics is limited to simple equations.

The author is indebted to several of the manufacturing companies who have coöperated in the matter of supplying photographs, cuts and material for the text; and particularly to Professor H. E. Clifford of The Harvard Engineering School, for his many suggestions and for the care and pains which he has taken in the matter of editing the manuscripts.

C. L. D.

HARVARD UNIVERSITY, CAMBRIDGE, MASS.
January, 1920.

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CHAPTER I

MAGNETISM AND MAGNETS

1. Magnets and magnetism are involved in the operation of practically all electrical apparatus. Therefore an understanding of their underlying principles is essential to a clear conception of the operation of all such apparatus.

2. Magnetic Materials.—Iron (or steel) is far superior to all other metals and substances as a magnetic material, and is practically the only metal used for magnetic purposes. Cobalt and nickel (and some of their alloys) possess magnetic properties, which are far inferior to those of iron. Liquid oxygen is also attracted to the poles of magnets.

3. Natural Magnets.—Magnetic phenomena were first noted by the ancients. Certain stones, notably at Magnesia, Asia Minor, were found to have the property of attracting bits of iron, hence the name *magnets* was given to these magic stones. The fact that such stones had the property of pointing north and south, if suspended freely, was not discovered until the tenth or twelfth century. The practical use of such a stone in navigation gave it the name of *Lodestone* or leading stone. Natural magnets are composed of an iron ore known in metallurgy as magnetite, having the chemical composition Fe_3O_4.

4. Artificial Magnets.—If a piece of hardened steel be rubbed with lodestone, it will be found to have acquired a very appreciable amount of magnetism, which it will retain indefinitely.

Such a steel magnet is called an *artificial magnet.* Artificial magnets commonly derive their initial excitation from an electric current as will be shown later. If a piece of soft steel or soft iron be similarly treated, it retains but a very small portion of the magnetism initially imparted to it.

These properties make it desirable to use hardened steel when a permanent magnet is desired and to use soft iron or steel when it is essential that the magnetism respond closely to changes of magnetizing force. It is found that even hardened steel ages or loses some of its magnetism with time. Where a high degree of permanency is desired, as in electrical instruments, or even in magnetos, the magnets are aged artificially.

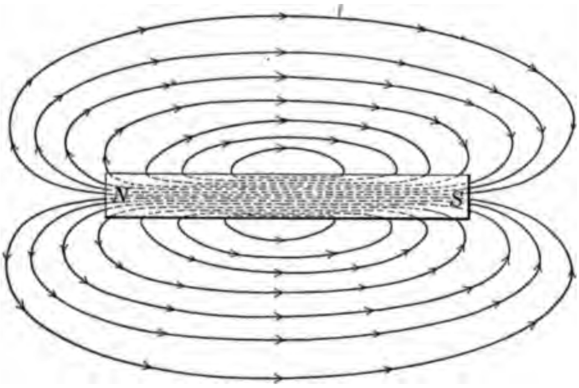


Fig. 1.—Magnetic field about a bar magnet.

5. Magnetic Field.—It is found that magnetism manifests itself as if it existed in lines, called *lines* of *magnetism* or *lines* of *induction.* The region in space through which these lines pass is called the *magnetic field.* Further, if the lines of induction of such a field be determined experimentally, it is found that they seem to emanate from one region of the magnet and enter some other region as shown in Fig. 1. These regions are called the *poles* of the magnet. The two poles are distinguished by the position which they seek if suspended freely. The one which points north is called the *north-seeking pole* or *north pole* for short, and the other the *south-seeking pole,* or *south pole.* In practice it is assumed that the lines of induction leave the magnet at the north pole and re-enter it at the south pole. Within the

magnet the lines of induction continue from the south to the north pole so that each line of induction forms a closed loop. The plane half way between the poles is the *neutral zone* or *equator* of the magnet. No magnetic force is apparent at this point. The entire path through which the lines of induction pass is called the *magnetic circuit*.

6. Effect of Breaking a Bar Magnet.—Neither a north pole nor a south pole can exist alone. For every north pole there exists an equal (but opposite) south pole. If an ordinary bar magnet be broken at the middle, or at various points, each frag-

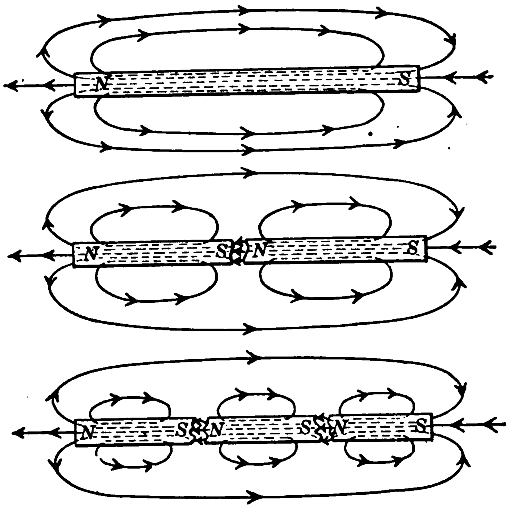


FIG. 2.—Effect of breaking a bar magnet.

ment will constitute a bar magnet having its north and its south pole lying in the same respective directions as those of the original magnet. This phenomenon is easily explained by noting that the lines of induction still continue to pass from one fragment to the next adjacent one, and in so doing constitute north and south poles as shown in Fig. 2. In experimental work, this phenomenon may be easily illustrated by magnetizing a highly tempered steel knitting needle and breaking it at various points.

7. Weber's Theory.—An explanation of the appearance of north and south poles upon breaking a magnet, and other phenomena

occurring in the magnetization of iron, is offered by Weber's Theory which has been expanded by Ewing. The molecules of a magnet are assumed to be an indefinitely great number of very small magnets as shown in Fig. 3 (a). Under ordinary conditions these small magnets are arranged in a haphazard way, as shown at (a), so that the various north and south poles all neutralize one another, and no external effect is produced. Upon the application of a magnetizing force, however, the small magnets tend to arrange themselves so that their axes are parallel

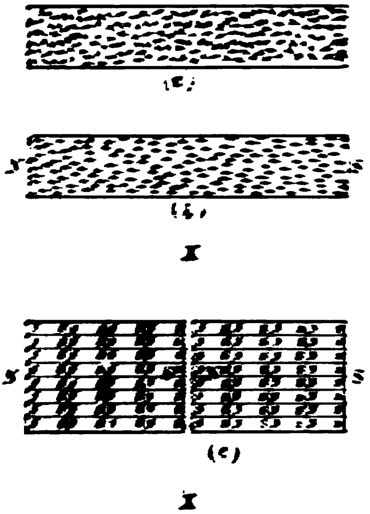


FIG. 3.—Weber's molecular theory of magnets.

and their north poles are all pointing in the same general direction as the magnetizing force. This is shown in Fig. 3 (b). It is evident that if the magnet be cut along the line XX, Fig. 3 (c), a new north and a new south pole will result, which, before the fracture took place, neutralized each other.

This theory is further substantiated by grinding a permanent magnet into very small particles. Each of the small particles possesses the properties of the bar magnet, each having its own north and its own south pole. Further, the theory offers a rational explanation of saturation, hysteresis, etc., occurring in iron subjected to a magnetizing force. This will be considered later.

8. Consequent Poles.—Consequent poles are occasionally found in bar magnets where different portions have been rubbed by a north pole, or a south pole, or when exciting coils, acting in opposition, have been placed upon the bar. Consequent poles are in reality due to the fact that the bar consists of two

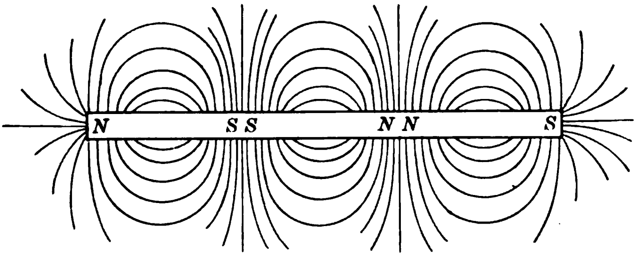


FIG. 4.—Consequent poles.

or more magnets arranged so that two north or two south poles exist in the same portion of the magnet. This is illustrated in Fig. 4. The magnetic field shown in Fig. 11, page 11, is in a way illustrative of the field resulting from consequent poles. In this case, however, two bar magnets are used and a small air gap exists between the adjacent north poles.

9. Magnetic Force.—When a freely suspended north pole is brought in the vicinity of another north pole, it is repulsed, whereas, if a south pole is brought in the presence of a north pole, it is immediately attracted toward the north pole. South poles are also found to repel one another. From this it may be stated that *like poles repel one another and unlike poles attract one another.*

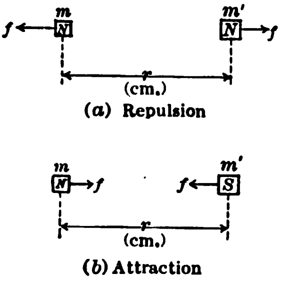


FIG. 5.—Repulsion and attraction between magnetic poles.

10. Pole Strength.—The force of attraction (or repulsion) between two given poles is found to be inversely as the square of the distance between the poles, provided that the dimensions of the poles are small compared with the distance between them. *A unit magnetic pole is one of such strength that if placed at a distance of one centimeter in*

free space from a similar pole of equal strength will repel it with a force of one dyne.

Pole strength is measured by the number of unit poles which, if placed side by side, would be equivalent to the pole in question.

The force f, existing between poles may be formulated as follows:

$$f = \frac{m\,m'}{r^2} \text{ dynes} \tag{1}$$

where m and m' are the respective pole strengths (in terms of a unit pole) of two magnetic poles, placed a distance r cm. apart, as shown in Fig. 5. This force may be attraction or repulsion according as the poles are unlike or like.

Example.—Two north poles, one having a strength of 500 units and the other a strength of 150 units, are placed a distance of 4 inches apart. What is the force in grams acting between these poles, and in what direction does it act?

$$4 \text{ in.} = 4 \times 2.54 = 10.16 \text{ cm.}$$

$$f = \frac{500 \times 150}{(10.16)^2} = \frac{75{,}000}{103.2} = 728 \text{ dynes}$$

$$\frac{728}{981} = 0.741 \text{ gram. Poles repel each other. } \textit{Ans.}$$

11. Lines of Force.—Thus far the magnetic field has been studied only with respect to the lines of magnetism or induction. If a single north pole be placed in such a field two effects will be observed.

1. This pole will be urged along the lines of induction.
2. The force urging this pole will be greatest where the lines of induction are the most dense, and, moreover, the force will be proportional to the number of lines per unit area taken perpendicular to the lines in the field in which the pole finds itself.

From these statements it can be seen that *lines of force*, similar to lines of induction, can be drawn, to represent the forces at the various points in the magnetic field. In much of the literature on the subject lines of induction and lines of force are used indiscriminately. The fallacy of so doing is immediately apparent upon considering a solid bar magnet. The lines of induction pass completely through the solid metal of the magnet, whereas the lines of force terminate at the poles. To be sure, a magnetic force does exist within the magnet, but this force can be determined only by making a cavity in the magnet, and the force

acting under these conditions is quite distinct from that indicated by the number of lines of induction passing through the bar. In air, however, the lines of force and the lines of induction coincide.

12. Field Intensity.—It has been stated that the force acting upon a magnetic pole placed in a magnetic field is proportional to the number of lines of induction at that point. *Unit field intensity is defined as the field strength which will act upon a unit pole with a force of one dyne.* One *line of force* perpendicular to and passing through a square centimeter represents unit field intensity. Field intensity is usually represented by the symbol H. It is evident that if a pole of m units be placed in a field of intensity H, the force acting on this pole is

$$f = m \times H \text{ dynes} \tag{2}$$

A pole placed in such a field must be of such small magnitude that it will have no appreciable disturbing effect upon the magnetic field.

13. Flux Density.—Flux density is the number of lines of induction per unit area, taken perpendicular to the induction. In free space, flux density and field intensity are the same, numerically, but within magnetic material the two are entirely different. The two should not be confused. The unit of flux density (one line per sq. cm.) is often called the *gauss*, but the expression "lines per square centimeter" and "lines per square inch" are more often used in practical work when speaking of flux density.

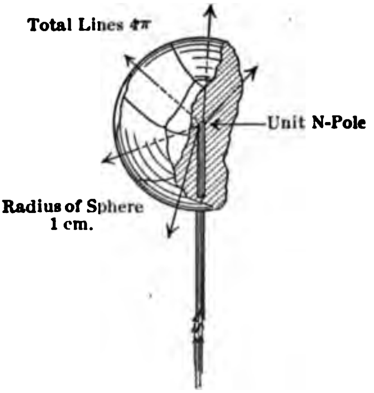


FIG. 6.—Lines of force emanating from a unit N-pole.

By definition the force exerted by a unit pole upon another unit pole at centimeter distance in air is always one dyne. The field intensity on a spherical surface of one centimeter radius must then be unity and can be represented by one line per square centimeter over the entire spherical surface as shown in Fig. 6.

Since there are 4π square centimeters upon the surface of a unit sphere, each unit pole must have radiating from it $4\pi = 12.6$ lines of force. Fig. 6 represents a portion of a spherical surface of one centimeter radius and shows roughly the passage of one line of force through each square centimeter of surface, each line originating in the unit north pole. This also explains the appearance of the 4π term so often encountered in magnetic formulæ. A pole having a strength of m units will radiate $4\pi m$ lines of force.

Example.—A total flux of 200,000 lines passes in air between two parallel pole faces, each 8 cm. square. The field is uniformly distributed. With what force (grams) will a pole, having a strength of 100 units, be acted upon if placed in this field?

Flux density $= \frac{200{,}000}{8 \times 8} = 3{,}120$ lines per sq. cm. or 3,120 gausses. Being in air this value of flux density also equals the field intensity, H.

$$f = m \times H = 100 \times 3{,}120 = 312{,}000 \text{ dynes}$$

$$\frac{312{,}000}{981} = 319 \text{ grams. } \textit{Ans.}$$

Example.—A pole having a strength of 400 units is placed at the center of a sphere having a radius of 3 cm. What is the flux density at the surface of the sphere and what force will be exerted on a pole of 10 units placed at the surface of the sphere?

Total lines emanating from pole $= 400 \times 4\pi = 5{,}020$ lines.

Area of surface of sphere $= 4\pi r^2 = 4\pi 9 = 113$ sq. cm.

Flux density $= \frac{5020}{113} = 44.4$ gausses.

Force upon pole of 10 units $= 44.4 \times 10 = 444$ dynes.

As a check, the force may also be determined by the law of inverse squares (see Par. 10).

$$f = \frac{mm'}{r^2} = \frac{400 \times 10}{3 \times 3} = 444 \text{ dynes.}$$

14. The Compass Needle.—The compass consists of a hardened steel needle or small bar, permanently magnetized and accurately balanced upon a sharp pivot. The north-seeking end or north pole points north, and the south-seeking end points south. The north pole of the needle is usually colored blue or given some distinguishing mark. With the exception of a few used for lecture purposes, the needle is enclosed in an air-tight case for mechanical protection. Mariners' compasses are mounted carefully upon gimbals, so that they always hang level. Upon steel ships, heavy iron balls placed near the compass

are necessary to compensate for the magnetic effect of the ship itself.

By means of the compass the polarity of a magnet is readily determined. The south pole of the compass points to

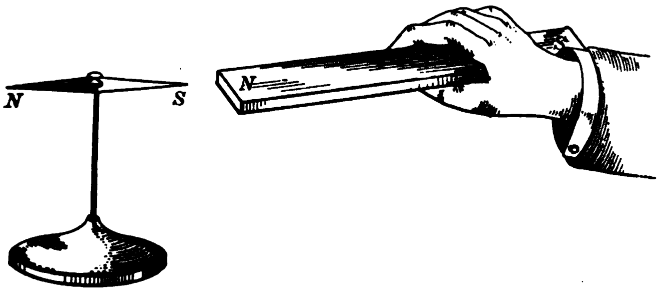


Fig. 7.—Compass needle and bar magnet.

the *north* pole of the magnet as shown in Fig. 7. Likewise, the *north* pole of the compass points to the *south* pole of the magnet. This action of the compass needle follows immediately from the law that like poles repel and unlike poles attract each other.

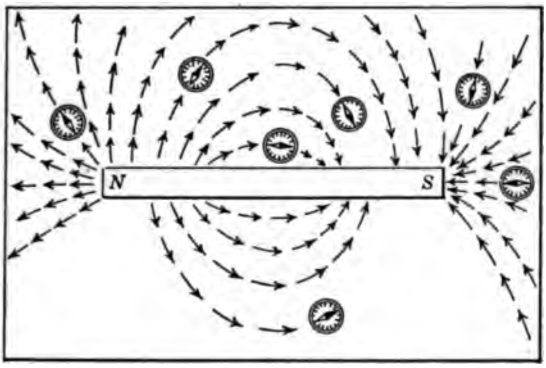


Fig. 8.—Exploring the field about a bar magnet with a compass.

This is very useful in practical work for it enables one to determine the polarity of the various poles of motors and generators and to show if the exciting coils are correctly connected.

Further, the compass needle always tends to set itself in the direction of the magnetic field in which it finds itself, the north end of the needle pointing in the direction of the lines of force or magnetic lines. This is illustrated in Fig. 8. By placing a

small compass at the various points in the region of a magnet, and drawing an arrow at each point, the arrow pointing in the same direction as the needle, the field around the magnet may be mapped out as shown in Fig. 8. In mapping out a field in this way it must be remembered that the earth's field may exert considerable influence on the compass needle in addition to the effect of the field being studied.

15. Magnetic Figures.—If a card be placed over a magnet and iron filings be sprinkled over the card, a magnetic figure is obtained. The filings at each point set themselves in the direction

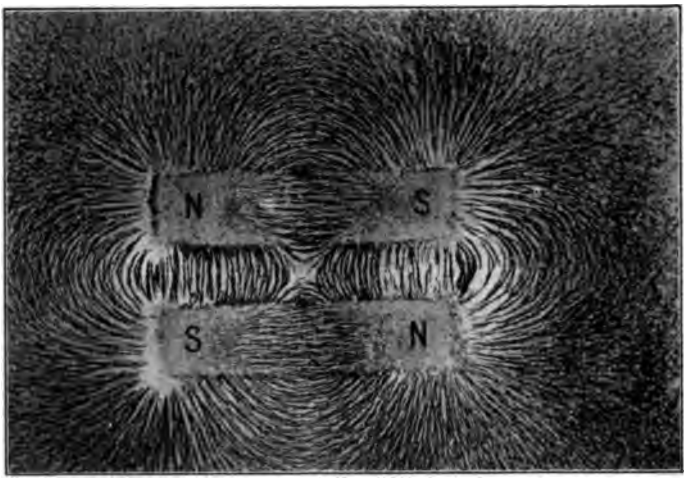


Fig. 9.—Magnetic figure, unlike poles adjacent.

of the lines of force at that point, and the resultant figure shows in very close detail the character of the magnetic field. Fig. 9 shows the magnetic field due to two bar magnets placed side by side and having unlike poles adjacent. On the other hand, Fig. 10 shows the field due to these same bar magnets when like poles are adjacent. It will be noted in Fig. 9 that the lines of force seem like elastic bands stretched from one pole to the other, acting to pull the unlike poles together. In Fig. 10 the lines of force from the two like poles appear to repel one another, indicating a state of repulsion between the poles. Fig. 11 shows the field obtained by placing the bar magnets end to end, having the two north poles adjacent.

16. Magnetic Induction.—If a magnet is brought near a piece of soft, non-magnetized iron, the piece of iron becomes

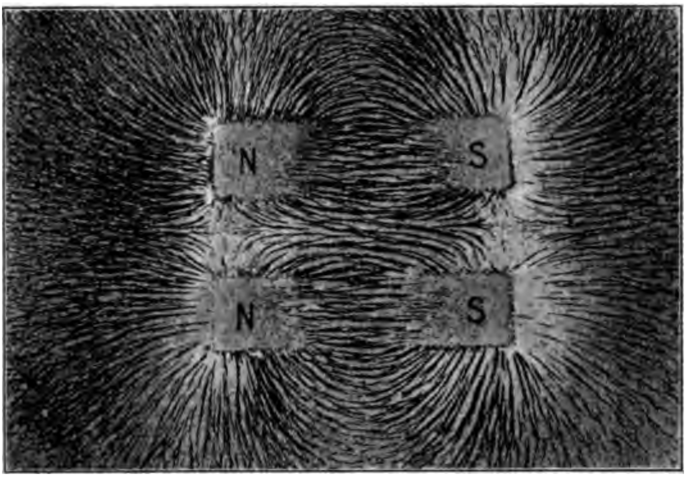


FIG. 10.—Magnetic figure, like poles adjacent.

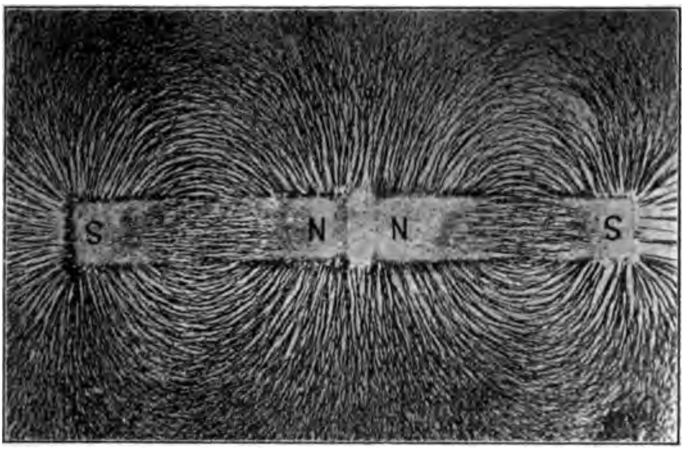


FIG. 11.—Magnetic figure, like N-poles adjacent.

magnetized by induction. If the north pole of the magnet is brought near the iron, a south pole is induced in that part of the iron nearest the inducing magnet, and if the south pole of the

magnet is brought near the iron a north pole is similarly induced. This is illustrated in Fig. 12*a*. From the foregoing, the ability of magnets to attract soft iron is readily understood. An opposite pole to that of the magnet is induced in the iron, and these two poles being of unlike polarity are then attracted toward each other.

It is sometimes noticed that if a comparatively weak north pole be brought into the vicinity of a strong north pole, attraction between the two results, rather than the repulsion which might be expected. This is no violation of the laws governing the at-

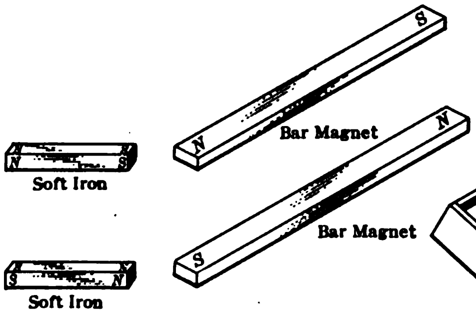


Fig. 12*a*.—Poles produced by magnetic induction.

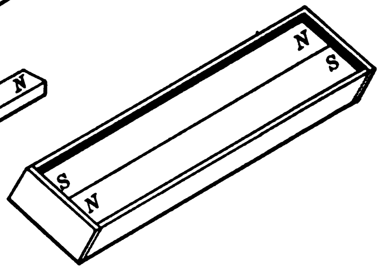


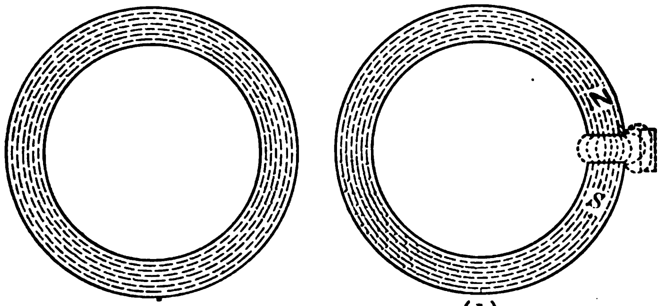
Fig. 12*b*.—Proper method of "keeping" bar magnets.

traction and repulsion of magnetic poles, but comes from the fact that the strong north pole induces a south pole which overpowers the existing weak north pole and results in attraction. In this way it is easy to reverse the polarity of a compass needle by holding one end too close to a strong magnetic pole of the same polarity.

For a similar reason, when two bar magnets are put away in a box, the adjacent ends should be of opposite polarity, as shown in Fig. 12*b*. They will retain their magnetism better under these conditions. When a horseshoe magnet is not in use a "keeper" of soft iron should be placed across the poles.

17. Law of the Magnetic Field.—*The magnetic field always tends to so conform itself that the maximum amount of flux is attained.* This offers further explanation of the attraction of iron to poles of magnets. The iron is drawn toward the magnet so that the magnetic lines may utilize it as a part of their return

path, since iron conducts these lines much better than the air. This is illustrated in the horseshoe magnet of Fig. 14. The armature is drawn toward the poles of the magnet, and the return



(a) (b)

FIG. 13.—Ring magnets.

path through the air is materially shortened, so that the number of magnetic lines is materially increased. The maximum flux exists when the armature is against the poles.

18. Other Forms of Magnets.—The simple bar magnet frequently is not suitable for practical work. For the same amount of material, other forms are more powerful and more compact. Fig. 13 (*a*) shows a closed ring magnet. All the magnetic flux is contained in the ring and little external effect is noted. This type is not very useful. However, if the ring be cut as shown in Fig. 13 (*b*), a north and a south pole are obtained. A piece of soft iron, if brought near this gap, will be strongly attracted and will tend to be drawn across the gap and thus shorten the length of the flux path.

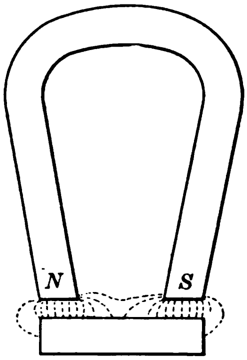


FIG. 14.—Horse-shoe magnet attracting a soft-iron armature.

The horseshoe magnet, shown in Fig. 14, is very useful, for two reasons. The two poles being near each other, a comparatively strong field exists. Further, if the function of the magnet is to exert a pull upon an armature, each pole is equally effective. Fig. 118, Chap. VII, shows a horseshoe magnet such as is used in Weston direct-current instruments.

19. Laminated Magnets.—It is found that thin steel magnets are stronger in proportion to their weight than thick ones. For a given amount of material a magnet made up of several laminations, as shown in Figs. 15 and 16, is more powerful than one made of a single piece of metal. Fig. 16 shows the form of horse-shoe magnet generally used for telephone and ignition magnetos.

20. Magnet Screens.—There is no known insulator for magnetic flux. No appreciable change in the flux or in the pull of a magnet is noticed if glass, paper, wood, copper, or other such material be placed in the magnetic field. However, it is often desirable to shield galvanometers and electrical measuring instruments from the earth's field and from stray fields due to generators, conductors carrying currents, etc. This is done by surrounding the instrument with an iron shell as shown in Fig. 17. This shell by-passes practically the entire flux and thus

Fig. 15.—Compound or laminated bar magnet.

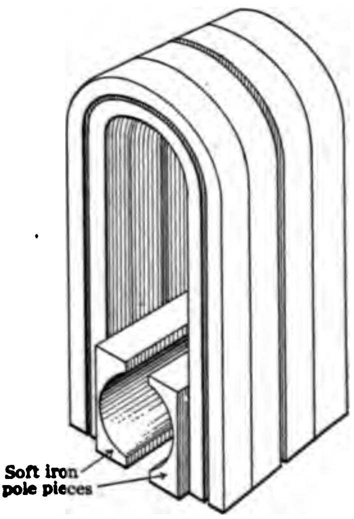


Fig. 16.—Compound horse-shoe magnet used in magnetos.

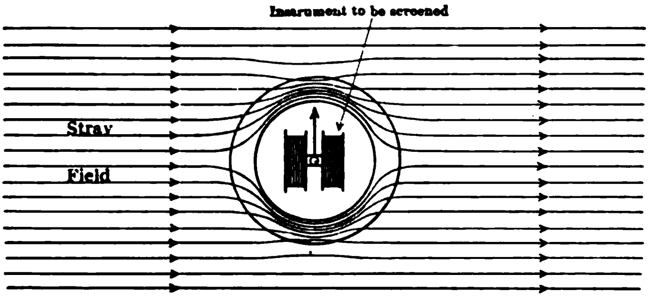


Fig. 17.—Magnetic screen.

prevents it from affecting the sensitive portions of the instrument. The smaller the openings in the shell, the more effective the screening becomes. Three or four shells, with air spaces between, are found to be more effective than one shell of the same total thickness. Such, however, are used only in connection with the screening of the most sensitive galvanometers.

21. Magnetizing.—A magnet may be magnetized by merely rubbing it with another magnet. The resulting polarity at any point is opposite to that of the last pole which came in contact with this point. Therefore, it is well to rub one end with the north pole of the inducing magnet and the other end with the south pole. This may be done simultaneously by the "divided touch" method shown in Fig. 18. It is advisable to rub both sides of the bar. Stronger magnets may be obtained by placing them between the poles of a very powerful electromagnet. Fig. 19 shows this method of magnetizing a horseshoe magnet. An armature or "keeper" should be placed across the poles of the horseshoe magnet before removing it from the electromagnet. Magnetization may also be produced by inserting the magnet in a suitable exciting coil and allowing a heavy current to flow in the coil. A few turns of low resistance wire may be wound around the magnet and connected in series with a fuse to the supply mains. Upon closing the switch, an enormous current passes temporarily, but the fuse blows immediately and prevents damage to the electric circuit. The heavy rush of current is usually sufficient to leave the steel in a strongly magnetized condition.

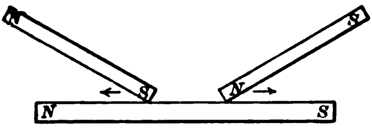


Fig. 18.—Divided touch method of magnetizing.

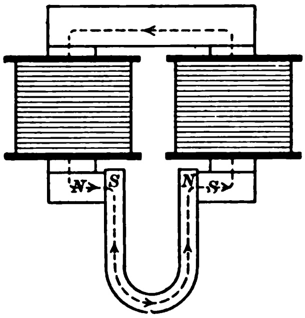


Fig. 19.—Magnetizing a horseshoe magnet with an electromagnet.

22. The Earth's Magnetism.—The earth behaves as a huge bar magnet, the poles of which are not far from the geographical

poles. The north magnetic pole (corresponding to the south pole of a magnet) is situated in Boothia Felix, about 1000 miles from the geographical north pole. The south magnetic pole has never been located but experiment points to the existence of two south poles. Due to the non-coincidence of the geographical and magnetic poles and to the presence of magnetic materials in the earth, the compass points to the true north in only a few places on the earth's surface. The deviation from the true north is called the declination, and magnetic maps are provided showing the declination at various parts of the earth. At New York it is about 9° west. The declination undergoes a gradual variation from year to year, called the variation change. A careful record is kept of this secular variation and scientific measurements, such as are made in astronomy, surveying, and navigation, must be corrected correspondingly. The needle undergoes a very small daily variation and an annual variation, due possibly to the influence of the sun and the moon.

A freely suspended and balanced needle does not take up a position parallel to the earth's surface, when under the influence of the earth's magnetism alone, but assumes a position making some angle with the horizontal. This angle is called the *dip* of the needle. At New York it is about 70° North. The dip undergoes changes similar to those in the variation. The field intensity (total, not horizontal) of the earth's field at New York is about 0.61 C.G.S. units, although this value changes slightly from time to time.

CHAPTER II

ELECTROMAGNETISM

23. Magnetic Field Surrounding a Conductor.—It had long been suspected that some relation existed between electricity and magnetism, but it remained for Oersted in 1819 to show that this relation not only existed but that it was a definite relation.

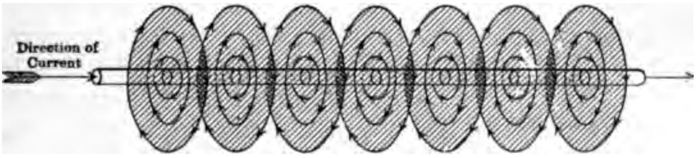


Fig. 20.—Magnetic field about a straight conductor.

If a compass be brought into the neighborhood of a single conductor carrying an electric current, the needle deflects, thus indicating the presence of a magnetic field. It is further observed that the needle always tends to set itself at right angles to the conductor. When it is held above the conductor, the

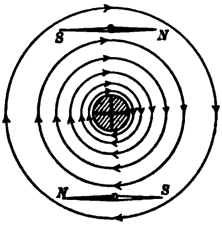


Fig. 21.—Lines of force surrounding a cylindrical conductor—current inwards.

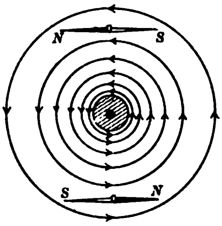


Fig. 22.—Lines of force surrounding a cylindrical conductor—current outwards.

needle points in a direction opposite to that which it assumes when held beneath the conductor. Further investigation shows that the magnetic flux exists in circles about the conductor (if there are no other conductors in the vicinity) as shown in

Figs. 20, 21 and 22. These circles have their centers at the center of the conductor and their planes are perpendicular to the conductor. If the current in the conductor be reversed, the direction in which the compass needle is deflected will be seen to reverse also, showing that the direction of this magnetic field is dependent upon the direction of the current. The relation of the two is shown in Fig. 20. The fact that the magnetic field exists in circles perpendicular to the conductor explains the reversal of the compass needle when moved from a point above the conductor to a point beneath it, for the direction of the field above the conductor must be opposite to that beneath the conductor. This is illustrated in Figs. 21 and 22.[1]

The experiment shown in Fig. 23 is illustrative of this concentric relation of the flux to the conductor. A conductor carrying a current is brought vertically down through a horizontal sheet of cardboard. Iron filings sprinkled on the cardboard form concentric circles. (A current of about 100 amperes is necessary to obtain distinct figures.) If four or more compasses are arranged as shown in Fig. 23, they will indicate, by the direction in which their needles point, that the magnetic lines are circles having the axis of the wire as a center.

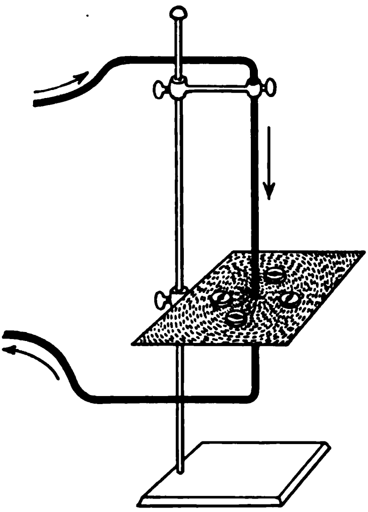


Fig. 23.—Investigation of the magnetic field surrounding a conductor.

24. Relation of Magnetic Field to Current.—A definite relation exists between the direction of the current in a conductor and the direction of the magnetic field surrounding the conductor. There are two simple rules by which this relation may be remembered.

[1] A circle having a cross inside (⊕) indicates that the current is flowing into the paper, and represents the feathered end of an arrow. A circle having a dot at the center (⊙) indicates that the current is flowing out of the paper, and represents the approaching tip of an arrow.

Hand Rule.—Grasp the conductor in the right hand with the thumb pointing in the direction of the current. The fingers will then point in the direction of the lines of force (Fig. 24).

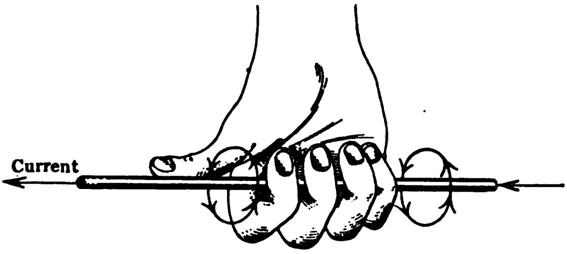


Fig. 24.—Hand rule.

Corkscrew Rule.—The direction of the current and that of the resulting magnetic field are related to each other as the forward travel of a corkscrew and the direction in which it is rotated.

This last rule is probably the most common and the most easily remembered. However, it must not be inferred from this rule that the magnetic field exists in spirals about the conductor. It exists actually in planes perpendicular to the conductor.

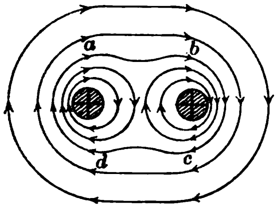


Fig. 25.—Magnetic field about two parallel conductors—current in same direction.

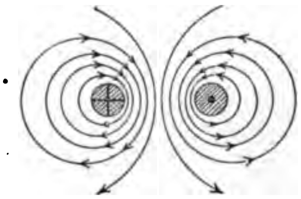


Fig. 26.—Magnetic field about two parallel conductors—current in opposite directions.

25. Magnetic Field of Two Parallel Conductors.—When each of two parallel conductors carries an electric current, flowing in the same direction, there is a tendency for the two conductors to be drawn together. The reason for this is obvious. In Fig. 25 the lines of force encircle each conductor in the same direction (corkscrew rule) and the resultant field is an envelope of lines tending to pull the conductors together. Further reason for this attraction is given by the rule of Par. 17 stating that the magnetic field tends to so conform itself that the

number of magnetic lines is a maximum. The pulling together of the conductors reduces the length of path *abcd* through which the lines must pass. The field due to each conductor separately is still circular in form but the resultant magnetic lines are no longer circular, as is shown in Fig. 25.

In Fig. 26 are shown the conditions which exist when two parallel conductors carry current in opposite directions. The magnetic lines are circles, but these circles are not concentric either with one another or with the conductor. The lines are crowded between the conductors and therefore tend to push the conductors farther apart. Again, when the conductors separate, the area through which the flux passes is increased, so that the magnetic circuit in this case also tends to so conform itself that the magnetic flux is a maximum.

From the foregoing, the following rules may be formulated. *Conductors carrying current in the same direction tend to be drawn together; conductors carrying current in opposite directions tend to be repelled from each other.*

All electric circuits tend to take such a position as will make their currents parallel and flowing in the same direction.

This effect is especially pronounced in modern large capacity power systems. Bus-bars have been wrenched from their clamps; transformer coils have been pulled out of place and transformers wrecked by the forces produced by the enormous currents arising under short circuit conditions.

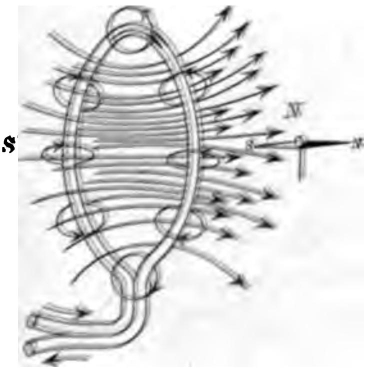


FIG. 27.—Magnetic field produced by a single turn.

26. Magnetic Field of a Solenoid.—If a wire carrying a current be bent into a loop a field similar to that shown in Fig. 27 is obtained. This magnetic field has a north pole and a south pole which possess all the properties of similar poles of a short bar magnet. A compass needle placed in this field assumes the direction shown, the north pole pointing in the direction of the magnetic lines.

27. The Solenoid.—An electric conductor wound in the form of a helix and carrying current is called a *solenoid*. A simple solenoid and the magnetic field produced within it when current flows through the conductor is shown in Fig. 28. The solenoid may be considered as consisting of a large number of the turns

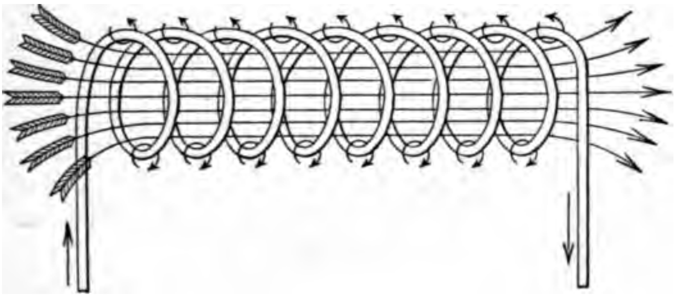


FIG. 28.—Magnetic field produced by a helix or solenoid.

shown in Fig. 27 placed together. The solenoid winding may consist of several layers as shown in Fig. 30.

The relation of the direction of the flux within the solenoid to the direction in which the current flows in the helix may be determined by the hand rule, or by the corkscrew rule of Par. 24.

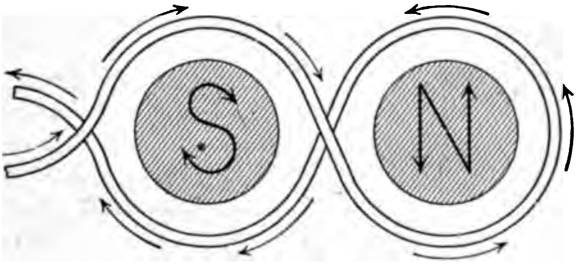


FIG. 29.—Relation of magnetic pole to direction of exciting current.

Another simple method, is shown in Fig. 29, where the arrows at the ends of the "*N*" and the "*S*" show the direction of current in the coil. For example, when looking down upon a north pole the current direction in the coil will be counter-clockwise as shown by the "*N*;" when looking down upon a south pole the direction of the exciting current will be clockwise as shown by the "*S*."

28. The Commercial Solenoid.—The solenoid is used in practice for tripping circuit breakers (Par. 237), for operating contactors in automatic motor starters (Par. 219), for operating voltage regulating devices (Par. 207), for arc lamp feeds (Chap. XIII, Vol. II), for operating valves, and for numerous other purposes. In practically all instances a soft iron (or steel) plung-

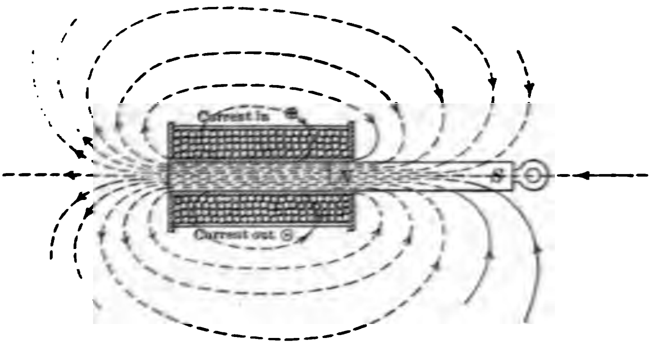


FIG. 30.—Simple solenoid and plunger.

er or armature is necessary to obtain the tractive pull required of the solenoid. The operation of a solenoid and plunger is indicated in Fig. 30. The flux due to the solenoid produces magnetic poles on the plunger. The pole nearer the plunger will be of such sign that it will be urged along the lines of force, (see Par. 11) and in such a direction as to be drawn within the solenoid.

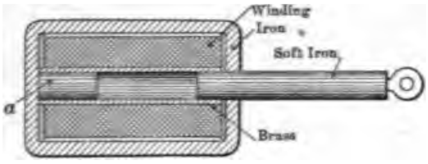


FIG. 31.—"Iron clad" solenoid and plunger with stop.

A position of equilibrium is reached when the center of the plunger reaches the center of the solenoid (Fig. 30). Fig. 31 shows an "iron-clad" solenoid commonly used for tractive work. The iron-clad feature increases the range of uniform pull and produces a very decided increase of pull as the plunger

approaches the end of the stroke. When a stop "*a*" is used, the solenoid becomes a *plunger electromagnet.* This changes

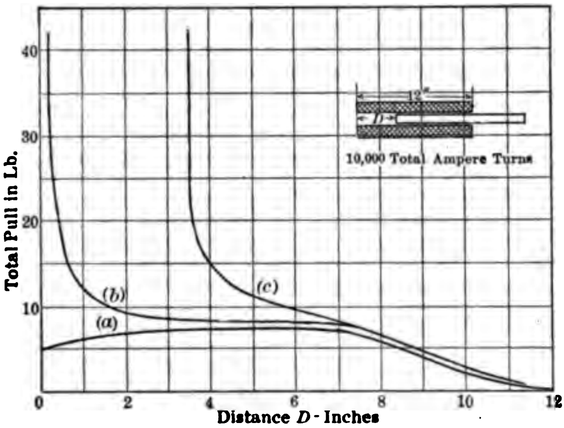


Fig. 32.—Pull of solenoid on plunger.

the characteristics of the solenoid in that the maximum pull now occurs when the end of the plunger is near the stop. Fig. 32 shows the results of solenoid tests made by C. R. Underhill.[1] Curve (*a*) is the pull upon the plunger of a simple solenoid like that of Fig. 30; curve (*b*) shows the pull when this solenoid is iron-clad as in Fig. 31 but without a stop; curve (*c*) shows the effect of the "stop" on the pull. It will be noted that the iron-clad feature and the stop have but little effect except near the end of the stroke.

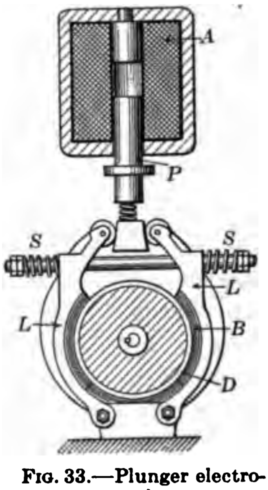


Fig. 33.—Plunger electromagnet operating a crane brake.

An important practical application of the solenoid occurs in the braking of elevators and cranes. When the power is removed from the lifting motor or when the power is interrupted due to a broken wire or other accident, the brake must be applied immediately. One method of accomplishing this is shown in Fig. 33. When

[1] "Standard Handbook, Section 5."

the power, for any reason, is interrupted, the plunger P of the solenoid A drops, due partly to gravity and partly to the action of the springs S. The springs S immediately force the levers L against the brake bands B, pressing these against the brake drum D, thus effecting the braking action. When the power is applied to the lifting motor, the plunger P is pulled up, thus releasing the brake. A plunger electromagnet is most suitable for this purpose because the stroke is short and the pull must be positive.

29. The Horseshoe Solenoid.—The use of an armature in connection with solenoids is well illustrated by the relay or the sounder used in telegraphy, and also by electric bells, buzzers, etc. To increase the effectiveness of such devices two solenoids

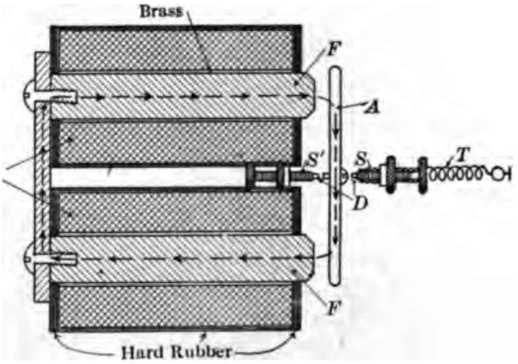


FIG. 34.—Telegraph relay.

are used, each being placed on one of the legs of a horseshoe magnet. When the coils C (Fig. 34) become excited, the iron armature A is attracted because of the tendency of the magnetic lines to make their path of minimum length. As a rule, the armature A is not allowed to close the magnetic circuit completely, for under these conditions the magnetic lines still exist after the excitation is removed, preventing rapid release of the armature. The stop S' prevents the armature making contact with the cores FF and thus completely closing the magnetic circuit. The contacts D close any secondary circuit that the relay may be operating. The spring T draws the armature back against a stop S when the excitation is removed.

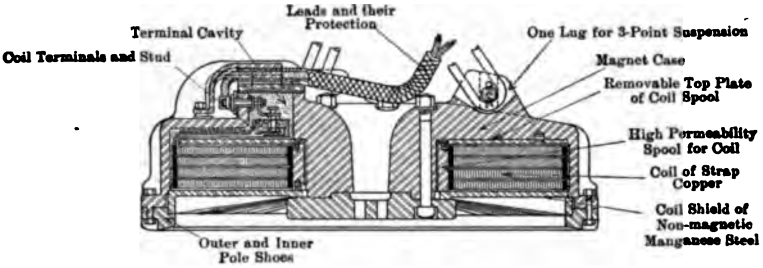


FIG. 35.—Cross-section of a lifting magnet.

FIG. 36.—Cutler-Hammer 36-inch magnet, handling heavy castings.

30. The Lifting Magnet.—Lifting magnets are used commercially to handle iron and steel in various forms. A very appreciable saving of time and labor is effected by their use, because chains and slings for holding the load are not necessary. They are very useful for handling steel billets in rolling mills, but the billets cannot be picked up when red hot as they lose their magnetic properties at this temperature. Magnets are especially useful in loading and unloading steel rails, for an entire layer may be picked up and laid down again without being disarranged. Lifting magnets effect a very great saving of labor when small pieces of iron, such as scrap iron, are handled, for they will pick up large quantities at every lift. Without a magnet each individual piece would have to be moved by hand. Fig. 35 shows in cross-section a typical Cutler-Hammer lifting magnet. Fig. 36 shows a lifting magnet in actual operation.

Formulæ for the holding force of electromagnets are given in Par. 149.

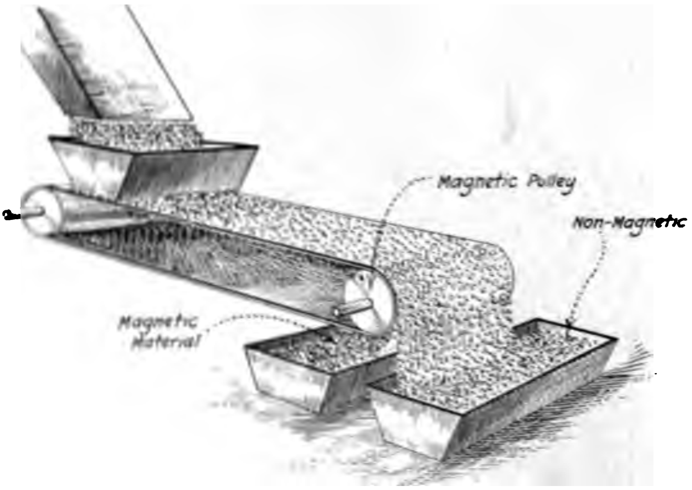


Fig. 37. Magnetic separator.

It should be understood that the magnet itself does little or no work in the lifting, but merely serves as a holding device. The actual work is performed by the engine or motor which operates the steel ropes or chains attached to the magnet.

31. Magnetic Separator.—Another important application of magnetic principles is found in the magnetic separator shown in Fig. 37. It is especially designed to remove steel and iron from coal, rock, ore, etc., but it may be used for separating steel shot from molding sand, iron chips from machine shop turnings, etc. The material is fed on an endless belt running at a speed of about 100 ft. per minute. The belt passes over a magnetized pulley. The non-magnetic material immediately drops off into a hopper, but the magnetic material is held by the pulley until the belt leaves the pulley when the material drops into another hopper. The pulley is magnetized by concentric exciting coils, to which current is carried by means of slip rings.

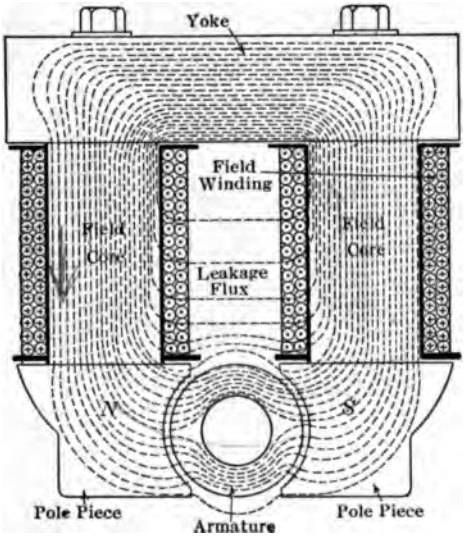


Fig. 38.—Magnetic circuit and field windings of an Edison bi-polar generator.

32. The Magnetic Circuits of Dynamos.—One of the most important uses of electromagnets is in the magnetic fields of motors and generators. An early and simple type of such a magnetic circuit is illustrated by the Edison bi-polar generator, shown in Fig. 38. The type of magnetic circuit shown is very inefficient, because of its great length, in comparison with its sectional area. There results a considerable magnetic leakage which re-

duces, therefore, the amount of flux passing through the armature. Moreover, the flux in taking the shortest path tends to crowd through the upper half of the armature. This tends to produce unsatisfactory commutation.

The magnetic circuit of a bi-polar generator of modern design is shown in Fig. 39. Because of the symmetry of the magnetic circuit the flux divides evenly through the two sides of the armature. The long air path existing between the pole shoes

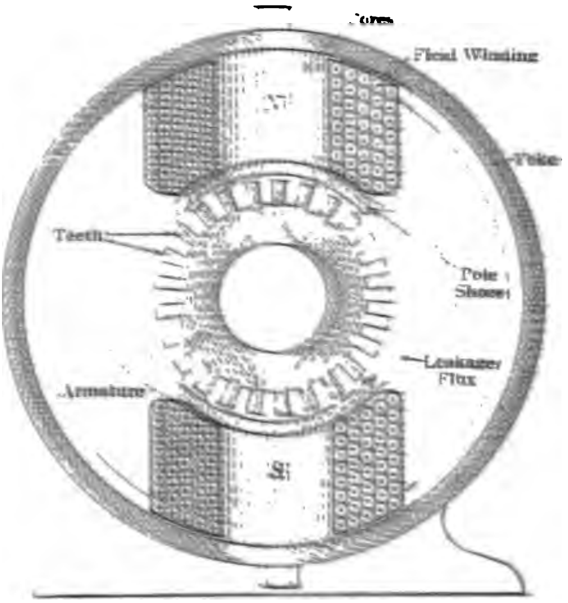


Fig. 39.—Magnetic circuit and field windings of a modern bi-polar generator.

reduces the magnetic leakage to a minimum. It is to be noted that the flux in the cores divides as it passes into the yoke. Consequently the yoke need only be one-half the cross-section of the field cores. Direct-current machines of the bi-polar type are made usually in small units.

Fig. 40 shows the more complex magnetic circuits of a multi-polar generator having eight poles. It is to be noted that the poles are alternately north and south. Again the flux passing through the field cores divides, both upon reaching the yoke and upon reaching the armature path and the cross-section of the

yoke need only be one-half that of the cores. In both Fig. 39 and Fig. 40 the magnetic leakage is very materially reduced

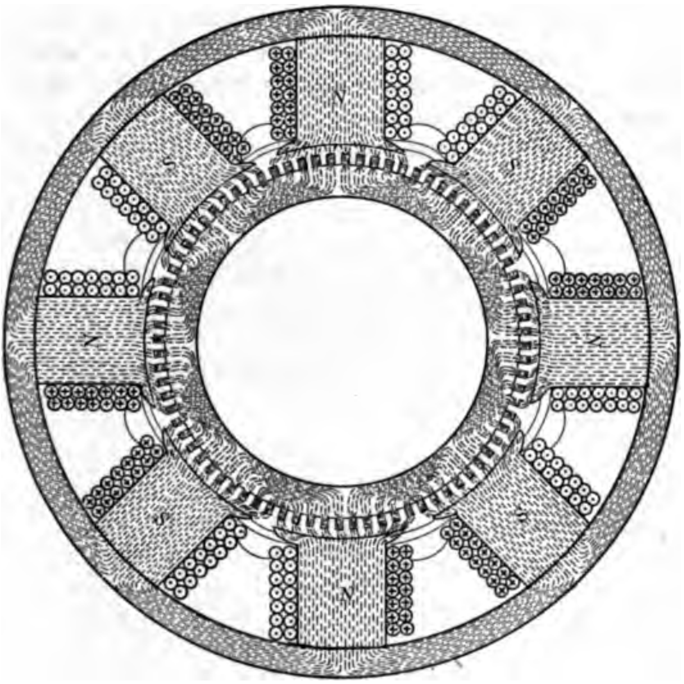


FIG. 40.—Magnetic circuits of a multi-polar generator.

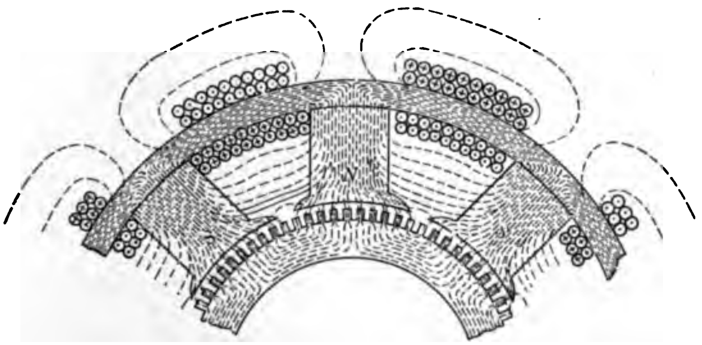


FIG. 41.—Magnetic leakage produced by incorrect position of exciting coils.

by placing the exciting ampere turns as near the armature as possible. This result is not secured in the Edison bi-polar

generator of Fig. 39. To illustrate, Fig. 42 shows the same generator as that in Fig. 41, but with the exciting coils placed upon the yoke. They still act upon the magnetic circuits, but because of their remoteness from the armature, a large magnetic leakage results around the outside of the yoke and through the interpolar space, resulting in a smaller percentage of the total flux passing through the armature.

It is to be understood that of itself magnetic leakage does not lower the efficiency of a machine, since to maintain a constant magnetic field does not require an expenditure of energy. However, in order that the necessary number of magnetic lines may reach the armature both the yokes and the cores must have sufficiently large cross-sections to carry the leakage flux in addition to the useful armature flux. This in turn increases the amount of field copper. Therefore a large magnetic leakage results in a larger, heavier and more expensive machine than would otherwise be necessary.

CHAPTER III

RESISTANCE

33. Electrical Resistance.—The current flowing through an electric circuit depends not only upon the electromotive force impressed across the circuit, but upon the circuit properties as well. If, for example, a wire be connected across the terminals of a battery a current will obviously flow through this wire. If a poor contact be made at a battery terminal or at some other point in the circuit the current will drop in value, even with the same electromotive force acting. Also considerable heat will be dissipated at the point of poor contact. This property of tending to prevent the flow of current and at the same time causing heat dissipation is called *resistance.*

Resistance in the electric circuit may be likened in its effect to friction in mechanics. For example, if a street-car is running at a uniform speed on a level track, friction tends to prevent the moving of the car. The power which is used in moving the car is converted by friction into heat. Friction tends to impede the flow of water in a pipe or in a flume, some of the energy of the water being expended in overcoming this friction. The loss of energy is represented by a loss of head. This energy loss is largely absorbed by the water and therefore careful measurements would show a slight increase in its temperature.

As will be shown in the next chapter, the energy loss which occurs when an electric current passes through a resistance, is directly proportional to the amount of resistance.

All[1] substances have resistance, but the resistance of some sub-

[1] Professor Kamerlingh-Onnes of Leyden, in a recent experiment was able to produce a circuit in which an electric current showed no diminution in strength 5 hours after the electromotive force had been removed. The current was induced magnetically in a short-circuited coil of lead wire at −270° C. in the presence of liquid helium and the inducing source then removed. Liquid helium has the lowest temperature known, being in the neighborhood of absolute zero (−273° C.). This experiment indicates that the resistance of the lead was practically zero at this extremely low temperature.

stances is many times greater than that of others. This leads to the classification of substances into either conductors or insulators. Even silver, one of the best conductors, has appreciable resistance, and glass or porcelain, among the best insulators known, will allow a certain amount of current to pass and therefore are not perfect insulators. The best conductors are the metals, silver coming first and copper second. Carbon and ordinary water also may be classed as conductors. Distilled or pure water, however, is a good insulator. Oils, glass, silk, paper, cotton, fiber, ebonite, paraffin, rubber, etc., may be considered as non-conductors or good insulators. Wood either dry or impregnated with oil is a good insulator, but wood containing moisture is a partial conductor.

34. Unit of Resistance.—The ohm is the practical unit of resistance and is defined as that resistance which will allow one ampere to flow if one volt is impressed across its terminals.

An ohm also has such a value that one ampere flowing through it for one second dissipates as heat one joule of energy.

The resistance of insulating substances is ordinarily of the magnitude of millions of ohms, so that it is awkward to express this resistance in terms of a unit as small as the ohm. The *megohm*, equal to 1,000,000 (10^6) ohms, is the unit ordinarily used under these conditions. (The prefix "mega" means million.)

On the other hand, the resistance of bus-bars and short pieces of metals may be so low that the ohm is too large a unit for conveniently expressing it. Under these conditions the *microhm* is used as the unit, and is equal to $\frac{1}{1,000,000}$ of an ohm (10^{-6}). (The prefix "micro" means one millionth.)

35. Resistance and Direction of Current.—The resistance of a body of given material depends not only on its size and shape, but upon the direction in which the electric current flows through it.

This may be illustrated by the reservoirs and pipes shown in Fig. 42. Two equal reservoirs A and B are to be emptied through pipes P and P' respectively. The pipe P is twice as long as the pipe P' but of one-half the cross-section. Therefore both pipes have the same volume. It is evident that reservoir B will be emptied much quicker than A, because pipe P' has twice

the cross-section of pipe P, and therefore offers less resistance per unit of length. Further, the length of P' is only half that of P, and this again makes the friction of P' half that of P even if the cross-sections were equal.

Now consider the two conductors A and B (Fig. 43) each of the same material. Conductor A has twice the length of conductor B but only one-half the cross-section. Therefore each

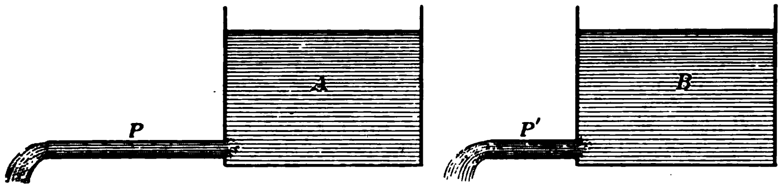


FIG. 42.—Water discharge through different-sized pipes.

conductor contains the same amount of material. It is evident, however, that the resistance per unit length of conductor A is twice that per unit length of B. Then if conductors A and B were of the same length conductor A would have twice the resistance of conductor B. However, conductor A is twice the length of B, and therefore must have 2×2 or 4 times the resistance of B.

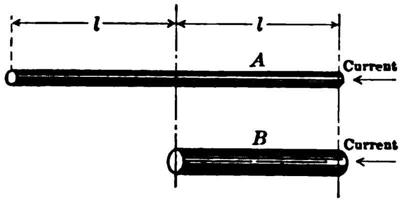


FIG. 43.—Two conductors of equal volume.

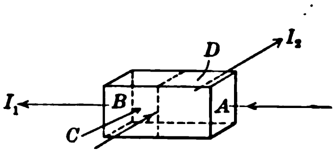


FIG. 44.—Rectangular prism as a conductor.

When specifying the resistance of a body or a substance the *direction* in which the current flows must be specified. Consider the rectangular prism of Fig. 44, composed of two cubes, each a centimeter on edge. Assume that the current flows in the direction of I_1 from the end A through the solid to the end B. It encounters the resistance of each of two cubes successively. If it encounters a *total* resistance of 4 microhms, the resistance of each cube then must be 4/2 or 2 microhms. If, however,

the current flows in the direction of I_2, between the two oblong surfaces C and D of the solid, it finds two paths in parallel, each 1 sq. cm. in cross-section, and each having a length of 1 cm. In virtue of the two paths the resistance per cm. length of the path CD is but one-half the resistance per cm. of AB. But, the path AB is 2 cm. in length, therefore the path CD must have a resistance of one-fourth that of path AB or 1 microhm. Consequently in specifying the resistance of a solid, the *direction* of the current flow should be specified unless this direction is obvious.

36. Specific Resistance or Resistivity.—From the deductions of Par. 35 the following rule for resistance may be stated:

The resistance of a homogeneous body of uniform cross-section varies directly as its length, and inversely as its cross-section.

That is,

$$R = k\frac{L}{A} \tag{3}$$

where R is the resistance in ohms, L is the length in the direction of the current flow, A is the area at right angles to the current flow, and k is a constant of the material known as its *resistivity* or *specific resistance.*

If L is 1 cm. and A is 1 cm. square, the substance in question must have the form of a cube, 1 cm. on an edge and

$$R = k\frac{1}{1 \times 1}$$

or

$$R = k$$

k is called the *specific resistance* or the *resistivity* of the substance, in this case per cm. cube. k may be expressed in terms of an in. cube or in other units as will be shown later. The resistivity of copper is 1.724 microhms, or 1/580,000 ohm, per cm. cube. It is evident that the cube is a perfectly definite unit of resistivity for the resistance between any two opposite faces is the same. The resistivities of various substances are given in Par. 43. Knowing the specific resistance in terms of the cm. cube, the resistance of a wire, bar, etc., may be readily computed from formula (3).

Example—Determine the resistance of 3,000 ft. of annealed 0000 copper wire having a diameter of 0.460 in., the specific resistance of copper being 1.724 microhms (= 0.000001724 ohm) per cm. cube (20° C.). (See Par. 42.)

$$3{,}000' = 3{,}000 \times 12 \times 2.54 = 91{,}500 \text{ cm.}$$

$$\text{Cross-section} = \frac{\pi}{4}(0.460 \times 2.54)^2 = 1.07 \text{ sq. cm.}$$

$$R = k\frac{L}{A} = (0.000001724) \times \left(\frac{91{,}500}{1.07}\right) = 0.1472 \text{ ohm.} \quad \textit{Ans.}$$

37. Volume Resistivity.—Since the volume of a body

$$V = LA$$

where L is its length and A its uniform cross-section, equation (3) may be written

$$R = k\frac{L}{A} = k\frac{L^2}{V} = k\frac{V}{A^2} \tag{4}$$

That is:

The resistance of a conductor varies direcly as the square of its length when the volume is fixed.

The resistance of a conductor varies inversely as the square of its cross-section when the volume is fixed.

Example:

A kilometer of wire having a diameter of 11.7 mm. and a resistance of 0.031 ohm is drawn down so that its diameter is 5.0 mm. What does its resistance become?

The original cross-section of the wire

$$A_1 = \frac{\pi}{4}\, 11.7^2 = 107.5 \text{ sq. mm.}$$

The final cross-section

$$A_2 = \frac{\pi}{4}\, 5.0^2 = 19.64 \text{ sq. mm.}$$

Applying equation (4)

$$R_1 = k\frac{V}{(107.5)^2} = 0.031 \text{ ohm}$$

$$R_2 = k\frac{V}{(19.64)^2}$$

Since the volume of the wire does not change during the drawing process and the resistivity constant k remains the same,

$$\frac{R_2}{R_1} = \frac{R_2}{0.031} = \frac{k\dfrac{V}{(19.64)^2}}{k\dfrac{V}{(107.5)^2}}$$

$$R_2 = 0.031\,\frac{(107.5)^2}{(19.64)^2} = 0.031\,\frac{11{,}560}{386} = 0.93 \text{ ohm.} \quad \textit{Ans.}$$

38. Conductance.—Conductance is the reciprocal of resistance and may be defined as being that property of a circuit or of a material which tends to permit the flow of an electric current. The unit of conductance is the reciprocal ohm or *mho*. Conductance is usually expressed by g.

$$g = \frac{1}{R} \tag{5}$$

also

$$g = k'\frac{A}{L} \tag{6}$$

where k' is the *specific conductance* or the *conductivity* of a substance, A the uniform cross-section and L the length.

The conductivity of copper is 580,000 mhos per cm. cube.

Example.—Determine the conductance of an aluminum busbar 0.5 in. thick, 4 in. wide, and 20 ft. long.

The conductivity of aluminum is 61 per cent. that of copper and copper has a conductivity of 580,000 mhos per cm. cube.

The conductivity of aluminum is:

$$k' = 0.61 \times 580{,}000 = 354{,}000 \text{ mhos/cm. cube}$$

The cross-section of the bus-bar:

$$A = 0.5 \times 4 \times 2.54 \times 2.54 = 12.9 \text{ sq. cm.}$$

The length $L = 20 \times 12 \times 2.54 = 610$ cm.

The conductance:

$$g = 354{,}000 \times \frac{12.9}{610} = 7{,}490 \text{ mhos.} \qquad \textit{Ans.}$$

39. Per Cent. Conductivity.—Until very recently the per cent. conductivity of copper has been based upon results obtained in 1862 by Matthiesen, who made careful measurements of the resistance of supposedly pure copper. He found the resistivity to be 1.594 microhms per cm. cube at 0° C. In view of the uncertainty of the quality of his copper, the Bureau of Standards has recently made a large number of measurements upon commercial copper. Its recommendation that the standard of resistivity be 1.724 microhms per cm. cube at 20° C. has been adopted internationally. The per cent. conductivity for carefully refined copper may exceed 100. Comparison to obtain per cent. conductivity should be made at 20° C.

Example.—A copper rod 4 ft. long and having a diameter of 162 mils has a resistance of 0.0016 ohm at 20° C. What is its per cent. conductivity?

$$\text{Cross-section} = \frac{\pi}{4}(0.162 \times 2.54)^2 = 0.133 \text{ sq. cm.}$$

$$\text{Length} = 4 \times 12 \times 2.54 = 122 \text{ cm.}$$

$$\text{Resistance per cm. cube} = \frac{0.0016 \times 0.133}{122} = 0.000001740 \text{ ohm}$$
$$= 1.740 \text{ microhms.}$$

$$\text{Per cent. conductivity} = \frac{1.724}{1.740} = 99\%. \qquad \textit{Ans.}$$

40. Resistances in Series and in Parallel.—If a number of resistances r_1, r_2, r_3, etc., Fig. 45, are connected in series, that is, end to end, the total resistance of the combination is:

$$R = r_1 + r_2 + r_3 + \ldots \tag{7}$$

That is:

In a series circuit the total resistance is the sum of the individual resistances.

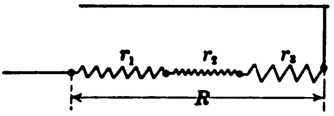


FIG. 45.—Resistances in series.

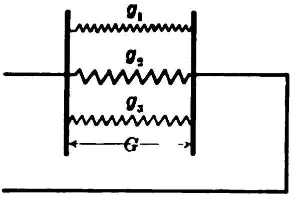


FIG. 46.—Conductances in parallel

If a number of conductances g_1, g_2, g_3, etc., are connected in parallel, Fig. 46, the total conductance of this portion of the circuit must be equal to the sum of the individual conductances, that is,

$$G = g_1 + g_2 + g_3 + \ldots \tag{8}$$

Since,

$$G = \frac{1}{R}, \quad \text{equation (8) may be written}$$

$$\frac{1}{R} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} \tag{9}$$

That is:

In a parallel circuit, the reciprocal of the total resistance is equal to the sum of the reciprocals of the individual resistances.

For a circuit having two resistances in parallel, r_1 and r_2, the joint resistance

$$R = \frac{r_1 r_2}{r_1 + r_2}; \tag{10}$$

for three resistances in parallel the joint resistance is

$$R = \frac{r_1 r_2 r_3}{r_1 r_2 + r_2 r_3 + r_3 r_1} \tag{11}$$

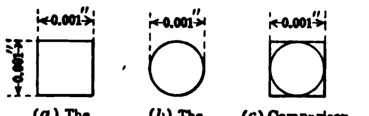
Example.—Determine the total resistance of a circuit having 4 branches, the individual resistances of which are 3, 4, 6 and 8 ohms, respectively.

$$\frac{1}{R} = \frac{1}{3} + \frac{1}{4} + \frac{1}{6} + \frac{1}{8} = 0.333 + 0.250 + 0.167 + 0.125$$
$$= 0.875 \text{ mho.}$$
$$R = \frac{1}{0.875} = 1.142 \text{ ohms.}$$

41. The Circular Mil.—In the English and American wire tables the circular mil is the standard unit of wire cross-section.

The term *mil* means one-thousandth; for example, a millivolt = $1/1000$ volt. A mil is *one thousandth* of an inch. A square mil is a square, each side of which is one mil (0.001 in.), as shown in Fig. 47*a*. The area of a square mil is $0.001 \times 0.001 = 0.000001$ sq. in.

A circular mil is the area of a circle whose *diameter* is *one mil* (0.001 in.), Fig. 47*b*, and is usually written C.M. or *cir. mil.* As will be seen from Fig. 47*c*, a cir. mil is a smaller area than a square mil. The area in sq. in. of a cir. mil $= \frac{\pi}{4}(0.001)^2 = 0.0000007854$ sq. in.



(*a*) The square mil (*b*) The circular mil (*c*) Comparison of the square mil and the circular mil

FIG. 47.

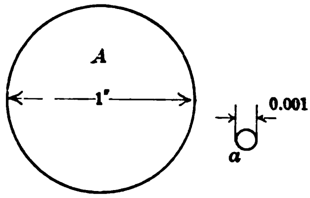


FIG. 48.—Cross-section expressed in cir. mils.

The cir. mil is the unit with which the cross-section of wires and cables is measured, just as the square foot is the unit by which larger areas such as floors, land, etc., are measured. The advantage of the circular mil as a unit is that circular areas

measured in terms of this unit bear a very simple relation to the diameters.

A, in Fig. 48, represents the cross-section of a wire having a diameter of 1 in. Required: to determine its area in cir. mils.

$$\text{The area, } A \qquad = \frac{\pi}{4}(1)^2 \text{ sq. in.}$$

$$\text{The area, } a, \text{ of a cir. mil} = \frac{\pi}{4}(0.001)^2 \text{ sq. in.}$$

The ratio of $\frac{A}{a}$ obviously gives the number of cir. mils in A.

Therefore

$$\frac{A}{a} = \frac{\frac{\pi}{4}(1)^2}{\frac{\pi}{4}\,0.000001} = 1{,}000{,}000 \text{ cir. mils.}$$

The general relation may be written:

$$\text{Cir. mils} = \frac{D_1^2}{(0.001)^2} = 1{,}000{,}000\,(D_1)^2 = D^2 \qquad (12)$$

where

D_1 is the diameter of the wire in *inches*.
D is the diameter of the wire in *mils*.

The matter may be summed up in two rules:

To obtain the number of cir. mils in a solid wire of given diameter express the diameter in mils and then square it.

To obtain the diameter of a solid wire having a given number of cir. mils, take the square root of the cir. mils and the result will be the diameter of the wire in mils.

Example.—00 wire (A.W.G.) has a diameter of 0.3648 in. What is its cir. milage?

$$0.3648 \text{ in.} = 364.8 \text{ mils}$$
$$(364.8)^2 = 133{,}100 \text{ cir. mils.} \quad \textit{Ans.}$$

Example.—A certain wire has a cross-section of 52,640 cir. mils. What is its diameter?

$$\sqrt{52{,}640} = 229.4 \text{ mils} = 0.2294 \text{ in.} \quad \textit{Ans.}$$

42. The Cir.-mil-foot.—Another convenient unit of resistivity, especially in the English system, is the cir.-mil-foot. This

unit is the resistance of a wire having a cross-section of 1 cir. mil and a length of 1 ft., as shown in Fig. 49. The resistance of a cir.-mil-foot of copper at 20° C. is 10.37 ohms. (In practical work this resistance may frequently be taken as 10 ohms.) Knowing this resistivity the resistance of any length and size of wire may be determined by formula (3).

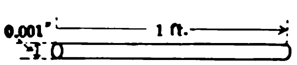


Fig. 49.—The circular-mil foot.

Example.—What is the resistance of a 750,000 C.M. copper cable, 2500 ft. long?

If the cable had a cross-section of 1 cir. mil it would have a resistance of 2500 × 10.37 = 25,900 ohms. However, the cross-section is actually 750,000 cir. mils, therefore,

$$R = \frac{25,900}{750,000} = 0.0346 \text{ ohm}$$

or formula (3) may be used directly

$$R = 10.37 \frac{2,500}{750,000} = 0.0346 \text{ ohm.}$$

When applying formula (3), L must be expressed in *feet* and A in *cir. mils.*

43. Table of Resistivities

Metals	Cm. cube (microhms)	Cir.-mil-foot (ohms)
Aluminum	2.828	17.0
Bismuth	108.0	648
Copper (drawn)	1.724	10.37
German silver	33.3 to 48.0	200 to 290
Ia Ia	49.0	294
Iron:		
Electrolytic	9.96	59.8
Cast	75 to 95	450 to 570
Lead	19	114
Manganin	41.4 to 73.8	248 to 443
Mercury	94.07	565
Nichrome	100 to 110	600 to 660
Nickel	10.67	640
Phosphor-bronze	3.95	23.7
Platinum	9.0 to 15.5	54 to 93
Silver	1.5 to 1.7	9.0 to 10.2
Steel:		
Soft	15.9	95.4
Glass hard	45.7	274
Silicon (4 per cent.)	51.15	308
Transformer	11.09	665

44. Temperature Coefficient of Resistance.—The resistance of the non-alloyed metals increases very appreciably with the temperature. As the temperature of the windings of electric machinery is necessarily much higher than that of the surrounding air, it is important to know the relation between temperature and resistance. The relation may be expressed as follows:

$$R_t = R_0 (1 + at) \tag{13}$$

where R_t is the resistance at the temperature t, R_0 the resistance at 0° C. and a is the *temperature coefficient of resistance* at 0°. For copper a is 0.00427 and for most of the unalloyed metals is sensibly of this value. The above is equivalent to saying that the resistance increases 0.427 of 1 per cent. for each degree Centigrade increase of temperature above 0°. For instance, suppose a coil has a resistance of 100 ohms at 0° C. For every degree increase of temperature the resistance will increase

$$100 \times 0.00427 \text{ ohm or } 0.427 \text{ ohm.}$$

At 40° C. the increase of resistance will be $40 \times 0.427 = 17.08$ ohms, and the resistance at 40° will be $100 + 17.08 = 117.08$ ohms.

If the resistance at some definite temperature other than 0° C. is known, ordinarily the resistance at 0° C. must first be found before the resistance at other temperatures can be determined.

For this purpose formula (13) may be put in the form

$$R_0 = \frac{R_t}{1 + at} \tag{14}$$

Example.—The resistance of an electromagnet winding of copper wire at 20° C. is 30 ohms. What is its resistance at 80° C.?

The resistance at 0° C.

$$R_0 = \frac{30}{1 + 0.00427 \times 20} = \frac{30}{1.085} = 27.65 \text{ ohms}$$

$$R_{80} = 27.65\,(1 + 0.00427 \times 80) = 37.11 \text{ ohms.} \quad \textit{Ans.}$$

This process of working back to 0° is a little inconvenient, although it is fundamental and easy to remember. Table 48 gives the temperature coef-

ficients of copper at the various temperatures other than 0° C. With this table available the above problem involves but one computation.

Example.—The temperature coefficient of copper at 20° initial temperature from Table 4B is 0.00393. The rise in temperature = 80° − 20° = 60°. Then the resistance at 80° C.

$$R_{80} = 30(1 + 0.00393 \times 60) = 37.07 \text{ ohms.}$$

45. If the resistance of copper at ordinary temperatures be plotted against the temperature the result is a practically straight line. If this line be extended, it will intersect the zero resistance line at −234.5° C. (an easy number to remember),

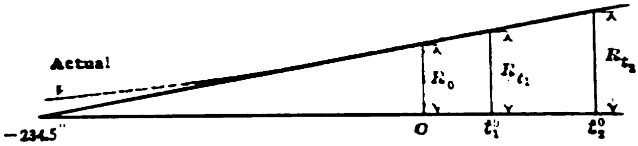


Fig. 50.—Variation of resistance with temperature.

as shown in Fig. 50. This is equivalent to saying that between ordinary limits of temperature copper behaves as if it had zero resistance at −234.5° C. (Actually the curve bends at these extremely low temperatures, as shown by the dotted line, Fig. 50.) This gives a convenient method for determining temperature-resistance relations.

By the law of similar triangles, Fig. 50,

$$\frac{R_0}{234.5^\circ} = \frac{R_{t_1}}{234.5^\circ + t_1} \qquad (15)$$

$$\frac{R_{t_1}}{234.5^\circ + t_1} = \frac{R_{t_2}}{234.5^\circ + t_2} \qquad (16)$$

Applying this equation to the previous example,

$$\frac{30}{234.5^\circ + 20^\circ} = \frac{R_{80}}{234.5^\circ + 80^\circ}$$

$$R_{80} = 30\,\frac{234.5^\circ + 80^\circ}{234.5^\circ + 20^\circ} = 30\,\frac{314.5}{254.5} = 37.1 \text{ ohms.} \quad \textit{Ans.}$$

46. Alloys.—Certain alloys, notably manganin and nickel-iron alloys, have practically zero temperature coefficients and are very useful as resistances for measuring instruments, where a change of resistance introduces error.

Table 47 gives the temperature coefficients at 20° C. for various materials. Table 48 gives the temperature coefficient of copper at various temperatures.

47. Temperature Coefficients of Resistance

	Per. degree C. at 20° C.
Aluminum	0.00388
Carbon (incandescent lamp) (−)	0.003
Graphite (−)	0.0006 to (−) 0.0012
German silver	0.00031 to 0.00020
Ia Ia	0.000005
Iron	0.00635
Manganin	0.000011 to 0.000039
Mercury	0.00072
Nichrome	0.00016 to 0.00044
Nickel	0.00622
Phosphor-bronze	0.0009
Platinum	0.00367
Silver	0.00377
Steel	0.00635

48. Temperature Coefficients of Copper at Different Initial Temperatures

(From formula $1/(234.5 + t)$)

Initial temperatures	Increase in resistance per 1° C.
0	0.00427
5	0.00418
10	0.00409
15	0.00401
20	0.00393
25	0.00385
30	0.00378
35	0.00371
40	0.00364
45	0.00358
50	0.00352

49. The American Wire Gage (A.W.G.).—The A.W.G. (formerly Brown & Sharpe Gage) is based upon a constant ratio of cross-section between wires of successive gage numbers. The following approximate relations make it a comparatively simple matter to determine the weight or resistance of any gage number without reference to the table: (1) No. 10 wire has a diameter of 0.1 in. and a resistance of 1 ohm per 1,000 ft. (2) The resistance of the wire doubles with every increase of 3 gage numbers. (3) Therefore the resistance increases $\sqrt[3]{2} = 1.26$ ($1\frac{1}{4}$) times for each successive gage number and $(1.26)^2 = 1.6$ times for every two numbers. (4) The resistance is multiplied or divided by 10 for every difference of 10 gage numbers. (5) The weight of 1,000 ft. of No. 2 wire is 200 lb.

Example.—What is the resistance and weight of 1,000 ft. of 0000 wire?

The resistances will decrease as follows:

Gage No.	10	7	4	1	000	
Resistance......	1	0.5	0.25	0.125	0.0625	(rules 1 and 2)

Resistance of 0000 = 0.0625/1.25 = 0.050 ohm (rule 3).
Weight of 1,000 ft. No. 2 = 200 lb.
Weight of 1,000 ft. 00 = 400 lb.
Weight of 1,000 ft. 0000 = 400 × 1.6 = 640 lb. (rule 5, 2 and 3).
The example might have been worked more quickly by rule 4.
Resistan e of 1,000 ft. of No. 10 = 1 ohm.
Resistance of 1,000 ft. of 0 = 0.1 ohm (rule 4)
Resistance of 1,000 ft. of 0000 = 0.050 ohm (rule 3).

50. Working Table, Standard Annealed Copper Wire, Solid[1]

American Wire Gage (B. & S.). English Units

Gage No.	Diameter in mils	Cross-section		Ohms per 1000 ft.		Ohms per mile	Pounds per 1,000 ft.
		Circular mils	Square inches	25° C. (=77° F.)	65° C. (=149° F.)	25° C. (=77° F.)	
0000	460.0	212,000.0	0.166	0.0500	0.0577	0.264	641.0
000	410.0	168,000.0	0.132	0.0630	0.0727	0.333	508.0
00	365.0	133,000.0	0.105	0.0795	0.0917	0.420	403.0
0	325.0	106,000.0	0.0829	0.100	0.116	0.528	319.0
1	289.0	83,700.0	0.0657	0.126	0.146	0.665	253.0
2	258.0	66,400.0	0.0521	0.159	0.184	0.839	201.0
3	229.0	52,600.0	0.0413	0.201	0.232	1.061	159.0
4	204.0	41,700.0	0.0328	0.253	0.292	1.335	126.0
5	182.0	33,100.0	0.0260	0.319	0.369	1.685	100.0
6	162.0	26,300.0	0.0206	0.403	0.465	2.13	79.5
7	144.0	20,800.0	0.0164	0.508	0.586	2.68	63.0
8	128.0	16,500.0	0.0130	0.641	0.739	3.38	50.0
9	114.0	13,100.0	0.0103	0.808	0.932	4.27	39.6
10	102.0	10,400.0	0.00815	1.02	1.18	5.38	31.4
11	91.0	8,230.0	0.00647	1.28	1.48	6.75	24.9
12	81.0	6,530.0	0.00513	1.62	1.87	8.55	19.8
13	72.0	5,180.0	0.00407	2.04	2.36	10.77	15.7
14	64.0	4,110.0	0.00323	2.58	2.97	13.62	12.4
15	57.0	3,260.0	0.00256	3.25	3.75	17.16	9.86
16	51.0	2,580.0	0.00203	4.09	4.73	21.6	7.82
17	45.0	2,050.0	0.00161	5.16	5.96	27.2	6.20
18	40.0	1,620.0	0.00128	6.51	7.51	34.4	4.92
19	36.0	1,290.0	0.00101	8.21	9.48	43.3	3.90
20	32.0	1,020.0	0.000802	10.4	11.9	54.9	3.09
21	28.5	810.0	0.000636	13.1	15.1	69.1	2.45
22	25.3	642.0	0.000505	16.5	19.0	87.1	1.94
23	22.6	509.0	0.000400	20.8	24.0	109.8	1.54
24	20.1	404.0	0.000317	26.2	30.2	138.3	1.22
25	17.9	320.0	0.000252	33.0	38.1	174.1	0.970
26	15.9	254.0	0.000200	41.6	48.0	220.0	0.769
27	14.2	202.0	0.000158	52.5	60.6	277.0	0.610
28	12.6	160.0	0.000126	66.2	76.4	350.0	0.484
29	11.3	127.0	0.0000995	83.4	96.3	440.0	0.384
30	10.0	101.0	0.0000789	105.0	121.0	554.0	0.304
31	8.9	79.7	0.0000626	133.0	153.0	702.0	0.241
32	8.0	63.2	0.0000496	167.0	193.0	882.0	0.191
33	7.1	50.1	0.0000394	211.0	243.0	1,114.0	0.152
34	6.3	39.8	0.0000312	266.0	307.0	1,404.0	0.120
35	5.6	31.5	0.0000248	335.0	387.0	1,769.0	0.0954
36	5.0	25.0	0.0000196	423.0	488.0	2,230.0	0.0757
37	4.5	19.8	0.0000156	533.0	616.0	2,810.0	0.0600
38	4.0	15.7	0.0000123	673.0	776.0	3,550.0	0.0476
39	3.5	12.5	0.0000098	848.0	979.0	4,480.0	0.0377
40	3.1	9.9	0.0000078	1,070.0	1,230.0	5,650.0	0.0299

NOTE 1.—The *fundamental resistivity* used in calculating the tables is the International Annealed Copper Standard viz., 0.15328 ohm (meter, gram) at 20° C. The *temperature coefficient* for this particular resistivity is α_{20} = 0.00393, or α_0 = 0.00427. The *density* is 8.89 grams per cubic centimeter.

NOTE 2.—The values given in the table are only for annealed copper of the standard resistivity. The user of the table must apply the proper correction for copper of any other resistivity. Hard-drawn copper may be taken as about 2.7 per cent. higher resistivity than annealed copper.

NOTE 3.—Pounds per mile may be obtained by multiplying the respective vaules above by 5.28.

[1] From *Circular of the Bureau of Standards*, No. 31.

51. Bare Concentric Lay Cables of Standard Annealed Copper

English Units

A. W. G. No.	Circular mils	Ohms per 1,000 ft. 25° C. (= 77° F.)	Ohms per 1,000 ft. 65° C. (= 149° F.)	Pounds per 1,000 ft.	Standard concentric stranding: Number of wires	Standard concentric stranding: Diameter of wires, in mils	Standard concentric stranding: Outside diameter, in mils
	2,000,000	0.00539	0.00622	6,180	127	125.5	1,631
	1,700,000	0.00634	0.00732	5,250	127	115.7	1,504
	1,500,000	0.00719	0.00830	4,630	91	128.4	1,412
	1,200,000	0.00899	0.0104	3,710	91	114.8	1,263
	1,000,000	0.0108	0.0124	3,090	61	128.0	1,152
	900,000	0.0120	0.0138	2,780	61	121.5	1,093
	850,000	0.0127	0.0146	2,620	61	118.0	1,062
	750,000	0.0144	0.0166	2,320	61	110.9	998
	650,000	0.0166	0.0192	2,010	61	103.2	929
	600,000	0.0180	0.0207	1,850	61	99.2	893
	550,000	0.0196	0.0226	1,700	61	95.0	855
	500,000	0.0216	0.0249	1,540	37	116.2	814
	450,000	0.0240	0.0277	1,390	37	110.3	772
	400,000	0.0270	0.0311	1,240	37	104.0	728
	350,000	0.0308	0.0356	1,080	37	97.3	681
	300,000	0.0360	0.0415	926	37	90.0	630
	250,000	0.0431	0.0498	772	37	82.2	575
0000	212,000	0.0509	0.0587	653	19	105.5	528
000	168,000	0.0642	0.0741	518	19	94.0	470
00	133,000	0.0811	0.0936	411	19	83.7	418
0	106,000	0.102	0.117	326	19	74.5	373
1	83,700	0.129	0.149	258	19	66.4	332
2	66,400	0.162	0.187	205	7	97.4	292
3	52,600	0.205	0.237	163	7	86.7	260
4	41,700	0.259	0.299	129	7	77.2	232

52. Conductors.—Although silver is a better conductor than copper, its use as a conductor is very limited because of its cost. In a few instances it is used where a delicate but highly conducting material is necessary, such as in the brushes and occasionally in the commutator of watt-hour meters. Copper, because of its high conductivity and moderate cost, is used more extensively

as a conductor than any other material. It has many good qualities such as ductility, high tensile strength, not easily abraided, not corroded by the atmosphere, and it can be readily soldered.

Aluminum has only 61 per cent. of the conductivity of copper, but for the same length and weight, it has about twice the conductance of copper. It is softer than copper, its tensile strength is much less, and it cannot be readily soldered. It is not affected by exposure to the atmosphere. The large diameter for a given conductance prohibits its use where an insulating covering is required. Aluminum is used extensively as a conductor for high voltage transmission lines, where its lightness and large diameter are an advantage. It is used to some extent for low voltage bus-bars as it offers much greater radiating surface than copper of the same conductance. The price of aluminum is held about 10 per cent. less than that of copper of the same conductance.

Iron and steel have about 9 times the resistance of copper for the same cross-section and length. The large cross-section for a given conductance prohibits their use where an insulating covering is necessary and the increased weight prevents their use in most cases where the conductors must be placed on poles. In view of their low cost per pound, they are cheaper than copper as simple conductors. They are most commonly used as resistors in connection with rheostats and for third rails of electric railways. Iron and steel ordinarily must be protected from oxidation by galvanizing or other protective covering. Copper-clad steel consists of a steel wire coated or covered with a layer of copper, fused or welded to the steel. The advantages claimed for it are that it possesses the high tensile strength of steel, combined with the high conductivity of copper. Further, the copper protects the steel from corrosion. Its field is the transmission line conductor, where long spans make high tensile strength necessary. It is also used as an overhead ground wire on such lines.

CHAPTER IV

OHM'S LAW AND THE ELECTRIC CIRCUIT

The exact nature of electricity is not known, but recent investigations indicate that it consists of infinitesimal charges called electrons. When these electrons are forced to travel in the same direction an electric current results. The flow of electricity through a circuit resembles in many ways the flow of water through pipes, for it acts as an incompressible fluid would act, undergoes pressure drop, etc., as will be shown later.

53. Electromagnetic Units. *Current.*—The unit of electric current is the *ampere* and represents the *rate of flow* of electricity. It corresponds in hydraulics to the rate of flow of water, expressed as cubic feet per second, gallons per minute, etc.

The ampere is defined by an act of Congress, 1894, as follows: "The unit of current shall be what is known as the international ampere, which is one-tenth of the unit of current of the centimeter-gram-second system of electromagnetic units and is the practical equivalent of the unvarying current, which when passed through a solution of nitrate of silver in water in accordance with standard specifications, deposits silver at the rate of one thousand one hundred and eighteen millionths (0.001118) of a gram per second."

Quantity.—The unit of quantity is the *coulomb.* This is equal to the quantity of electricity conveyed by one ampere in one second. The coulomb is analagous to the unit quantity of water in hydraulics, such as the cubic foot, the gallon, etc.

From this definition it is evident that an electric current may be expressed in *coulombs per second* rather than in *amperes.*

Difference of Potential and Electromotive Force (emf.).—Difference of potential and electromotive force tend to cause a flow of electricity. The unit of potential difference or of emf. is the *volt,* and is defined as that potential difference which when impressed across the terminals of a resistance of one ohm will cause a current of one ampere to flow. The *international volt* is now more specifically defined as $1/1.01830$ of the voltage of a normal Weston cell. (See Par. 89.)

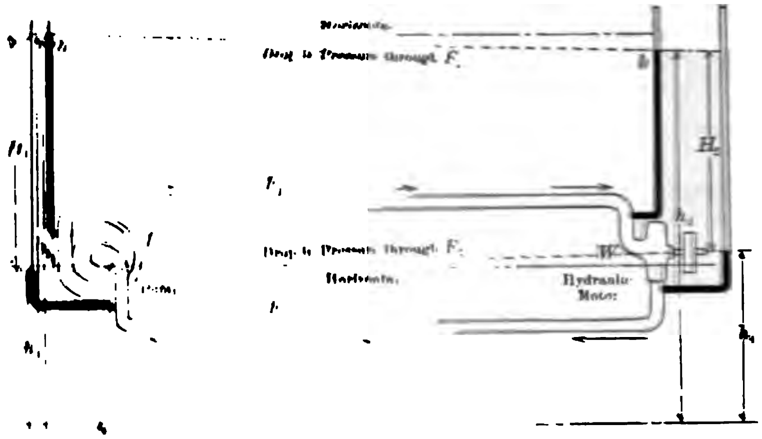
The mechanical analogy of potential is pressure. The difference in hydraulic pressure between the ends of a pipe causes or tends to cause the flow of water. The pressure of water behind the dam tends to cause water to flow through the penstock or through any leaks. The pressure in a boiler tends to cause steam to flow through the pipes, valves, etc. Likewise difference of potential tends to cause current to flow.

Resistance.—The ohm, the unit of resistance, has already been defined in Chap. III as that resistance which will allow one ampere to flow if one volt is impressed across its terminals. The *international ohm* is specifically defined as the resistance of a column of mercury at the temperature of melting ice (0° C.), 14.4521 grams in mass, of a constant cross-sectional area and of a length of 106.300 cm.

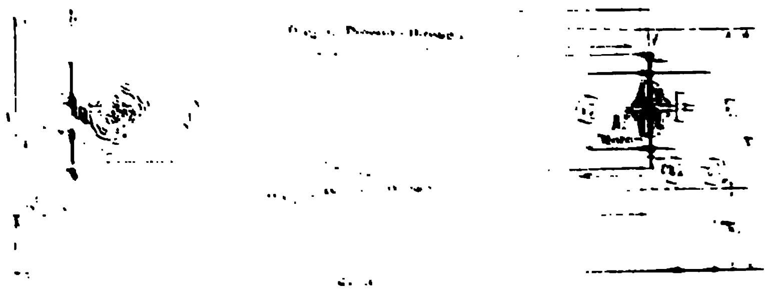
54. Nature of the Flow of Electricity.—The flow of electricity through a circuit resembles in many ways the flow of water through a closed system of pipes. For example, in Fig. 51 water enters the mechanically driven centrifugal pump P at a pressure h_1 (represented by the length of a column of mercury) above the point of zero pressure shown by the line h_0. In virtue of the action of the pump blades, its pressure through the pump is increased from h_1 to h_2, representing a net increase of pressure H_1. The water then flows out along pipe F_1 to the hydraulic motor W. Because of the friction loss in the pipe F_1, the pressure at the motor terminals h_3 is slightly less than h_2. In other words, a pressure of $h_2 - h_3$ is required to overcome the frictional resistance of the pipe F_1. The line ab shows the pressure drop along the pipe, this pressure drop being uniform.

In Fig. 52 the mechanically driven electrical generator G raises the potential of the current entering its negative terminal, from v_1 to v_2 where v_1 and v_2 are measured from the earth whose potential is ordinarily assumed as zero. (The various voltages are measured with voltmeters v'_1, v'_2, etc.) The generator, in raising the potential of this portion of the circuit from v_1 to v_2, produces a net increase in pressure $v_2 - v_1 = V_1$. The current now flows out through the line L_1 to the + terminal of the motor. Because of the line resistance, the potential drops from v_2 at the generator to v_3 at the motor in practically the same manner that the water pressure dropped in pipe F_1 (Fig. 51). A voltage

[illegible] is necessary to force the current through the line L_1. The line $a'b'$ shows the actual voltage at each point along the wire, the distance of $a'b'$ from the ground line being proportional to the voltage at each point. This voltage drop is uniform.



[illegible] water through a hydraulic motor and pipe system.



It is to be noted that H_2, the net pressure at the motor terminals, is less than the pressure H_1 at the pump, by the sum of the pressures necessary to overcome the friction in the two pipes F_1 and F_2.

In a similar manner, the pressure of the electric current in passing through the motor M, drops from v_3 to v_4, representing a net drop in pressure V_2. A large percentage of this voltage V_2 is necessary to overcome the back emf. of the motor. v_4 is necessarily greater than v_1, or the current could not flow along L_2 back to the negative terminal of the generator. It is to be noted that, as in the case of Fig. 51, the net potential difference V_2 at the motor M is less than that at the generator V_1 by the amount lost in the drop in potential due to the resistance of both the outgoing and the return wire.

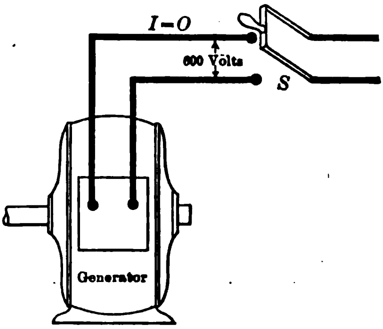


FIG. 53.—Illustrating the existence of potential difference without current.

Difference of potential is therefore the equivalent of pressure and *tends* to send current through a circuit; current is quantity of electricity per second. Potential difference may exist with no current flow, in the same manner that a boiler may have a very high steam pressure with no steam flow, due to all the valves being closed. Likewise a generator, Fig. 53, may have a very high potential difference at its terminals, yet because the switch S is open, no current flows.

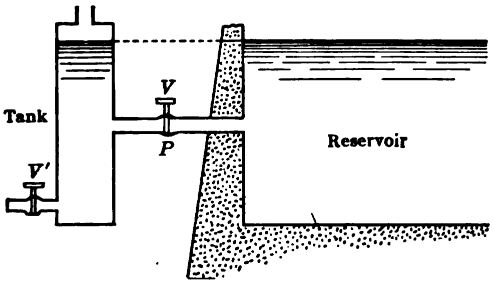


FIG. 54.—Tank and reservoir at the same pressure.

55. Difference of Potential.—In order that current may flow between two points, there must be a *difference of potential* between the two points as shown in Fig. 52. This is further illustrated in Fig. 54. A large reservoir and a small tank are connected by a

pipe P. The water level in the tank and in the reservoir is the same, that is. there is pressure in each but there is no difference in pressure between them. When the valve V is opened, no water flows from the reservoir to the tank. However, if the valve V' is opened, allowing the water level in the tank to fall, a difference of pressure between the two tanks will result and water will now flow from the reservoir to the tank.

Fig. 55 shows two batteries A_1 and A_2 each having an emf. of 2 volts. The positive terminal of A_1 has a potential of $+2$ volts above its negative terminal; likewise the $+$ terminal of A_2 has a potential of $+2$ volts above its negative terminal. The negative terminals of both batteries are at the same potential

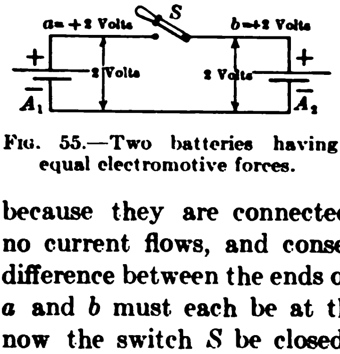


Fig. 55.—Two batteries having equal electromotive forces.

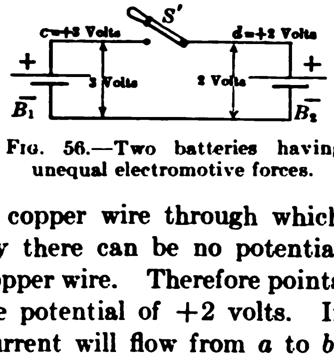


Fig. 56.—Two batteries having unequal electromotive forces.

because they are connected by a copper wire through which no current flows, and consequently there can be no potential difference between the ends of the copper wire. Therefore points a and b must each be at the same potential of $+2$ volts. If now the switch S be closed, *no* current will flow from a to b, because there is no *difference* of *potential* between a and b.

In Fig. 56 the emf. of battery B_1 is 3 volts and therefore the potential of its positive terminal is 3 volts above that of its negative terminal. The emf. of battery B_2 is 2 volts and therefore the potential of its positive terminal is 2 volts above that of its negative terminal. The negative terminals are at the same potential. If this potential be assumed as zero, the point c is at a potential of $+3$ volts whereas the potential of d is $+2$ volts. Therefore the point c is at a potential of $3 - 2$ or 1 volt higher than d. When switch S' is closed, a current will flow from c to d, in virtue of c being at a higher potential than d.

56. Measurement of Voltage and Current.—Voltage or potential difference is ordinarily measured with a voltmeter. It is only occasionally that *absolute potential* is of interest. Ordinarily *difference of potential* is the quantity desired. The voltmeter

therefore should be connected *across* or *between* the wires whose difference of potential is to be measured, as shown in Fig. 57.

Current is ordinarily measured with an ammeter. As current is the *quantity* of *electricity* per second passing in the wire, the ammeter must be connected so that only the current to be measured passes through it. This is accomplished by opening one of the wires of the circuit and inserting the ammeter, just as a water meter is inserted in a pipe when it is desired to measure the flow of water in the pipe. When the ammeter is so connected, the current passing through to the load is measured by the ammeter. (See Fig. 57.) *Never connect an ammeter across the line.*

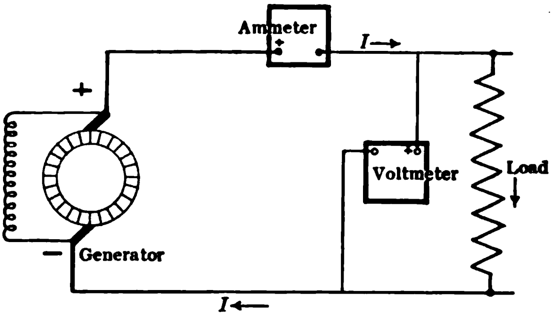


FIG. 57.—Proper method of connecting a voltmeter and an ammeter.

57. Ohm's Law.—Ohm's Law states that for a steady current the current in a circuit is directly proportional to the electromotive force acting on the circuit and is inversely proportional to the resistance of the circuit.

The law may be expressed by the following equation if the current I is in *amperes*, the emf. E is in *volts*, and the resistance R is in *ohms*.

$$I = \frac{E}{R} \tag{17}$$

That is, the current in amperes in a circuit is equal to the emf. of the circuit in volts divided by the resistance of the circuit in ohms. Potential difference may be represented by either the letter "V" or "E," V usually meaning terminal voltage and E electromotive force or induced voltage.

Example.—The resistance of the field winding of a shunt motor is 30 ohms. What current will flow through the winding when it is connected across 115-volt mains?

$$I = \frac{E}{R} = \frac{115}{30} = 3.83 \text{ amp.} \quad \textit{Ans.}$$

By transformation equation (17) becomes

$$E = IR \tag{18}$$

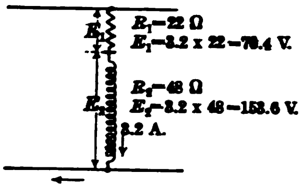


FIG. 58.—Voltage drops across a generator field and its rheostat.

That is, the voltage across any part of a circuit is equal to the product of the current in amperes and the resistance in ohms, when the current is steady.

Example.—The resistance of the field winding of a shunt generator is 48 ohms and the resistance of its rheostat is 22 ohms. (See Fig. 58.) If the field current is 3.2 amp., what is the voltage across the field winding terminals, the voltage across the rheostat, and the voltage across the generator terminals?

$E_1 = IR_1 = 3.2 \times 48 = 153.6$ volts across field terminals

$E_2 = IR_2 = 3.2 \times 22 = 70.4$ volts across rheostat

Total 224.0 volts at terminals.

Also

$E = I(R_1 + R_2) = 3.2\ (22 + 48) = 224.0$ volts (check). *Ans.*

Again, if equation (17) be solved for the resistance the result is

$$R = \frac{E}{I} \tag{19}$$

That is, the resistance of a circuit is equal to the voltage divided by the current, when the current is steady. This formula is very useful in making resistance measurements. (See Par. 118.)

Example.—The voltage across the terminals of a generator field is 220 volts and the field current is 4 amperes. What is the resistance of the field circuit?

$$R = \frac{E}{I} = \frac{220}{4} = 55 \text{ ohms.} \quad \textit{Ans.}$$

58. The Series Circuit.—As was stated in Par. 39, if several resistances are connected in series, the total resistance is the sum of the individual resistances. That is,

$$R = r_1 + r_2 + r_3, \text{ etc.} \tag{20}$$

and the current

$$I = \frac{E}{R} = \frac{E}{r_1 + r_2 + r_3} \tag{21}$$

Example.—A 50-ohm relay is connected in series with a resistance tube of 30 ohms and with a small pilot lamp having a resistance of 5 ohms. The operating voltage is 115 volts. What current flows in this relay circuit?

$$I = \frac{115}{50 + 30 + 5} = \frac{115}{85} = 1.35 \text{ amp.} \quad \textit{Ans.}$$

59. The Parallel Circuit.—In Par. 39, the relation of total resistance to the component resistances in a parallel circuit was proved by transforming conductances into resistances. This equation may be proved by Ohm's Law as follows: Consider the circuit of Fig. 59, consisting of resistances r_1, r_2, and r_3 in parallel across the voltage E. Let I_1 = the current in resistance r_1, I_2 = current in r_2, and I_3 = the current in r_3.

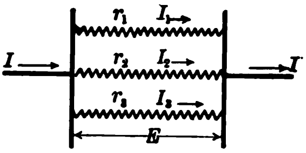


Fig. 59.—A parallel circuit.

Then

$$\left.\begin{aligned} I_1 &= \frac{E}{r_1} \\ I_2 &= \frac{E}{r_2} \\ I_3 &= \frac{E}{r_3} \end{aligned}\right\} \quad \text{(equation 17)}$$

Adding these together:

$$I_1 + I_2 + I_3 = \frac{E}{r_1} + \frac{E}{r_2} + \frac{E}{r_3} = E\left(\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}\right)$$

Let the total current be $I = I_1 + I_2 + I_3$.
Let the equivalent resistance be R

$$I = \frac{E}{R}$$

Substituting I for $I_1 + I_2 + I_3$

$$I = \frac{E}{R} = E\left(\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}\right)$$

or

$$\frac{1}{R} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} \tag{22}$$

That is, *the reciprocal of the equivalent resistance of a parallel circuit is the sum of the reciprocals of the individual resistances.*

If but two resistances are involved,

$$R = \frac{r_1 r_2}{r_1 + r_2} \tag{23}$$

If three resistances are involved,

$$R = \frac{r_1 r_2 r_3}{r_1 r_2 + r_2 r_3 + r_3 r_1} \tag{24}$$

Example.—Determine the total current in a circuit consisting of 4 resistances of 4, 6, 8, and 10 ohms respectively, connected in parallel across a 10-volt battery.

$$\frac{1}{R} = \frac{1}{4} + \frac{1}{6} + \frac{1}{8} + \frac{1}{10} = 0.25 + 0.167 + 0.125 + 0.10$$

$$= 0.642 \text{ mho}$$

$$R = \frac{1}{0.642} = 1.56 \text{ ohms}$$

$$I = \frac{10}{1.56} = 6.42 \text{ amp.} \quad \textit{Ans.}$$

FIG. 60.—Division of current in a two-branch parallel circuit.

60. Division of Current in a Parallel Circuit.—In Fig. 60, two resistances, R_1 and R_2, are connected in parallel across the voltage E. Then:

$$I_1 = \frac{E}{R_1}$$

$$I_2 = \frac{E}{R_2}$$

or

$$\frac{I_1}{I_2} = \frac{E/R_1}{E/R_2} = \frac{R_2}{R_1} \tag{25}$$

That is, in a parallel circuit of two branches, the currents are inversely as the resistances. (This does not apply to the division of current through the field and armature of a shunt motor when the motor is running.)

Example.—A current of 12 amp. divides between two paths in parallel, part passing through a branch having a resistance of 8 ohms, the other branch having a resistance of 12 ohms. How much current passes through each branch?

Let I_1 be the current in the 8-ohm branch and I_2 be the current in the 12-ohm branch.

$$\frac{I_1}{I_2} = \frac{12}{8} \quad \text{(eq. 25)} \tag{1}$$

Also

$$I_1 + I_2 = 12 \tag{2}$$

$$I_1 = I_2 \frac{12}{8} = I_2 \frac{3}{2} \quad \text{from} \quad (1)$$

substituting in (2)

$$I_2 \frac{3}{2} + I_2 = 12$$

$$\frac{5I_2}{2} = 12 \qquad I_2 = 4.8 \text{ amp.} \quad \textit{Ans.}$$

$$I_1 = 4.8 \frac{3}{2} = 7.2 \text{ amp.} \qquad \textit{Ans.}$$

If the circuit consists of three branches (Fig. 61)

R_1, R_2, and R_3,

the respective currents may be found as follows:

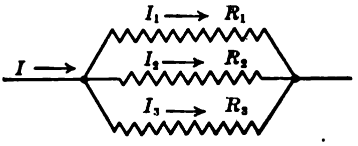


FIG. 61.—Division of current in a three-branch parallel circuit.

Let I be the total current $= I_1 + I_2 + I_3$.

It can be shown that the current in each branch is given by:

$$I_1 = I\left(\frac{R_2R_3}{R_1R_2 + R_2R_3 + R_3R_1}\right) \tag{26}$$

$$I_2 = I\left(\frac{R_3R_1}{R_1R_2 + R_2R_3 + R_3R_1}\right) \tag{27}$$

$$I_3 = I\left(\frac{R_1R_2}{R_1R_2 + R_2R_3 + R_3R_1}\right) \tag{28}$$

(Note the cyclic order of the subscripts.)

Example.—A current of 25 amperes passes through a parallel circuit of three resistances of 2.5, 4 and 6 ohms, respectively. How does the current divide?

Current in 2.5 ohms.

$$I_1 = 25\,\frac{4.0 \times 6.0}{(2.5 \times 4.0) + (4.0 \times 6.0) + (6.0 \times 2.5)}$$

$$= 25\,\frac{24}{10 + 24 + 15} = 12.25 \text{ amp.}$$

$$I_2 = 25\,\frac{6.0 \times 2.5}{49} = 7.65 \text{ amp.}$$

$$I_3 = 25\,\frac{2.5 \times 4.0}{49} = 5.10 \text{ amp.}$$

Total 25.00 amp. (check).

61. The Series-parallel Circuit.—A circuit may consist of groups of parallel resistances in series with other resistances or groups of resistances as shown in Fig. 62. In this case, each group of parallel resistances is first combined into its equivalent single resistance by equation (22) and the whole is then treated as a series circuit.

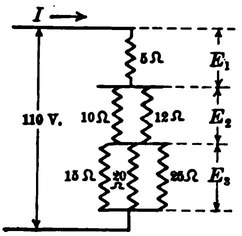


FIG. 62.—Series-parallel circuit.

Example.—Determine the total current in the circuit shown in Fig. 62; determine the voltage across each portion of the circuit; determine the current in each resistance.

Combine first the 10 and 12 ohm resistances into a resistance R_1

$$\frac{1}{R_1} = \frac{1}{10} + \frac{1}{12} = 0.10 + 0.0833 = 0.1833 \text{ mho}$$

$$R_1 = 5.45 \text{ ohms}$$

Likewise combining the group of three resistances into R_2

$$\frac{1}{R_2} = \frac{1}{15} + \frac{1}{20} + \frac{1}{25} = 0.0667 + 0.050 + 0.040$$
$$= 0.1567 \text{ mho}$$

$$R_2 = \frac{1}{0.1567} = 6.39 \text{ ohms}$$

$$I = \frac{110}{5 + 5.45 + 6.39} = \frac{110}{16.84} = 6.54 \text{ amp.}$$

$$E_1 = 6.54 \times 5.0 = 32.7 \text{ volts}$$
$$E_2 = 6.54 \times 5.45 = 35.6 \text{ volts}$$
$$E_3 = 6.54 \times 6.39 = 41.7 \text{ volts}$$
Total 110.0 volts (check).

Current in 10 Ω $= \frac{35.6}{10} = 3.56$ amp.

Current in 12 Ω $= \frac{35.6}{12} = 2.97$ amp.

Total 6.54 amp. (check).

Current in 15 Ω $= \frac{41.7}{15} = 2.78$ amp.

Current in 20 Ω $= \frac{41.7}{20} = 2.09$ amp.

Current in 25 Ω $= \frac{41.7}{25} = 1.67$ amp.

Total 6.54 (check).

62. Electrical Power.—The unit of electrical power is the *watt* and is defined as the power developed by one ampere in falling through a potential difference of one volt. Watts are

therefore equal to the product of the volts and the amperes. Thus the power

$$P = EI \text{ watts.} \tag{29}$$

Since in a circuit containing resistance only $E = IR$ (equation 18), equation (29) may be written

$$P = (IR)\,I = I^2R \tag{30}$$

Substituting for I its value $\left(I = \frac{E}{R}\right)$ in equation (29)

$$P = \frac{E^2}{R} \tag{31}$$

Equation (29) is useful when the volts and the amperes are known; equation (30) is useful when the current and the resistance are known; and equation (31) is useful when the voltage and the resistance are known.

Example.—The resistance of a 150-scale voltmeter is 12,000 ohms. What power is consumed by this voltmeter when it is connected across a 125-volt circuit?

Since the voltage and the resistance are known, equation (31) is most convenient:

$$P = \frac{(125)^2}{12{,}000} = 1.3 \text{ watts. } \textit{Ans.}$$

This may be checked by equation (29)

$$I = \frac{125}{12{,}000} = 0.0104 \text{ amp.}$$

$$P = 125 \times 0.0104 = 1.3 \text{ watts (check).}$$

The watt is often too small a unit for commercial use and the *kilowatt* (equal to 1,000 watts) is used when large amounts of power are being considered. It is often necessary to transform from mechanical horsepower to electrical power and conversely, and a knowledge of the relation of the two is therefore useful:

$$746 \text{ watts} = 1 \text{ horsepower} \tag{32}$$

$$0.746 \text{ kw.} = 1 \text{ horsepower} \tag{33}$$

and

$$1 \text{ hp.} = 3/4 \text{ kw. very nearly} \tag{34}$$

$$1 \text{ kw.} = 4/3 \text{ hp. very nearly} \tag{35}$$

Example.—An electric motor takes 28 amperes at 550 volts and has an efficiency of 89 per cent. What horsepower does it deliver?

$$\text{Input} = 28 \times 550 = 15{,}400 \text{ watts}$$

$$\text{Output} = 15{,}400 \times 0.89 = 13{,}700 \text{ watts}$$

$$13{,}700/746 = 18.35 \text{ hp. at the pulley. } \textit{Ans.}$$

63. Electrical Energy.—Power is the *rate of doing work*, or is the *rate of expenditure of energy*. Therefore electrical energy is equal to the product of electrical power and time. The unit of electrical energy is the *watt-second* or *joule*.

$$W = E\,I\,t \text{ watt-seconds}$$

if t is in seconds, E is in volts, and I is in amperes.

The watt-second is ordinarily too small a unit for commercial purposes, so the larger unit, the *kilowatt-hour* (kwh.) is commonly used. 1 kilowatt-hour = 1,000 × 60 × 60 = 3,600,000 joules or watt-seconds.

The difference between power and energy (or work) should be clearly borne in mind. Power is *rate* of doing work, just as velocity is rate of motion. On the other hand, energy is the total work done and is equal to the power multiplied by the time during which it acts just as distance covered is the velocity or rate of motion multiplied by the time. To speak of a train traveling at a rate of 40 miles per hour gives no information as to the total distance which the train travels. Likewise, to speak of 50 kilowatts does not state the amount of energy that is involved. The statement "electricity is sold for so many cents per kilowatt" is incorrect. The correct expression is "electrical *energy* is sold for so many cents per kilowatt-*hour*." To illustrate:

Example.—If energy is sold for 10c per kilowatt-hour (kwh.), how many kilowatts may be purchased for 20c? This question as it stands cannot be answered, since the *time* is not given. If, however, it is assumed that the power is to be used for 1 hour:

20c/10c = 2 kw.-hr. available
2 kw.-hr./1 hr. = 2 kw. *Ans.*
If used in 0.5 hr., 2kw.-hr. /0.5 hr. = 4 kw. *Ans.*
If used in 0.001 hr., 2 kw.-hr./0.001 hr. = 2,000 kw. *Ans.*

so that the 20c could purchase *any number* of *kilowatts*, depending on the time during which the power is supplied.

In a similar way, horsepower is *rate of doing work* and is equivalent to 33,000 ft.-lb. *per minute* and *not* to 33,000 ft.-lb. A motor developing $\frac{1}{8}$hp. could do 33,000 ft.-lb. of work if allowed 8 minutes in which to do it. When speaking of *work* in connection with horsepower, the *horsepower-hour* is the unit ordinarily used.

Example.—How many watt-seconds are supplied by a motor developing 2 hp. for 5 hours?

$$2 \times 5 = 10 \text{ hp.-hr.}$$
$$10 \text{ hp.-hr.} \times 746 = 7{,}460 \text{ watt-hours}$$
$$7{,}460 \times 3{,}600 = 2.68 \times 10^6 \text{ watt-seconds.} \quad \textit{Ans.}$$

64. Heat and Energy.—It is well known that heat may be converted into mechanical and electrical energy, and, conversely, that electrical and mechanical energy may be converted into heat. The complete cycle of energy transformation is well illustrated by a steam power plant. The energy is brought to the plant in the coal, as *chemical energy*. The ingredients of the coal combine with the oxygen of the air, thus converting the chemical energy into *heat energy*. A certain percentage of this heat is transferred to the boiler and produces steam. The expansion of the steam in the engine cylinders, or through the buckets and blades of the turbine, converts the heat energy of the steam into *mechanical energy*. This mechanical energy drives the generator, which converts a large percentage of this energy into *electrical energy*. A portion of this electrical energy is transformed into heat in the wires, bus-bars, etc. Finally, the remainder is used to supply lamps, propel electric cars, operate motors, and some may be used for chemical processes. Ultimately all the energy appears again as heat or else is converted into chemical or other forms of energy.

The following table shows approximately what becomes of each 100 heat units existing initially in the coal in the most efficient modern power plants, using superheaters, condensers, and large units.

Efficiency of Energy Conversion

	Form of energy	Efficiency (per cent.)	Heat units converted
Coal	Chemical	..	100.0
Boiler	Heat	80	80.0
Turbine	Mechanical	25	20.0
Generator	Electrical	95	19.0
Distribution system (to point of utilization)	Electrical	85	16.2
Small motors	Mechanical	65 (av.)	10.5
Lamps	Light	2	0.32

Fig. 63[1] (by R. A. Philip) shows graphically the flow of power from the boiler to the point of utilization. It is apparent that even in the most modern power plants the over-all efficiency is very low.

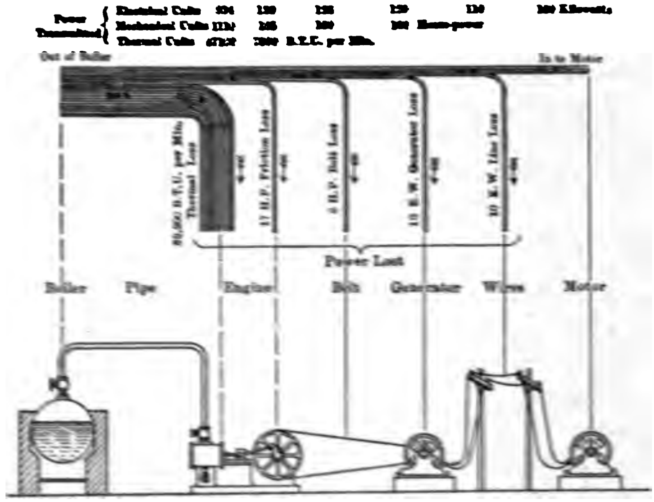


FIG. 63.—Energy flow—thermal, mechanical and electrical transmission.

65. Thermal Units.—The unit of heat in the English system is the B.t.u. (British thermal unit) and is equal to the amount of heat required to raise one pound of water 1° F. It is equal to 778 ft.-lb. (called the Mechanical Equivalent of Heat).

In the C.G.S. system the heat unit is the gram-calorie and is equal to the amount of heat required to raise one gram of water 1° C.[2] A gram-calorie is equal to 4.2 watt-seconds or joules.

By Joule's Law the heat developed in a circuit is:

$$W = \frac{1}{4.2} I^2Rt = 0.24\, I^2Rt \text{ calories} \tag{36}$$

where t is in seconds, I in amperes and R in ohms.

Example.—10 horsepower is delivered by a pump circulating 400 gallons of water per minute through a certain cooling system. How many degrees F. is the temperature of the water raised by the action of the pump?

$$10 \text{ hp.} = 10 \times 33,000 = 330,000 \text{ ft.-lb. per min.}$$
$$330,000 \div 778 = 424 \text{ B.t.u. per min.}$$
$$400 \text{ gal.} = 400 \times 8.34 = 3,336 \text{ lb.}$$
$$424 \div 3,336 = 0.127° \text{ F. Ans.}$$

[1] A. I. E. E. Trans. Vol. XXXIV (1915), page 779.
[2] See Appendix A, page 407.

Example.—An incandescent lamp taking 0.5 amp. from 110-volt mains is immersed in a small tank of water, containing 2,000 c.c. of water. Neglecting radiation, by how many degrees per minute is the temperature of the water raised?

$$W = 0.24 \times 0.5 \times 110 \times 60 = 792 \text{ calories per minute.}$$
$$792/2{,}000 = 0.396° \text{ C. } \textit{Ans.}$$

66. Potential Drop in a Feeder Supplying One Concentrated Load.—Fig. 64 shows a feeder (consisting of a positive and a negative wire) supplying a motor load. The feeder is connected to bus-bars having a constant potential of 230 volts. The feeder is 1,000 ft. long and consists of two 250,000 C.M. conductors. The maximum load on the feeder is 250 amperes. It is required to determine the voltage at the motor terminals and the efficiency of transmission.

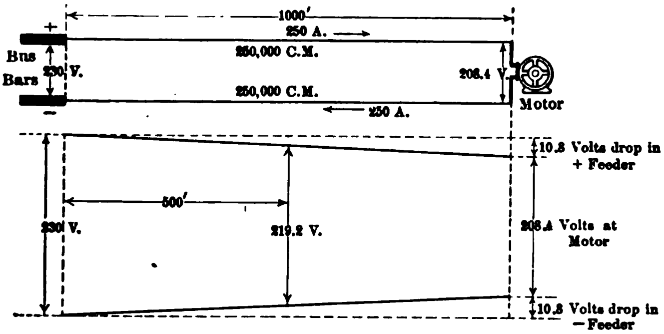


FIG. 64.—Voltage drop in a feeder due to a single load.

As was stated in Par. 54, the voltage at the motor must be less than that at the bus-bars because of the voltage lost in supplying the resistance drop in the feeder.

From Table 51, the resistance of 1,000 ft. of 250,000 C.M. cable is 0.0431 ohm. As was shown in Par. 54, the net voltage at the receiving end of the line is less than the voltage at the sending end by the voltage loss in both the *outgoing* and the *return* wire. Therefore the drop in 2,000 ft. of cable must be taken, the total resistance being 0.0862 ohm.

The current is 250 amperes.

By equation (18) the voltage drop in the line:

$$E' = 250 \times 0.0862 = 21.55 \text{ volts.}$$

Therefore the voltage at the motor terminals is

$$230 - 21.6 = 208.4 \text{ volts.} \quad \textit{Ans.}$$

In Fig. 64 the voltage drop along the line is shown graphically. The voltage at the sending end of the line is 230 volts, and there is a uniform drop in each wire, this drop increasing uniformly to 10.8 volts, making a total voltage loss of 21.6 volts. The potential difference between the two wires 500 ft. from the sending end will be 230 − 10.8 = 219.2 volts as shown.

The power delivered to the motor = 208.4 × 250 watts.
The power delivered to the line = 230 × 250 watts.

$$\text{The efficiency of the line} = \frac{\text{output}}{\text{input}} = \frac{208.4 \times 250}{230 \times 250} = \frac{208.4}{230}$$
$$= 90.8 \text{ per cent.}$$

With one concentrated load the efficiency of transmission is given by the voltage at the load divided by the voltage at the sending end of the line.

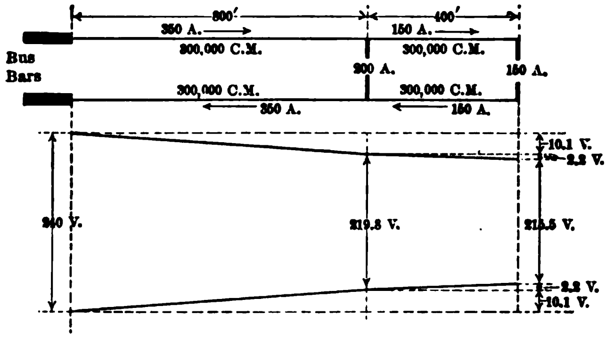


FIG. 65.—Voltage drops in a feeder supplying two loads.

67. Potential Drop in a Feeder Supplying Two Concentrated Loads at Different Points.—In Fig. 65 a 300,000 C.M. feeder supplies 200 amp. to a load 800 ft. from the bus-bars, and 150 amp. to a load 400 ft. farther on. If the bus-bar voltage is maintained constant at 240 volts, determine the voltage at each load, the total line loss and the efficiency of transmission.

From Table 51, the resistance of 1,000 ft. of 300,000 C.M. cable is 0.0360 ohm. The resistance of 800 ft. = 800/1,000 × 0.0360 = 0.0288 ohm.

Voltage drop to the 200-amp. load

$$E' = 350\ (2 \times 0.0288) = 20.16 \text{ volts.}$$

Voltage at 200-amp. load

$$E_1 = 240 - 20.2 = 219.8 \text{ volts.} \quad \textit{Ans.}$$

Resistance of one cable from the 200-amp. load to the 150-amp. load $= 400/1{,}000 \times 0.0360 = 0.0144$ ohm.

Voltage drop from 200-amp. load to 150-amp. load

$$E'' = 150\ (2 \times 0.0144) = 4.32 \text{ volts.}$$

Voltage at 150-amp. load

$$E_2 = 219.8 - 4.3 = 215.5 \text{ volts.} \quad \textit{Ans.}$$

The voltage distribution along this line is shown graphically in Fig. 65.

To determine the efficiency:

Line loss to 200-amp. load

$$P_1 = (350)^2\ (2 \times 0.0288) = 7{,}060 \text{ watts (equation 30).}$$

Line loss from 200-amp. load to 150-amp. load

$$P_2 = (150)^2\ (2 \times 0.0144) = 649 \text{ watts (equation 30).}$$

Total line loss

$$P_1 + P_2 = 7{,}060 + 649 = 7{,}709 \text{ watts or } 7.709 \text{ kw.}$$

Efficiency =

$$\frac{\text{input} - \text{losses}}{\text{input}} = \frac{(240 \times 350) - 7{,}709}{240 \times 350} = \frac{76{,}290}{84{,}000} = 90.8 \text{ per cent.}$$

68. Estimation of Feeders.—It was stated in Par. 42 that a cir.-mil-foot of copper has a resistance of 10.37 ohms. In many cases it is sufficiently exact to assume a value of 10 ohms. Assume the current density in a feeder to be 1,000 cir. mils per ampere, or 0.001 amp. per cir. mil. Call this the *normal* current density. (Bus-bars and large feeders operate at a density very nearly equal to this.)

The voltage drop through a cir.-mil-foot carrying 0.001 amp. is:

$$E = IR = 0.001 \times 10 = 0.01 \text{ volt.}$$

Another cir.-mil-foot, carrying 0.001 amp., will also have a drop of 0.01 volt between its ends. If these be placed side by side, the drop across the two will still be 0.01 volt. With any number of wires, each having 1 cir. mil cross-section, a length of 1 ft.

and a current of 0.001 amp., the drop across the ends of e wire will be 0.01 volt. The wires may be separated or t may be made into a cable.

In Fig. 66 (*a*) are shown four separate conductors, each cir.-mil-foot and each carrying 0.001 amp. The voltage ac each must be 0.01 volt. In Fig. 66 (*b*) these same four conduc are grouped together and as each carries 0.001 amp., the t current must be 0.004 amp. The voltage drop across the gr

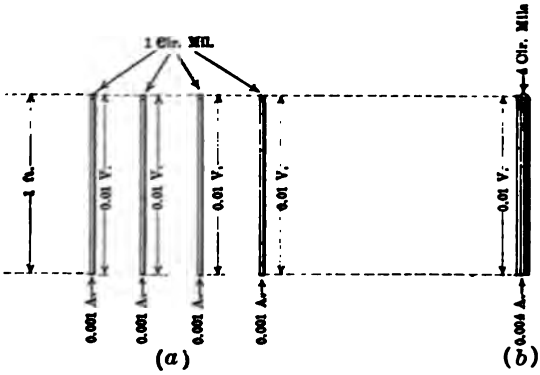


Fig. 66.—Voltage drop in a cir.-mil foot.

is still 0.01 volt. If any number of cir.-mil-ft. conductors e carrying 0.001 amp. are added in parallel to the group of 66 (*b*), the drop remains 0.01 volt.

From the foregoing the following rule may be deduced:

The voltage drop per foot of copper conductor is always (*volt provided that the current density is* 0.001 *amp. per cir. Further, if the density is other than* 0.001 *amp. per cir. mil, voltage drop will be in direct proportion to the current den* This last follows from equation (18).

Example.—A motor 800 ft. from the power house is to take 500 from 230-volt bus-bars. What size cable is necessary in order that voltage drop shall not exceed 20 volts?

A cable to operate at the normal density must have

$$500 \times 1{,}000 = 500{,}000 \text{ C.M.}$$

The total voltage drop then becomes

$$0.01 \times 800 \times 2 = 16 \text{ volts.}$$

The allowable drop is 20 volts, so a smaller cable may be used.

$$500{,}000 \times \frac{16}{20} = 400{,}000 \text{ C.M.} \quad \textit{Ans.}$$

This makes the actual current density

$$\frac{500}{400} = 1.25 \text{ amp. per } 1{,}000 \text{ cir. mils.}$$

69. Power Loss in a Feeder.—The method of Par. 68 may be used to estimate the power loss in a copper conductor. At the normal density

$$P' = I^2R = (0.001)^2 10 = 0.00001 \text{ (or } 10^{-5}\text{) watt per cir.-mil-ft.}$$

The total power loss at the *normal density*

$$P_0 = 0.00001 \times \text{C.M.} \times l$$

where C.M. is the conductor cross-section in cir. mils and l its length in feet.

The *actual* power loss is proportional to the *square* of the ratio of the actual to the normal density.

That is,

$$P = P_0D^2$$

where P is the actual power loss, P_0 the power loss at the normal density and D is the actual current density in amperes per 1,000 cir. mils.

Example.—Determine the power loss in the example of Par. 68

$P_0 = 0.00001 \times 400{,}000 \times 800 \times 2 = 6{,}400$ watts at the normal density.

The actual power loss

$$P = 6{,}400 \times (1.25)^2 = 10{,}000 \text{ watts} = 10 \text{ kw.} \quad \textit{Ans.}$$

The foregoing gives an easy and rapid method of solution of many problems. It is sufficiently exact for most practical problems.

CHAPTER V

BATTERY ELECTROMOTIVE FORCES—KIRCHOFF'S LAWS

70. Battery Electromotive Force and Resistance.—If a voltmeter be connected across the terminals of a battery (Fig. 67), the switch S being open, the instrument will record a certain voltage E. If now the switch S be closed, allowing the current I to flow, the instrument will record a voltage V which is less than E.

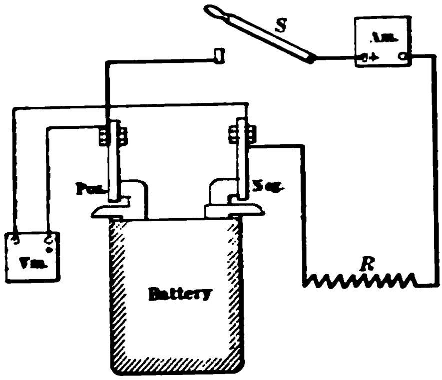


FIG. 67.—Connections for measuring battery resistance.

The voltage E, measured when the battery delivers no current, is the *internal voltage* or the *electromotive force* of the battery; the voltage V, measured when a current I flows, is known as the *terminal voltage* of the battery for that particular current value.

The difference between the open-circuit voltage E and the voltage V, measured when current is being taken from the battery, is the *voltage drop* in the battery due to the passage of current through the battery resistance. Every cell has a certain resistance, lying for the most part in the electrolyte, but partly in the battery plates and terminals. When the external circuit is closed so that current can flow, a certain voltage is required to send this current through the battery resistance, just as voltage is required to send current through an external resistance.

If the voltage E, measured at the battery terminals when the circuit is open, drops to V when the circuit is closed, the voltage

$e = (E - V)$ is the voltage drop through the cell due to the passage of the current I. Let the cell resistance be r. Then, by Ohm's Law,

$$E - V = e = Ir \quad \text{(by equation 18)}$$

or

$$r = \frac{e}{I} = \frac{E - V}{I} \quad \text{(by equation 19)} \tag{37}$$

$$E = V + Ir \tag{38}$$

That is, the internal resistance of the battery is equal to the open circuit voltage minus the closed circuit terminal voltage divided by the current flowing when the circuit is closed.

Example.—The open circuit voltage of a storage cell is 2.20 volts. The terminal voltage measured when a current of 12 amp. flows is found to be 1.98 volts. What is the internal resistance of the cell.

The voltage drop through the cell

$$E - V = 2.20 - 1.98 = 0.22 \text{ volt}$$

Then

$$r = \frac{0.22}{12} = 0.0183 \text{ ohm.} \quad \textit{Ans.}$$

In making a measurement of this character, it must be remembered that under open circuit conditions even the ordinary voltmeter takes some current. If the cell capacity is small (as in the case of a Weston cell) the voltmeter current alone may reduce the terminal voltage to a value one-half, or even less, of the open circuit voltage. Under these conditions the voltmeter cannot be used to measure the electromotive force of the cell.

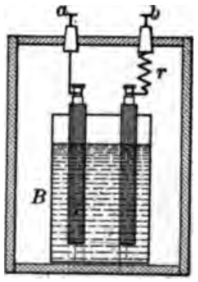


FIG. 68.—The internal resistance of a cell.

Moreover, it is impossible to measure directly the internal voltage of the battery when the battery delivers current, for the voltage drop occurs *within* the cell itself. Fig. 68 represents these conditions so far as their effect on the external circuit is concerned. A battery cell B is enclosed in a sealed box. Its resistance r is considered as removed from the cell itself and connected external to the cell, but within the sealed box. The cell then may be considered as having no resistance, its resistance having been replaced by r. The connections are brought through bushings in the box to

terminals a and b. When no current is being delivered by the cell, if a voltmeter be connected across the two terminals a and b, the instrument will measure the emf., E. If, however, a current I flows, the terminal voltage will drop from E to V, due to the voltage drop in the resistance r. Under these conditions it is impossible to measure E when the current is flowing, since the voltmeter can only be connected outside the resistance, through which the voltage drop occurs.

The voltage E and the resistance r are seldom constants but are more or less dependent upon the current. They are also affected by temperature, change in specific gravity of the electrolyte, polarization, etc.

71. Battery Resistance and Current.—As was shown in Par. 70, the resistance within the battery tends to reduce the flow of current. If, in Fig. 67, the switch be closed, the cell electromotive force E will be acting upon a circuit consisting of the internal resistance of the cell r and the resistance of the external circuit R. The resistances r and R being in series, the total resistance in the circuit is their sum. The current is

$$I = \frac{E}{r + R} \qquad (39)$$

The power lost in the battery is

$$P = I^2 r$$

If the cell is short circuited, R becomes zero and $I = \frac{E}{r}$. Under these conditions all the electrical energy developed by the cell is converted into heat within the cell itself.

Example—A battery-cell having an electromotive force of 2.2 volts and an internal resistance of 0.03 ohm is connected to an external resistance of 0.10 ohm. What current flows and what is the efficiency of the battery as used?

$$I = \frac{2.2}{0.03 + 0.10} = \frac{2.2}{0.13} = 16.9 \text{ amp.} \quad \textit{Ans.}$$

Power lost in the battery

$$P' = (16.9)^2 \times 0.03 = 8.57 \text{ watts.}$$

The useful power

$$P = (16.9)^2 \times 0.10 = 28.6 \text{ watts.}$$

P is equal to the total power developed by the battery minus the battery loss.

$$2.2 \times 16.9 = 37.2 \text{ watts}$$

$$P = 37.2 - 8.6 = 28.6$$

$$\text{Eff.} = \frac{28.6}{28.6 + 8.6} = 76.9 \text{ per cent.} \quad \textit{Ans.}$$

From the above, the following rule may be deduced: *The current in a circuit is equal to the total electromotive force acting in the circuit divided by the total resistance of the circuit.*

72. Batteries Receiving Energy.—If a resistance load be connected across a battery, current will immediately flow from the positive terminal of the battery and will return to the battery though its negative terminal. As has already been pointed out, the battery terminal voltage will be less than its open circuit value, due to the current flowing through the internal resistance of the battery. Under these conditions, the battery is a source of energy and is acting as a generator, that is, it *delivers* energy.

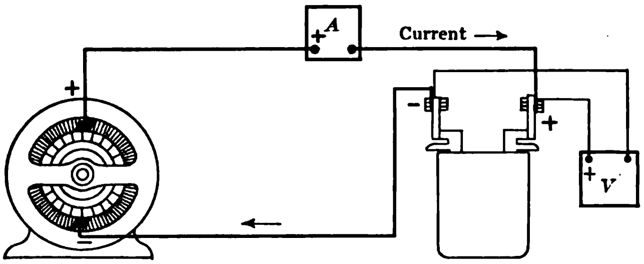


FIG. 69.—Generator charging a battery.

If current is forced to *enter* at the positive terminal of the battery, the battery will no longer be supplying energy but will be receiving energy. This energy must be supplied from some other source, as from another battery, or, as is more common, from a generator. The cell shown in Fig. 69 has an electromotive force of 2 volts, and a voltmeter V, connected across its terminals, indicates 2 volts when no current flows. If another source of electrical energy, such as a direct current generator, supply a potential difference of just 2 volts and its + terminal be connected to the + terminal of the battery and its − terminal connected to the − terminal of the battery, as shown in the figure, the voltmeter V will still read 2 volts and the ammeter A will read zero. That is, the battery neither delivers nor receives energy and no effect is noted other than those noted when the battery stood open-circuited. Under these conditions the battery is said to be "floating." If, however, the voltage of the generator be raised slightly, the ammeter A will indicate a current flowing from the

+ terminal of the generator *into* the + terminal of the battery, a direction just opposite to that which the current had when the battery *supplied* energy. The voltmeter will no longer read 2 volts, but will indicate a potential somewhat in excess of 2 volts.

What actually happens may be illustrated by a mechanical analogy. Fig. 70 shows a car standing on the track. A force of 400 lb. is necessary to overcome the standing friction of the car on the track. At one end of the car a force F is applied. Before the force F can move the car its value must at least equal 400 lb. When F is exactly 400 lb. the car will not move, just as no current flowed into the battery when the generator voltage was just equal to that of the battery. When the force F exceeds 400 lb., however, the car will move, the force effective in producing this motion being the amount by which F exceeds 400 lb. Thus, if F = 450 lb., 400 lb. of this is utilized in overcoming the 400 lb. opposing force due to friction and 50 lb. is effective in moving the car.

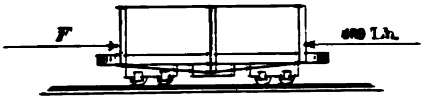


FIG. 70.—Force necessary to start a car.

In the case of the battery no current will flow until voltage in excess of the 2 volts is produced by the generator. Thus, if the generator voltage be raised to 2.4 volts, 2.0 volts of this is utilized to "buck" the 2.0 volts of the cell and 0.4 volt is effective in sending current into the cell. Thus, if the cell resistance be 0.1 ohm, the current will be

$$I = \frac{0.4}{0.1} = 4.0 \text{ amp.}$$

This assumes that the resistance of the leads is negligible.

Therefore, if E is the electromotive force of a battery, r its resistance and V the terminal voltage when current flows *in* at its positive terminal,

$$I = \frac{V - E}{r} \tag{40}$$

and

$$E = V - Ir \tag{41}$$

That is, the electromotive force of the cell is less than the terminal voltage by the amount of the resistance drop in the cell itself. These equations should be compared with equations (37) and (38), respectively.

Under these conditions, the cell is *receiving* electric energy, as is the case when a storage battery is being charged.

73. Battery Cells in Series.—Strictly speaking, a battery consists of more than one unit or cell. However, the term battery has come also to mean a single cell, when this cell is not acting in conjunction with others.

When cells are connected in series, their electromotive forces are added together to obtain the total electromotive force of the battery, and their resistances are added together to obtain the total resistance of the battery.

Thus, if several cells, having electromotive forces, E_1, E_2, E_3, E_4, etc., and resistances r_1, r_2, r_3, r_4, etc., are connected in series, the total electromotive force of the combination is

$$E = E_1 + E_2 + E_3 + E_4, \text{ etc.} \tag{42}$$

and the total resistance is

$$r = r_1 + r_2 + r_3 + r_4, \text{ etc.} \tag{43}$$

Equation (42) assumes that the cells are all connected + to − so that their electromotive forces are additive. If any cell be connected so that its electromotive force opposes the others, its voltage in equation (42) must be preceded by a minus sign.

If an external resistance R is connected across these cells in series, then by equation (38) the current is

$$I = \frac{E}{r + R} = \frac{E_1 + E_2 + E_3 + E_4, \text{ etc.}}{r_1 + r_2 + r_3 + r_4, \text{ etc.}, + R} \tag{44}$$

Example.—Four dry cells having electromotive forces of 1.30, 1.30, 1.35, and 1.40 volts and resistances of 0.3, 0.4, 0.2, and 0.1 ohm, respectively, are connected in series to operate a relay having a resistance of 10 ohms. What current flows in the relay?

$$I = \frac{1.30 + 1.30 + 1.35 + 1.40}{0.3 + 0.4 + 0.2 + 0.1 + 10} = \frac{5.35}{11.0} = 0.486 \text{ amp.} \quad \textit{Ans.}$$

A battery consisting of n equal cells in series has an emf. n times that of one cell, but has the current capacity of one cell only.

74. Equal Batteries in Parallel.—To operate satisfactorily in parallel all the batteries should have the same electromotive force. The behavior of batteries having unequal electromotive forces can be treated as special problems (see Par. 78).

Fig. 71 shows a battery of six cells, each having an electromotive force of 2.0 volts and a resistance of 0.2 ohm. It is clear that the emf. of the entire battery is no greater than the emf. of any one cell. The current, however, has 6 paths through which to flow. Therefore, for a fixed external current, the voltage drop in each cell is one-sixth that occurring if all the current passed through one cell. If the internal resistance of one cell is 0.2 ohm, the resistance of the battery as a whole must be $0.2/6 = 0.033$ ohm.

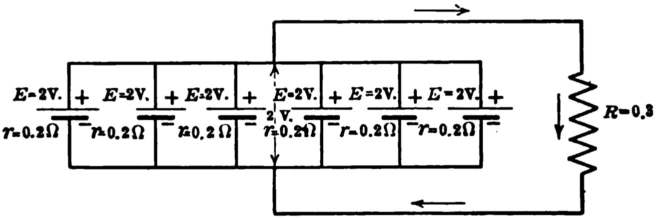


FIG. 71.—Parallel arrangement of equal batteries.

Example.—If the external resistance connected across the terminals of the battery in Fig. 71 is 0.3 ohm, what current flows?

Resistance of battery $= 0.2/6 = 0.033$ ohm.

$$I = \frac{2.0}{0.033 + 0.3} = \frac{2.0}{0.333} = 6 \text{ amp.} \quad \text{(eq. 39).} \quad \textit{Ans.}$$

If the emfs. are equal but the resistances of the cells are not all equal, but are r_1, r_2, r_3, r_4, etc., the battery resistance r is found by considering these resistances as being in parallel (equation (9), Chap. III).

$$\frac{1}{r} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} + \frac{1}{r_4} + \text{etc.} \tag{45}$$

Example.—A battery consists of 4 cells connected in parallel, each having an electromotive force of 2.0 volts, but resistances of 0.30, 0.25, 0.22, and 0.20 ohm respectively. If a resistance of 0.5 ohm is connected across the terminals of the battery, what current flows, and how much current does each cell supply? What is the voltage across the battery terminals?

$$\frac{1}{r} = \frac{1}{0.30} + \frac{1}{0.25} + \frac{1}{0.22} + \frac{1}{0.20} = 16.87 \text{ mhos.}$$

$$r = \frac{1}{16.87} = 0.0593 \text{ ohm.}$$

$$I = \frac{2.0}{0.0593 + 0.50} = \frac{2.0}{0.5593} = 3.58 \text{ amp.} \quad \textit{Ans.}$$

The terminal voltage

$$E_1 = IR = 3.58 \times 0.5 = 1.79 \text{ volts.} \quad \textit{Ans.}$$

The current in each cell may be found by means of equation (37).

$$\frac{2.0 - 1.79}{I_1} = 0.30$$

Solving $\quad I_1 = \frac{2.0 - 1.79}{0.30} = \frac{0.21}{0.3} = 0.7 \text{ amp.}$

Likewise $\quad I_2 = \frac{2.0 - 1.79}{0.25} = \frac{0.21}{0.25} = 0.84 \text{ amp.}$

$$I_3 = \frac{2.0 - 1.79}{0.22} = \frac{0.21}{0.22} = 0.95 \text{ amp.}$$

$$I_4 = \frac{2.0 - 1.79}{0.20} = \frac{0.21}{0.20} = \underline{1.05 \text{ amp.}}$$

Total current 3.54 (check). *Ans.*

That is, ***the current in any cell is equal to the voltage drop in the cell divided by the resistance of the cell.***

It will be found that the products of the current and the resistance of each cell are all equal.

$$0.7 \times 0.3 = 0.84 \times 0.25 \doteq 0.95 \times 0.22 = 1.05 \times 0.20$$

Cells connected in parallel ***must all have the same terminal voltage*** since all the positive terminals are tied together and all the negative terminals are tied together. If the emf.'s of the cells are all equal, the total battery emf. is equal to the emf. of but one cell. The total battery resistance may be found by the equation for resistances in parallel. The current in each cell is inversely proportional to the resistance of the cell ***if the electromotive forces are all equal.*** The current capacity of the battery is the sum of the current capacities of the individual cells.

75. Series-parallel Grouping of Cells.—Rows of series-connected cells may be so connected that the rows themselves are

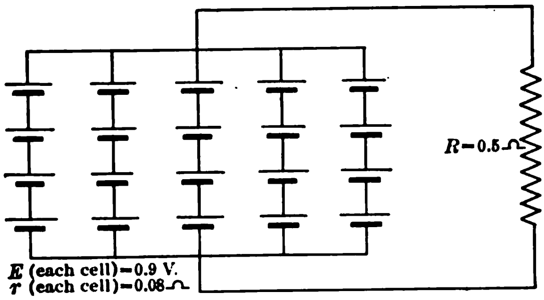


FIG. 72.—Series parallel-grouping of cells.

grouped in parallel. Fig. 72 shows a row of 4 cells in series, and five of these rows in parallel. If there are m equal cells in series in each row, then the emf. of each row must be

$$E = mE' \qquad \text{(by equation 42)}$$

where E' is the emf. of one cell.

The resistance of each row must be

$$r_1 = mr' \quad \text{(by equation 43)}$$

where r' is the resistance of one cell.

Since there are n rows in parallel, the resistance of the whole combination must be

$$r = \frac{r_1}{n} = \frac{m}{n} r' \tag{46}$$

If an external resistance R is connected to the battery, the current is

$$I = \frac{mE'}{\frac{m}{n} r' + R} \tag{47}$$

Example.—Let each of the cells of Fig. 72 have an emf. of 0.9 volt and an internal resistance of 0.08 ohm. If the external resistance R is 0.5 ohm, what current flows?

$$I = \frac{4 \times 0.9}{\frac{4}{5} 0.08 + 0.5} = \frac{3.6}{0.564} = 6.4 \text{ amp.} \quad \textit{Ans.}$$

76. Grouping of Cells.—(*a*) *To obtain the best economy,* group the cells so that the battery resistance is as low as possible. This usually means a large number of parallel connections. Under these conditions the life of the battery will be prolonged but the initial cost is excessive.

(*b*) *To obtain the maximum current with a fixed external resistance* make the internal resistance $(\frac{m}{n} r')$ of the battery equal to the external resistance. This is not economical, since only half of the energy developed by the battery is available in the external circuit; the other half is lost in the cells themselves. Under these conditions the battery delivers the maximum power.

(*c*) *To obtain quick action* for the intermittent operation of relays, bells, etc., group the cells in series if possible.

Example.—In the example of Par. 75, how should the cells be arranged to obtain the maximum current?

The total battery resistance $\frac{m}{n} 0.08$ must be equal to the external resistance.

That is,
$$\frac{m}{n} 0.08 = 0.5$$

Also
$$m \times n = 20 \qquad n = \frac{20}{m}$$

Solving
$$\frac{\frac{m}{20}}{\left(\frac{20}{m}\right)}0.08 = 0.5$$

$$m^2 = \frac{20}{0.08}\,0.5 = 125$$

$$m = 11 + \qquad \textit{Ans.}$$

The best arrangement is ten cells in series, and two rows in parallel. (Eleven cells in series would not operate satisfactorily if connected in parallel with the remaining nine cells in series.)

77. Kirchoff's Laws.—By means of Kirchoff's Laws it is possible to solve many circuit networks that would otherwise be difficult of solution.

(1) *In any branching network of wires, the algebraic sum of the currents in all the wires that meet at a point is zero.*

(2) *The sum of all the electromotive forces acting around a complete circuit is equal to the sum of the resistances of its separate parts multiplied each into the strength of the current that flows through it,* or *the total change of potential around any closed circuit is zero.*

The first law is obvious. It states that the total current *leaving* a junction is equal to the total current *entering* the junction. If this were not so electricity would accumulate at the junction.

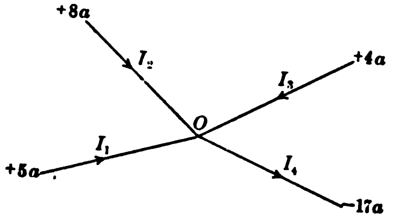


FIG. 73.—Illustrating Kirchoff's first law.

The law is illustrated by Fig. 73. Four currents, I_1, I_2, I_3, and I_4 meet at the junction 0. The first three currents flow *toward* the junction so have plus signs as they *add* to the quantity at the point 0. The last current I_4 flows away from the junction, so has a minus sign as it subtracts from the quantity at the point 0. Then

$$I_1 + I_2 + I_3 - I_4 = 0 \tag{48}$$

Assume that $I_1 = 5$ amp., $I_2 = 8$ amp. and $I_4 = 17$ amp.

Then $$5 + 8 + I_3 - 17 = 0$$

and $I_3 = +4$ amp., the plus sign indicating that the current flows toward the junction.

The second law is but another application of Ohm's Law (equation 18). The basis of the law is obvious; if one starts at a cer-

tain point in a circuit, and follows continuously around the paths of the circuit until the starting point is again reached, he must again have the same potential with which he started. Therefore the sources of electromotive force encountered in this passage must necessarily be equal to the voltage drops in the resistances, every voltage being given its proper sign.

This second law is illustrated by the following example.

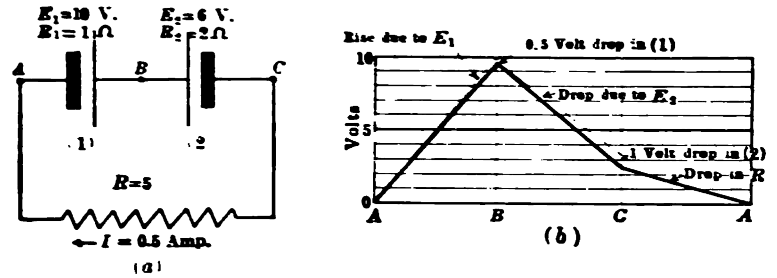


Fig. 74.—Voltage relations in an electric circuit.

Two batteries (Fig. 74), having electromotive forces of 10 and 6 volts and internal resistances of 1 and 2 ohms respectively, are connected in series opposing (their + terminals connected together) and in series with an external resistance of 5 ohms. Determine the current and the voltage at each part of the circuit.

Since the two batteries act in opposition, the net electromotive force of the two batteries is 10 − 6 = 4 volts.

The current is,

$$I = \frac{10 - 6}{1 + 2 + 5} = \frac{4}{8} = 0.5 \text{ amp.}$$

Consider the point A as being at reference potential. In passing from A to B there is a 10-volt rise in potential due to the electromotive force of battery No. 1, but around the circuit in the direction of the current flow there occurs a simultaneous 0.5-volt drop of potential due to the current flowing through the 1-ohm resistance of cell No. 1. Therefore the net potential at B is but 9.5 volts greater than that at A, as is shown in Fig. 74*b*. In passing from B to C there is a drop of 6 volts due to passing from the + to the − terminal of battery No. 2, and there is

also a further drop of 1 volt due to the current of 0.5 amp. passing through the 2-ohm resistance of battery No. 2. This makes the net potential at $C = 9.5 - 6 - 1 = +2.5$ volts. In passing from C to A there is a drop in potential of 2.5 volts due to the current of 0.5 amp. flowing through the 5-ohm resistance. When point A is reached the potential has dropped to zero.

Therefore the sum of all the electromotive forces in the circuit, taken with their proper signs, is equal to the sum of the Ir drops. This is illustrated as follows:

Electromotive forces		*Ir* drops		
Cell No. 1 =	+ 10 volts	Cell No. 1 =	$-0.5 \times 1 =$	− 0.5 volt
" No. 2 =	− 6 volts	" No. 2 =	$-0.5 \times 2 =$	− 1.0 volt
Total	+ 4 volts	5-ohm res. =	$-0.5 \times 5 =$	− 2.5 volt
		Total		− 4.0 volt

$$+4 + (-4) = 0$$

78. Applications of Kirchoff's Laws.—In the application of Kirchoff's second law to specific problems the question of algebraic signs may be troublesome and is a frequent source of error. If, however, the following rules are kept in mind no difficulties should occur.

A rise in voltage should be preceded by a + sign.
A drop in voltage should be preceded by a − sign.

For example, in passing *through a battery* from the − to the + terminal, the potential *rises* so that this voltage should be preceded by a + sign. On the other hand, when passing from the + terminal to the − terminal, the potential *drops*, so that a − sign should precede this voltage. These points are illustrated by Fig. 74.

When going through a resistance in the *same* direction as the current, the voltage drops, so that this voltage should be preceded by a − sign. A voltage due to passage through a resistance in the direction *opposite* to the current flow should be preceded by a + sign.

This is further illustrated by the electric circuit shown in Fig. 75. Three batteries having emf.'s E_1, E_2, and E_3 are con-

nected as shown in different parts of the network of resistances. R_1, R_2, R_3, R_4. The assumed directions for the various currents are indicated by the arrows. The battery resistances are assumed negligible as compared with the other circuit resistances.

Starting at the point *a*, and applying Kirchoff's second law to the path *abcda*, an equation may be written

$$+E_1 - I_1R_4 - I_2R_2 - E_2 - I_1R_1 = 0$$

Starting at *f* and passing along the path *fcbedf*:

$$-E_3 - I_3R_3 - I_2R_2 - E_2 = 0$$

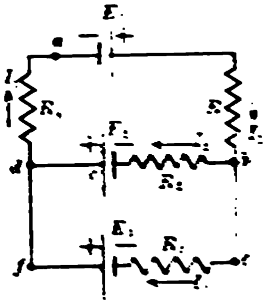


FIG. 75.—Application of Kirchoff's laws.

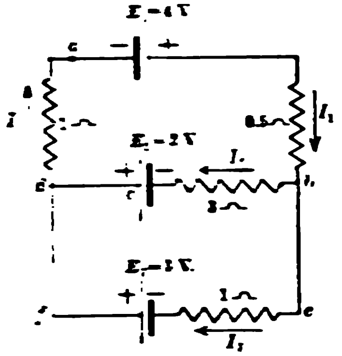


FIG. 76.—Application of Kirchoff's laws.

This gives but two equations for the determination of three unknown currents. Three equations are necessary. The third may be obtained by applying Kirchoff's first law to some junction as *b*.

$$+I_1 - I_2 - I_3 = 0$$

since I_1 is assumed to flow *toward* the junction and I_2 and I_3 *away* from the junction.

With these three equations it is possible to determine the three currents.

Example. Fig. 76 shows a network identical with that shown in Fig. 75, except that numerical values are used. The battery resistances are assumed to be small compared with the circuit resistances, and are neglected.

Considering path *abcda*,

$$+4 - I_1 0.5 - I_2 3 + 2 - I_1 1 = 0$$

or

$$1.5I_1 + 3I_2 = 6 \qquad (A)$$

Similarly, path *febcdf*, starting at *f*,

$$-3 + (I_3 1) - 3I_2 + 2 = 0$$

or

$$3I_2 - I_3 = -1 \qquad (B)$$

and at the junction *b*,

$$+I_1 - I_2 - I_3 = 0$$

or

$$I_1 = I_2 + I_3 \qquad (C)$$

Substituting I_1 (*C*) in (*A*)

$$1.5(I_2 + I_3) + 3I_2 = 6$$
$$4.5I_2 + 1.5I_3 = 6$$

and combining with (*B*)

$$9I_2 - 3I_3 = -3 \qquad (B)$$
$$9I_2 + 3I_3 = 12$$
$$-6I_3 = -15$$
$$I_3 = 2.5 \text{ amp.} \qquad \textit{Ans.}$$

Substituting this value in (*B*)

$$3I_2 - 2.5 = -1$$
$$3I_2 = 1.5$$
$$I_2 = 0.5 \text{ amp.}$$
$$I_1 = I_2 + I_3 = 3.0 \text{ amp.} \qquad (C) \quad \textit{Ans.}$$

79. Assumed Direction of Current.—In the solution of this type of problem, the question of assuming the proper direction of current often arises. The current may be *assumed* to flow in either direction. If the assumed direction of the current is not the actual direction, this current will be found to have a minus sign when the equations are solved.

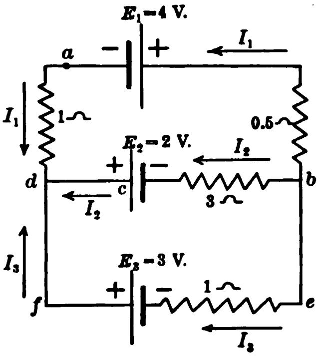


FIG. 77.—Application of Kirchoff's laws.

Example.—This is illustrated by assuming that the three currents of Par. 78 have such a direction that they all meet at point *d* as is shown in Fig. 77. This condition is of course impossible.

Considering circuit *abcda*, starting at *a*,

$$+4 + 0.5I_1 - 3I_2 + 2 + I_1 = 0$$
$$1.5I_1 - 3I_2 + 6 = 0$$

Similarly circuit *febcdf*, starting at *f*,

$$-3 + I_3 - 3I_2 + 2 = 0$$
$$I_3 - 3I_2 - 1 = 0$$

The three currents I_1, I_2, I_3 all flow toward junction *d*, therefore

$$I_1 + I_2 + I_3 = 0.$$

Substituting and solving

$$I_1 = -3 \text{ amp.} \quad \textit{Ans.}$$
$$I_2 = 0.5 \text{ amp.} \quad \textit{Ans.}$$
$$I_3 = 2.5 \text{ amp.} \quad \textit{Ans.}$$

The minus sign preceding I_1 signifies that this current flows in the opposite direction to that assumed and indicated by the arrow, Fig. 77. The + signs before I_2 and I_3 indicate that the assumed directions for these two currents were the actual direction of flow.

80. Further Applications of Kirchoff's Laws.—Kirchoff's laws might be applied to problems involving distribution systems, electric railways, etc., where power is fed to the loads through different feeders and from different substations. In practice, however, Kirchoff's laws are rarely directly applied to electric railway systems, since the widely fluctuating loads which are constantly shifting their location make it impossible to formulate a definite problem. Only occasionally is it necessary to apply these laws to power and lighting systems, since the feeder layout in such systems is usually determined by various operating considerations.

The following problem illustrates the possible application of these laws.

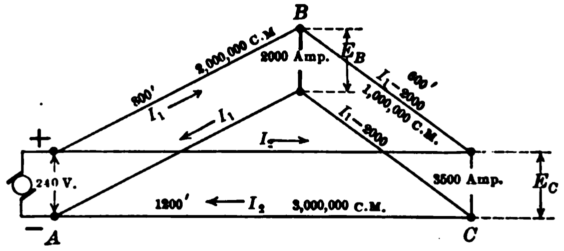


FIG. 78.—Ring-feeder system.

Example.—In Fig. 78 a 240-volt substation at *A* supplies two distributing centers *B* and *C*, by a ring system of feeders. Between *A* and *B*, a distance of 800 ft., two 1,000,000 C.M. feeders are paralleled; between *A* and *C*, a distance of 1,200 ft., three 1,000,000 C.M. feeders are paralleled; between

B and C, a distance of 600 ft., a 1,000,000 C.M. feeder is connected. Determine the current in each feeder and the voltage at each distributing center, when the load at B is 2,000 amp. and that at C is 3,500 amp.

Assuming 10 ohms per cir.-mil-foot:

$$\text{Resistance per wire } A \text{ to } B = \frac{800 \times 10}{2,000,000} = 0.004 \text{ ohm.}$$

$$\text{Resistance per wire } B \text{ to } C = \frac{600 \times 10}{1,000,000} = 0.006 \text{ ohm.}$$

$$\text{Resistance per wire } A \text{ to } C = \frac{1,200 \times 10}{3,000,000} = 0.004 \text{ ohm.}$$

Going from A to B to C, out on the positive and back on the negative conductor,

$$240 - I_1(0.004) - (I_1 - 2,000)0.006 - E_C - (I_1 - 2,000)0.006 - I_1(0.004) = 0$$

$$240 - I_1(0.02) + 24 = E_C \qquad (1)$$

Likewise going direct from A to C

$$240 - I_2(0.004) - E_C - I_2(0.004) = 0$$

$$240 - I_2(0.008) = E_C \qquad (2)$$

Equating (1) and (2)

$$240 - I_1(0.02) + 24 = 240 - I_2(0.008)$$

$$0.02I_1 - 0.008I_2 = 24 \qquad (3)$$

At the junction at C

$$I_1 - 2,000 + I_2 = 3,500$$

$$I_1 + I_2 = 5,500 \qquad (4)$$

Substituting in (3) for $I_1 = 5,500 - I_2$

$$0.02(5,500 - I_2) - 0.008I_2 = 24$$

$$110 - 0.02I_2 - 0.008I_2 = 24$$

$$0.028I_2 = 86$$

$$I_2 = 3,070 \text{ amp.} \quad \textit{Ans.}$$

$$I_1 = 2,430 \text{ amp.} \quad \textit{Ans.}$$

Voltage at C (equation 2)

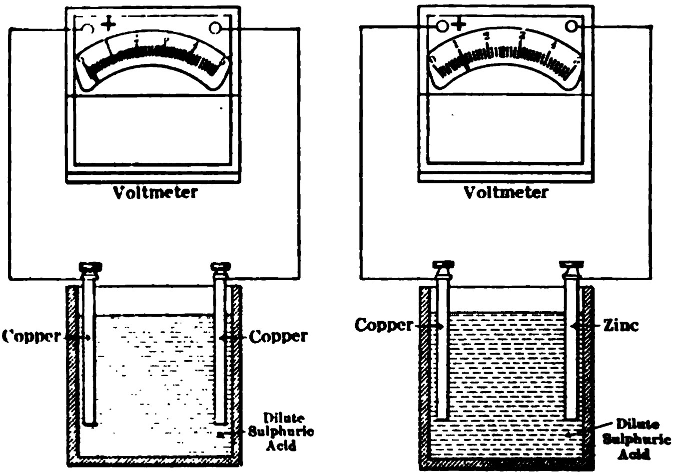
$$E_C = 240 - 3,070(0.008) = 215.44 \text{ volts.} \quad \textit{Ans.}$$

$$E_B = 240 - 2,430(0.008) = 220.56 \text{ volts.} \quad \textit{Ans.}$$

CHAPTER VI

PRIMARY AND SECONDARY BATTERIES

81. Principle of Electric Batteries.—If two copper strips or plates be immersed in a dilute sulphuric acid solution (Fig. 79*a*) and be connected to the terminals of a voltmeter, no appreciable deflection of the voltmeter will be observed. This shows that no appreciable difference of potential exists between the copper strips. If, however, one of the copper strips (Fig. 79*b*)



(*a*) (*b*)

FIG. 79. Simple primary cell.

be replaced by a zinc strip, the voltmeter needle will deflect and will indicate approximately one volt, showing that a potential difference now exists. It will be necessary to connect the copper to the + terminal of the voltmeter and the zinc to the − terminal in order that the voltmeter may read up scale. This shows that so far as the *external* circuit is concerned, the copper is positive to the zinc.

The above experiment may be repeated with various metals.

For example, carbon or lead may be substituted for the copper and a potential difference will be found to exist between each of these and the zinc, although it will not be of the same value as it was for the copper-zinc combination. Likewise other metals may be substituted for the zinc, and potetinal differences will be found to exist.

Furthermore, it is not necessary that sulphuric acid be used for the solution. Other acids such as hydrochloric, chromic, etc., may be substituted for the sulphuric; or even salt solutions such as common salt (sodium chloride), ammonium chloride (sal ammoniac), copper sulphate, zinc sulphate, etc., may be used.

In order to obtain a difference of potential between the two metal plates, but two conditions are necessary.

(1) The plates must be of different metals.

(2) They must be immersed in some electrolytic solution, such as an acid, alkali, or salt.

Again, if current be taken from the cell shown in Fig. 79 (*b*) by connecting a resistance across its terminals (Fig. 80), current will flow from the copper through the resistance *AB* and into the cell through the zinc. Inside the cell, however, the current will flow *from* the *zinc* through the solution *to* the *copper* as shown in Fig. 80. For the reason that current flows *from zinc to copper within the cell,* zinc is said to be electrochemically positive to copper. Therefore, when cnosidering such an electrolytic cell, the copper is positive to the zinc when the external circuit is considered, but the zinc is electro-positive to the copper when the plates and the solution alone are considered.

82. Definitions.—The metal strips or plates of a cell are called *electrodes.* The electrode at which current enters the solution (as the zinc, Fig. 80) is the *anode,* and the electrode at which current leaves the solution (as the copper, Fig. 80) is the *cathode.*

The solution used in a cell is called the *electrolyte.*

If current be taken from the cell under proper conditions and for a considerable time, the zinc plate will diminish in weight. This is true not only in the case of this particular cell, but in practically all cells the flow of current is accompanied by a loss in weight of at least one of the plates. Energy is stored in the cell *chemically,* and the electrical energy is delivered at the expense of

the plate which goes into solution. That is, one plate is either oxidized or converted into another chemical compound, this change being accompanied by a decrease of chemical energy of the system. Therefore *chemical energy* is converted into *electrical energy*, when the cell delivers a current.

Hence:

An electric cell or battery is a device for transforming chemical energy into electrical energy.

Such cells or batteries are divided into two classes: *primary cells* and *secondary cells*.

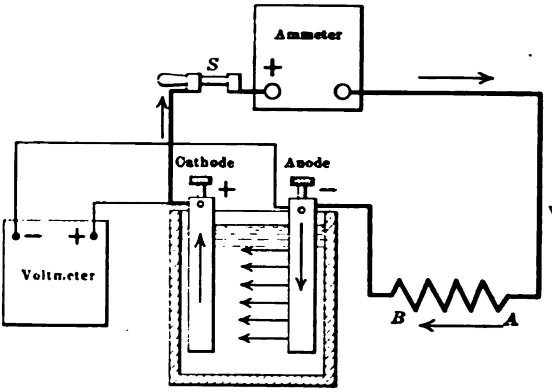


Fig. 80.—Current-flow in a single cell.

In a *primary cell* it is necessary from time to time to renew the electrolyte and the electrode which goes into solution by fresh solution and new plates, respectively.

In a *secondary cell* the electrolyte and the electrodes which undergo change during the process of supplying current are restored electrochemically by sending a current through the cell in the reverse direction.

83. Primary Cells.—Although it was stated in Par. 81 that there are many combinations of metals and solutions capable of generating an electromotive force and so forming a cell, only a limited number of such combinations are commercially practicable. The general requirements of a good cell are as follows:

(*a*) There must be little or no wastage of the materials when the cell is not delivering current.

(*b*) The electromotive force must be of such a magnitude as to enable the cell to deliver a reasonable amount of energy with a moderate current flowing.

(*c*) Frequent replacement of materials must not be necessary and such materials must not be expensive.

(*d*) The internal resistance and the polarization effects must not be excessive, otherwise the battery cannot supply even moderate values of current, at least for any appreciable time.

As an illustration, the cell shown in Fig. 79*b* would not be practicable, because both the copper and the zinc would waste away even were the battery delivering no current. Polarization (see Par. 85) would be excessive, and therefore the battery would be capable of delivering only a comparatively small current.

84. Internal Resistance.—As was pointed out in Chap. V, every cell or battery has an internal resistance which reduces the magnitude of the current and causes the terminal voltage to drop when current is taken from the cell. Such resistance lies in the electrodes, in the contact surface between the electrodes and the electrolyte, and in the electrolyte itself. This resistance may be reduced by changing the dimensions of the cell in the same way as would be done for any electric conductor. The cross-section of the path through which the current flows inside the cell should be made as large as is practicable. This means large area of electrodes in contact with the electrolyte. Also the cross-section of the plates must be large enough to carry the current to the cell terminals without excessive drop in voltage. Little difficulty is experienced in making this voltage drop negligible. It will be appreciated that larger electrodes mean a larger cell, with a greater current capacity. In addition to increasing the area of the electrodes, the resistance of the cell may be diminished by decreasing the distance between the plates. This reduces the length of the path through which the current flows within the cell and correspondingly reduces the cell resistance.

Increasing the size of the cell does not increase its electromotive force. This electromotive force depends only upon the material of the two electrodes, and the electrolyte. Thus, Fig. 81 shows two gravity cells, made up of the same materials, but differing materially in size. The cells are bucking one another,

that is, their + terminals are joined and their − terminals are joined. A galvanometer G connected in one of the leads reads zero, indicating that no current flows from the larger cell to the smaller.

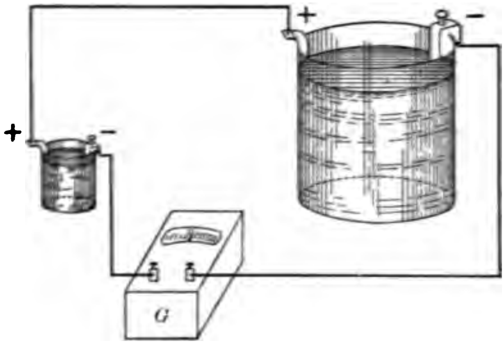


FIG. 81.—Equality of electromotive forces in cells of unequal sizes.

85. Polarization.—If a test be made to determine the fall of terminal voltage as current is taken from a cell, by connecting a voltmeter, ammeter, and an external resistance as in Fig. 80, the results will be somewhat as follows:

When the cell is on open circuit the voltmeter will indicate

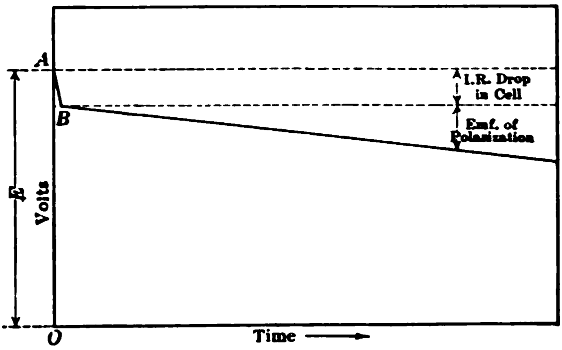


FIG. 82.—Drop of voltage in a cell due to polarization.

the cell electromotive force E, represented by the distance OA, Fig. 82. When the switch S is closed, current will flow and the voltage will drop immediately from OA to OB. The distance AB represents the voltage drop due to the internal resistance

of the cell and this has been considered earlier in some detail. As time elapses the terminal voltage will be observed to drop still further, even though the current be maintained constant. This further drop of voltage is due to *polarization.*

When the cell delivers current, small bubbles of hydrogen appear upon the positive plate or cathode, practically covering it. These bubbles have two effects:

They cause a substantial increase in the resistance at the contact surface between the cathode and the electrolyte.

Hydrogen acting in conjunction with the cathode or positive plate sets up an electromotive force which opposes that of the cell.

These two effects explain the reduction in the current capacity of many types of cells after they have delivered current for some time.

Remedies for Polarization.—These hydrogen bubbles may be removed mechanically by brushing them off or by agitating the electrolyte. This is impracticable under commercial conditions. If the plate be roughened, the bubbles form at the projections and come to the surface more readily.

The hydrogen bubbles may be removed chemically by bringing oxidizing agents, such as chromic acid or manganese peroxide, itno intimate contact with the cathode. The hydrogen readily combines with the oxygen of these compounds to form water (H_2O). This method is used in the bichromate cell, in the Le Clanché cell and in dry cells.

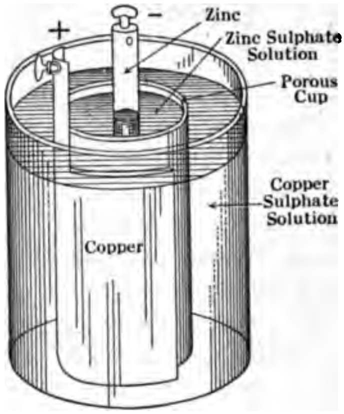


FIG. 83.—Daniell cell.

86A. Daniell Cell.—This cell, Fig. 83, is a two-fluid cell having copper and zinc as electrodes. It consists of a glass jar inside of which is a porous cup containing zinc sulphate solution or a solution of zinc sulphate and sulphuric acid. The anode or negative electrode is immersed in this electrolyte. The porous cup is placed in a solution of copper sulphate with copper sulphate crystals in the bottom of the jar.

The copper plate, which is cathode, surrounds the porous cup. The porous cup keeps the two solutions separated. As the copper is in a copper sulphate solution, there is no polarization. This cell is designed for use in a circuit which is kept continually closed. If left idle the electrodes waste away. When the cell is taken out of service for some time, the electrodes should be removed and the porous cup should be thoroughly washed. The electromotive force of this cell is about 1.1 volts.

86.B Gravity Cell.—The gravity cell is similar to the Daniell cell, except that gravity, rather than a porous cup, is depended upon to keep the electrolytes separated. This cell is shown in Fig. 84. The cathode, which is of copper, is made of strips riveted together and placed in the bottom of the cell together with copper sulphate crystals. A solution of copper sulphate is then poured to within a few inches of the top of the jar. The connection to the copper is usually an insulated copper wire fastened to the copper and carried out through the solution to the top of the jar. There should always be copper sulphate crystals at the bottom of the cell.

Fig. 84.—Gravity cell.

The anode is zinc, is usually rather massive and is cast in the form of a crow's foot and hung on the top of the jar. This is surrounded by a zinc sulphate solution. The solutions are kept separated by gravity. The copper sulphate is the heavier of the two solutions and therefore tends to remain at the bottom. The solutions should be poured in carefully for if the copper sulphate solution comes in contact with the zinc, copper will be deposited. This copper should be removed if by chance it becomes deposited in any way. In the operation of the cell the zinc goes into solution as zinc sulphate, and metallic copper comes out of the copper sulphate solution and is deposited upon the copper electrode. The cathode will therefore gain in weight whereas the anode will lose in weight. This is the reason for having the zinc electrode massive, and the copper electrode of very thin sheet copper, when the cell is set up initially.

Due to capillary action the electrolyte tends to creep up

over the top of the jar forming a crystalline deposit. To prevent this creeping, the top of the jar should be paraffined. To prevent evaporation the upper surface of the electrolyte may be covered with oil. When the cell is replenished, metallic zinc and copper sulphate are supplied and metallic copper and zinc sulphate are removed.

The gravity cell is a *closed circuit* battery, and the circuit should therefore be kept closed for the best results. Otherwise the copper sulphate will gradually mix with the zinc sulphate. The cell has been found very useful in connection with railway signals, fire alarm systems, and telephone exchanges, all closed circuit work, although the storage battery has replaced it in many instances. The electromotive force of the cell is practically that of the Daniell cell, being about 1.09 volts, but varies slightly with the concentration of the solutions.

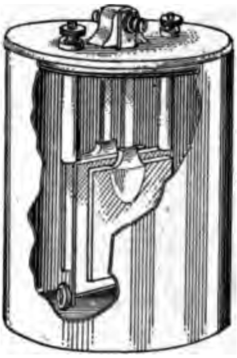


FIG. 85.—Edison-Lalande cell or Edison primary battery.

87. Edison-Lalande Cell.—The Edison-Lalande cell is still used to some extent. The cathode is of copper oxide and is suspended between two zinc plates which form the anode. All the plates are fastened to a porcelain cover by means of bolts which serve as binding posts as well as supports for the plates. The electrolyte is caustic soda ($NaOH$), one part by weight of soda to three of water. To prevent the soda being acted upon by the air, the electrolyte is covered with a layer of mineral oil. The copper oxide of the cathode gives up its oxygen very readily to the hydrogen which forms on it, thus preventing any substantial polarization. These cells are capable of delivering a very heavy current. The electromotive force is about 0.95 volt, and when delivering current the terminal voltage drops to 0.75 volt. There is little or no local action in this cell and it can therefore be used to advantage on both open circuit and closed circuit work. Its chief disadvantage is its low electromotive force.

88. Le Clanché Cell.—The Le Clanché cell is perhaps the most familiar type of primary battery, because of its wide applica-

tion. The cathode is molded carbon and the anode is amalgamated zinc. The electrolyte is sal ammoniac or ammonium chloride. This type of cell is suited only for open circuit work because of the rapidity with which it polarizes. The electromotive force is 1.4 volts, but because of the drop due to its internal resistance and that due to polarization, not over 1 volt per cell should be allowed in planning an installation. The most common method of reducing polarization is to bring manganese dioxide into intimate contact with the carbon. This gives up oxygen readily which unites with the hydrogen bubbles to form water.

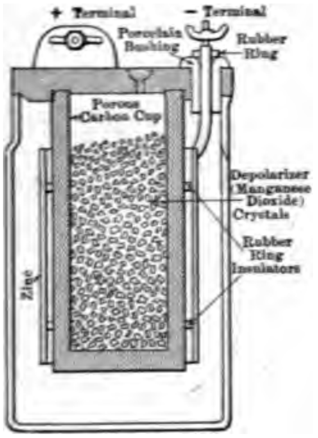


Fig. 86.—Porous cup Le Clanché cell.

In one type of Le Clanché cell a pencil zinc is suspended in the center of a hollow cylinder of carbon and manganese dioxide. An improved type, the porous cup cell, is shown in Fig. 86. In this form a hollow carbon cylinder is filled with manganese dioxide, and the zinc, bent into cylindrical form, surrounds the carbon cylinder, being separated therefrom by rubber rings.

The solution should consist of 3 ounces of sal ammoniac to 1 pint of water. A more concentrated solution produces zinc crystals on the zinc and carbon. To prevent the solution "creeping," the top of the cell is dipped in paraffin and the top of the carbon is covered with a black wax.

This cell owes it wide use to its simplicity, to the small amount of attention that it requires, and to the fact that it contains no injurious acids or alkalis. Its uses are for intermittent work, such as ringing door-bells, telephone work, and open circuit telegraph work.

89. Weston Standard Cell.—It is essential in practical work to be able to reproduce accurately standards of current, voltage, and resistance. Obviously if two of the above quantities are known, the third is readily obtainable by Ohm's Law. It is a matter of no great difficulty to make and reproduce resistance standards,

as such standards are nothing more than metals in strips and in other forms, carefully mounted and calibrated. Such standards are very permanent and their resistance remains constant indefinitely.

A standard of either current or voltage is much more difficult to reproduce and maintain than is the standard of resistance. Of the two, it has been found more practicable to produce and maintain a voltage standard rather than a current standard. This voltage standard is obtained in a *standard cell.* The electromotive force of a cell depends upon its materials and their impurities, the concentration of the electrolyte, the temperature, the polarization effects, etc. It is difficult, therefore, to select such materials for a cell as will enable it to be reproduced at different times and at various places with a high degree of accuracy. The Clark cell was the first of the standard cells to prove commercially successful. This had a cathode of mercury, an anode of zinc, and an electrolyte of mercurous sulphate and zinc sulphate. The objections to this cell were that the electromotive force changed very appreciably with the temperature and that this change lagged behind the change in temperature.

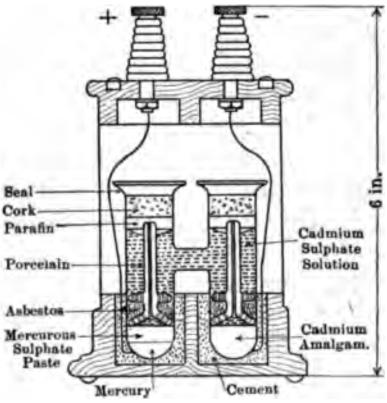


FIG. 87.—Weston standard cell.

In the Weston cell, cadmium is substituted for the zinc of the Clark cell. A cross-section of the portable form of Weston cell is shown in Fig. 87. The anode is mercury located at the bottom of one leg of an H-tube. Above this is mercurous sulphate paste. These materials are held in position by means of a porcelain tube, expanded at the bottom and packed with asbestos. This tube extends to the top of the cell and acts as a vent for any gases that are formed. In the bottom of the other leg of the H-tube is the cathode, of cadmium amalgam. This is held in place by another porcelain tube packed with asbestos. The electrolyte

is cadmium sulphate. The leads from the cathode and the anode are sealed into the tubes at the bottom. The top of the cell is sealed with cork, paraffin, and wax. The entire cell is mounted in a wood and metal case with binding posts at the top.

The cell is made in two forms, the *normal cell* and the unsaturated or *secondary* cell. In the normal cell, cadmium sulphate crystals are left in the bottom of the solution so that it is always saturated. Its electromotive force is affected slightly by temperature, but corrections can be accurately made. It is possible to reproduce such cells with electromotive forces differing by only a few parts in 100,000.

In the unsaturated cell, the solution is saturated at 4° C. and as no crystals are left in the solution, its concentration is substantially constant at other temperatures. Such cells have practically no temperature coefficient. They are not as accurately reproducible as is the normal cell. A certificate should accompany each one giving its electromotive force, which usually is about 1.0186 volts. The unsaturated type of cell rather than the normal cell is used almost entirely in practical work.

The terminal voltage of any cell differs from its electromotive force by the IR drop due to the cell resistance. As the resistance of a Weston cell is about 200 ohms, it is evident that if any appreciable current be taken from the cell its terminal voltage will be quite different from its electromotive force. The cell must be used, therefore, in such a manner that it delivers no appreciable current. By means of the so-called Poggendorf method, described in par. 125, the cell is used without delivering current. Not more than 0.0001 amp. should be taken, from the cell at any time. If appreciable current is taken, the electromotive force drops, but when the circuit is again opened the electromotive force slowly recovers its initial value.

90. Dry Cells.—Dry cells are a modification of the Le Clanché cell and as they are very light, portable, and convenient, they are rapidly replacing other types of cells. The word "dry cell" is really a misnomer, for no cell that is dry will deliver any appreciable current. In fact the chief cause of dry cells becoming exhausted is due to their actually becoming dry.

A cross-section of a typical dry cell is shown in Fig. 88. The anode is sheet zinc, made in the form of a cylinder with an open top, and acts as the container of the cell. The binding post is soldered to the top of the zinc. The zinc is lined with some non-conducting material such as blotting paper or plaster of paris. The anode consists of a carbon rod, and the mixture of coke, carbon, etc., which surrounds this rod. The rod itself varies in shape among various manufacturers. It is located axially in the zinc container and the binding post is secured to the top of it. The depolarizing agent, powdered manganese dioxide, is mixed with fienly crushed coke and pressed solidly into the container between the carbon and the non-conducting material which lines the zinc. It fills the cell to within about an inch of the top. Sal ammoniac, with perhaps a little zinc sulphate, is added and the cell then sealed with wax or some tar compound. The outside of the zinc is frequently lacquered, and the cells are always set in close-fitting cardboard containers.

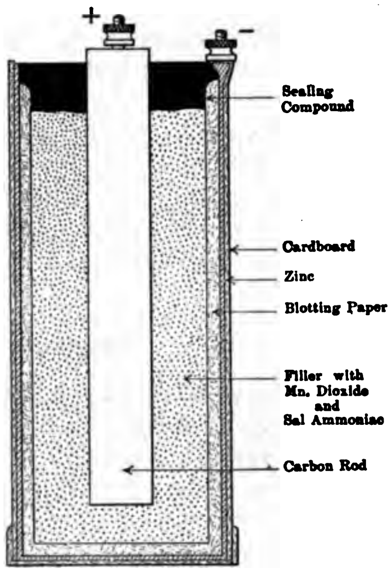


FIG. 88.—Sectional view—dry cell.

The electromotive force of a dry cell is about 1.5 volts when new but this drops to about 1.4 volts with time, even though the cell remains idle. A cell is practically useless after a year to 18 months, even if not used at all. The internal resistance of the cell is about 0.1 ohm when new and increases to several times this value with time. The polarization effect is large as compared with the internal resistance so that a low value of internal resistance is not important except as an indication of the condition of the cell. A method for testing the condition of a cell is to short-circuit it through an ammeter, when it should deliver an instantaneous value of 1.5/0.1 or 15 amp., if in good condition.

When new the current under these conditions may reach even 25 amp. When delivering appreciable current the terminal voltage is very nearly 1 volt.

One of the chief causes of a cell's becoming useless is the using up of the zinc as a result of electrochemical actions in the cell. This allows the solution to leak out and to dry up and the cell then becomes worthless. The life of a cell may be prolonged temporarily by introducing fresh solution, but the results are usually far from satisfactory.

As is well known, dry cells have many applications. Their field is limited to supplying moderate currents intermittently, but they are capable of supplying very small currents of the magnitude of 0.1 amp. continuously. They are used extensively for door bells, electric bells, buzzers, telephones, telegraph instruments, gas engine ignition, flash lamps, and for many other purposes.

STORAGE BATTERIES

91. Storage Batteries.—A storage or secondary cell (sometimes called an accumulator) involves the same principles as a primary cell, but the two differ from each other in the manner in which they are renewed. The materials of a primary cell which are used up in the process of delivering current are replaced by new materials, whereas, in the storage cell, the cell materials are restored to their initial condition by sending a current through the cell in a reverse direction. For this reason the electrochemical products resulting from the discharge of such a cell must remain within the cell. Therefore if a cell in its operation gives off material, usually in the form of gases, so that it cannot be brought back to its original condition with a reverse current, it is not suitable for a storage cell. For example, the Le Clanché cell gives off free ammonia gas and therefore cannot be used as a storage cell. The Daniell and gravity cells are both reversible and hecnc are theoretically capable of being used as storage cells; but as the active materials go into solution and do not all return during the reverse cycle, the life of such a cell would be limited. There are but two forms of storage cells in common use, the *lead-lead-acid* type and the *nickel-iron-alkali* type. In both of these cells the active materials do not leave the electrodes.

92. The Lead Cell.—The principle underlying the lead cell may be illustrated by the following simple experiment. Two plain lead strips (Fig. 89) are immersed in a glass of dilute sulphuric acid. These are connected in series with an incandescent lamp supplied from 115-volt direct current mains, or from a battery. When current flows through this cell bubbles of gas

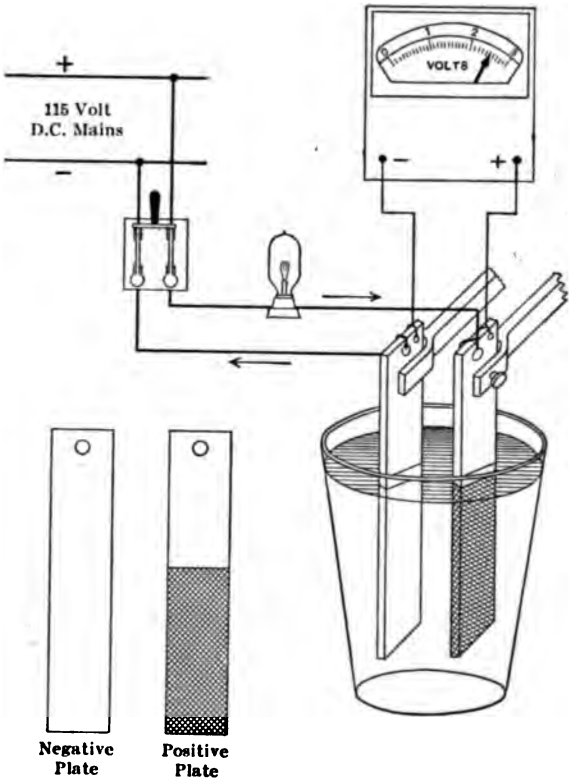


FIG. 89.—Forming the plates of an elementary lead storage cell.

will be given off from each plate, but it will be found that a much greater number come from one plate than the other. After a short time one plate will be observed to have changed to a dark chocolate color, and the other apparently will not have changed its appearance. A careful examination, however, will show that the metallic lead at the surface of the latter plate has started to change from solid metallic lead to spongy lead.

7

When the current is flowing as shown in Fig. 89 the voltmeter connected across the cell will register about 2.5 volts. If the current be interrupted by pulling the switch the voltmeter reading will fall to about 2.1 volts, and the cell will now be found to be capable of delivering a small current. This current is of sufficient magnitude to operate a small buzzer for a very short period, but the amount of energy that such a cell can deliver is very limited; even the small current taken by the voltmeter is sufficient to exhaust the cell in a very short time. As the cell **discharges** the voltage drops off slowly to about 1.75 volts, after which it drops more rapidly until it becomes zero and the cell is apparently exhausted. The color of the dark brown plate will now have become lighter and will more nearly resemble its initial lead color. After a short rest the cell will recover slightly and will again deliver current for a very brief period.

The plate which is a dark chocolate color in the above experiment is the positive plate or cathode and the one which is partially converted to spongy lead is the negative plate or anode. The bubbles which were noted come mostly from the negative plate and are free hydrogen gas. When the current is passed through such a cell the metallic lead of the positive plate becomes converted into lead peroxide, whereas the negative plate is not changed chemically, but is converted from solid lead into the spongy form which is softer and more porous than ordinary metallic lead. When the cell is discharged the lead peroxide of the positive plate is changed to lead sulphate and the spongy lead of the negative plate becomes a sulphate so that they both tend to become electrochemically equivalent.

The principle of the cell is the same as that of the primary cell. When the two lead plates are the same electrochemically, that is, when both are lead sulphate, no current flows. When the positive is converted to the peroxide and the negative to spongy lead by the action of an electric current, the two plates become dissimilar and an electromotive force exists between them. This electromotive force is about 2.1 volts, the excess of 0.4 volt observed in charging the cell being necessary to overcome the internal resistance and polarization effects. This simple experiment illustrates the principle underlying the operation of lead storage cells.

The chemical reactions which take place in a storage cell are as follows:

Battery discharged				Battery charged		
(+ plate)	(− plate)			(+ plate)	(− plate)	
$PbSO_4$ +	$PbSO_4$	+ $2H_2O$	$\leftrightarrows$	PbO_2	+ Pb +	$2H_2SO_4$
Lead sulphate +	lead sulphate	+ water	is changed to	Lead peroxide	+ lead +	sulphuric acid

The above equation shows the changes that occur when the battery is charged. The reverse takes place on discharge. It will be noted that when the battery is being charged the only change that takes place in the electrolyte is that water is converted into sulphuric acid. This accounts for the rise of specific gravity on charge. On discharge the sulphuric acid is dissociated, and reacts with the lead peroxide to form water. Therefore the specific gravity of the electrolyte decreases when the cell is discharging. When charging, free hydrogen is given off at the negative plate and oxygen at the positive plate. Because of the explosive nature of hydrogen, no flame should be allowed to come in proximity to a storage battery, when it is charging.

It would not be practicable to construct storage cells of plain lead sheets such as were used in this experiment. The current capacity of the cell would be so small that the cell could not deliver currents of commercial value for any length of time, unless the cell were made prohibitively large in order to secure the necessary plate area.

If the charging of the elementary cell, Fig. 89, were carried further, the dark lead peroxide of the positive plate would be observed to fall off in flakes and drop to the bottom of the tumbler. Therefore in a commercial cell provision must be made to minimize this flaking of the active material.

It was recognized very early that in order to make the storage cell commercial, a large plate area must be exposed to the action of the acid and a large amount of the lead must be converted into the peroxide and so become active material. There are two methods of obtaining this result, the Planté process and the Faure process. In the Planté process the active material on the plates is formed from the metallic lead by passing a current through the cell first in one direction and then in the reverse direction, which procedure works the lead on the surface of the plates into active

material. This process is slow but may be accelerated by adding certain acids to the sulphuric acid during the forming process. The Gould plate shown in Fig. 90 is made by this process. The plate is first passed under revolving steel wheels which convert its surface into ridges and furrows, increasing the surface area of the plate. As this process weakens the plate mechanically, certain portions of it are not acted upon by the wheels. These portions act as ribs which give support and mechanical strength to the plate and tend to prevent buckling. The active material is then formed electrically by the Planté process. The negative plate is made from the positive by reducing the peroxide to spongy lead by an electric current.

FIG. 90.—Gould ploughed plate, Planté process.

Another type of Planté plate, the Manchester type, is shown in Fig. 91. A grid made of lead and antimony is perforated. The active material consists of a corrugated lead ribbon, which is coiled into spirals and pressed into the perforations of the grid. The peroxide has a greater volume than the lead from which it is derived. Therefore when the cell is charged, these spirals expand and become more firmly embedded in the plate. The grid itself is not acted upon to any great extent, but serves as a

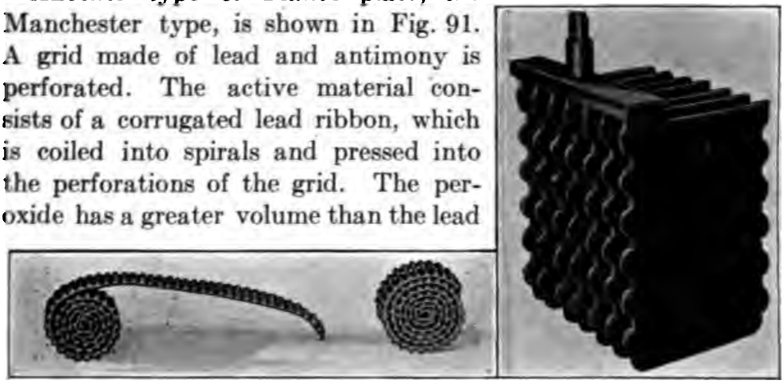


FIG. 91.—Planté (Manchester) positive group and button.

mechanical support. The advantage of this type of plate is its rigidity and mechanical strength. It has less overload capacity than other types and possibly the life is slightly less. The ordinary Planté positive, if properly cared for, should be good for from 1,800 to 2,400 complete cycles of charge and discharge. The negative should have about 25 per cent. greater life than this.

93. Faure or Pasted Plate.—This type of plate consists of a lead-antimony lattice work or skeleton into which lead oxide is

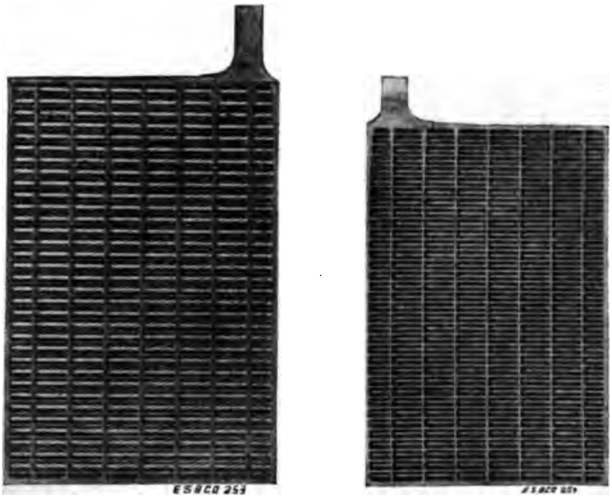


Fig. 92.—Pasted positive and negative plates.

applied in the form of a paste. The battery is then charged. The paste on the positive grid is converted into peroxide and that on the negative grid into spongy lead. Various types of pasted plates are shown in Fig. 92.

The chief advantage of the pasted plate is its high overload capacity, especially for short periods, together with its lesser size, cost, and weight for a given discharge rate. It is therefore very useful where lightness and compactness are necessary, such as in electrical vehicle batteries, ignition and starting batteries for gasoline cars, etc. The pasted type of positive has a much shorter life than the Planté type, due to a more rapid shedding of the active material. This life is approximately one-fourth that

of the Planté plates. Cells having a pasted plate for the negative and a Planté positive are common.

In all batteries there is one more negative than positive plate. This allows all the positives to be worked on *both* sides. Were any of the positives to be worked on one side only, the expansion of the active material, which occurs when it is converted to the peroxide on charge, would be unequal on the two sides of the plate and buckling would result.

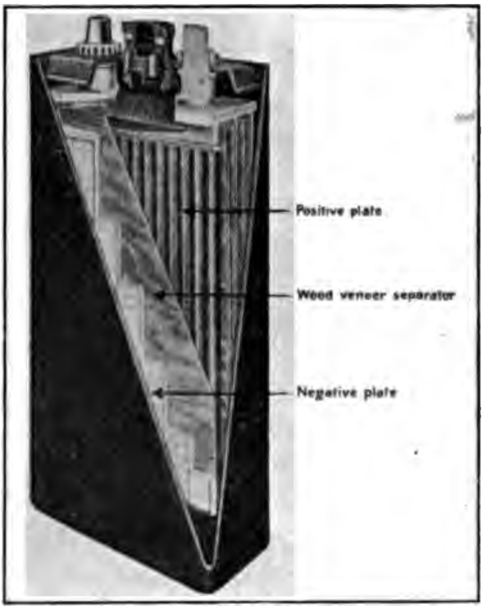


FIG. 93.—Cut away of an Iron-clad Exide cell.

"*Iron-clad Exide.*"—In order to overcome the erosion of active material from the positive plate the iron-clad exide has been developed. Its positive consists of a lead-antimony frame which supports a number of perforated hard rubber tubes. An irregular lead-antimony core passes through the center of each tube and serves as a collecting device for the current. The peroxide is pressed into the tubes, filling the space between the core and the inner wall of the tube. The perforations are so small that the peroxide does not drop out readily. An ordinary pasted plate is used for the negative plate of this cell. Although expensive,

this type of cell has a long life and can stand considerable rough usage. It is used principally to operate electric vehicles. A view of an iron-clad exide, cut away to show the assembly, is given in Fig. 93.

Storage batteries are divided into two general classes, *stationary batteries* and *portable batteries.*

94. Stationary Batteries.—The plates of this type of battery may be either of the Planté type or of the pasted type, depending on the nature of the service. For merely regulating duty, involving only moderate, though continual, charging and discharging, the Planté plate is preferable. Where a battery is installed for emergency service, to carry an enormous overload for a very short period during a temporary shut-down of the generating apparatus, the Faure or pasted plate is preferable. For a given floor area the pasted plate can discharge twice the current that the Planté plate can at the one-hour rate, and at less than the one-hour rate this ratio becomes greater. This is a very important factor in congested city districts where such batteries are usually located and where floor area is very valuable.

95. Tanks.—The containing tanks are of three general types: glass, earthenware, and lead-lined wooden tanks. Glass jars are used only for cells of small capacity, as they are expensive and have not the requisite mechanical strength in the larger sizes. Earthenware tanks have been used more as an experiment and will probably not come into general use. The wooden tanks must be strong and well made. They are lined with sheet lead. The seams of the lead lining must be sealed by burning the lead with a non-oxidizing flame. Solder should never be used. The wood should be painted with an acid-resisting paint, such as asphaltum. An occasional application of linseed oil will prevent decomposition due to the acid.

When glass jars are used, the plates are suspended by projecting lugs which rest on the edges of the jar. (See Fig. 100.) In the lead-lined tanks, the plates are similarly suspended upon two glass slabs, 3/8 in. thick, which rest on the bottom of the tank. (See Fig. 94.) The plates of like polarity are burned to a heavy lead strip or bus-bar to which the current-carrying lead is either burned or bolted. There should always be a liberal space between the plates and the bottom of the tank to allow the red lead

peroxide to accumulate without short-circuiting the plates. All types of stationary batteries should have a glass cover to reduce evaporation and to intercept the fine acid spray which occurs during the charging period.

96. Separators.—To prevent the positive and negative plates from coming in contact with one another, several types of separators have been tried. Very thin perforated hard rubber is still in use for small cells, but this is unsuitable for larger cells as the limited area of the perforations offers too much resistance to the passage of the current to the active material. Glass rods have

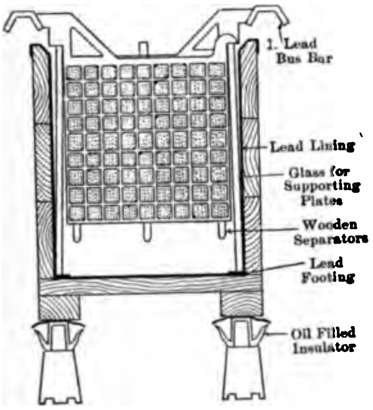


FIG. 94.—Lead-lined wooden tank storage cell.

FIG. 95.—Assembly of a wooden separator.

been suspended between the plates, but these are unsatisfactory because there is still opportunity for bits of peroxide dropping from the positive plate to lodge between the plates and cause a short circuit. Moreover, the rods are not a complete barrier between plates so that the expansion of the active material on either the positive or the negative plate may cause a short circuit. The most satisfactory separators are made of wood. These are very thin and are grooved vertically to permit the circulation of the electrolyte. They are specially treated to remove ingredients that would be detrimental to the electrolyte. The wood, after being treated, is not attacked by the acid. These separators should never be allowed to become dry, as they then decompose very readily. After being received, they should be kept wet

until installed. In larger sizes of batteries the separators are held in place by dowel pins. (See Fig. 95.)

97. Electrolyte.—The electrolyte should be chemically pure sulphuric acid. When fully charged the specific gravity should be 1.210 for Planté plates and not higher than 1.300 for pasted plates. This solution may be made from concentrated acid (oil of vitriol sp. gr. 1.84) by *pouring the acid into water* in the following ratios:

Parts Water to One Part Acid

Specific gravity	Volume	Weight
1.200	4.3	2.4
1.210	4.0	2.2
1.240	3.4	1.9
1.280	2.75	1.5

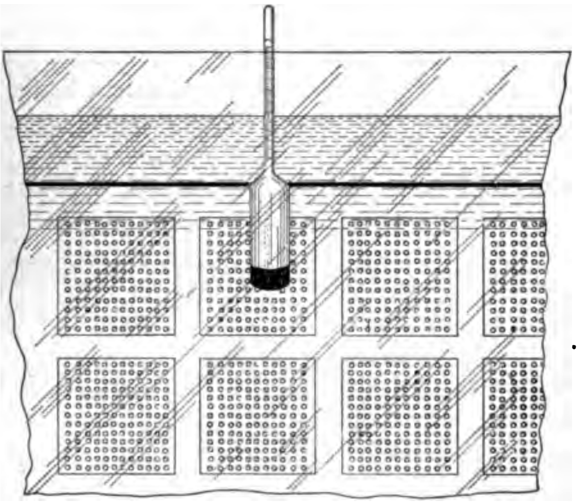


Fig. 96.—Measurement of specific gravity in a stationary battery.

A large amount of heat is evolved when acid and water are mixed. This results in a large amount of steam being generated if the water is added to the acid. This should be avoided as it may scatter the acid, break the container and even cause personal injury.

The specific gravity of a solution may be determined directly by the use of a hydrometer. This consists of a weighted bulb and a graduated tube which floats in the liquid as shown in Fig. 96. The bulb floats in the liquid whose specific gravity is to be

measured, and the specific gravity is read at the point where the surface of the liquid intercepts the tube. Such a tube may be left floating permanently in stationary batteries in a representative cell called a *pilot cell* (Fig. 96).

The small amount of liquid and the inaccessibility of vehicle and starting batteries make the use of such a hydrometer impossible. To determine the specific gravity with such batteries,

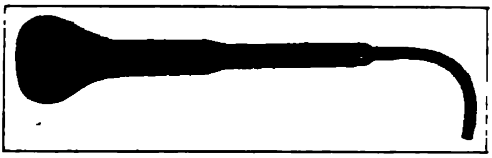


FIG. 97.—Syringe hydrometer.

the syringe hydrometer shown in Fig. 97 is used. The syringe contains a small hydrometer and when sufficient liquid is drawn into the syringe tube, the small hydrometer floats and may be read directly.

Fig. 98 shows the change in specific gravity during charge and discharge. This relation is very important, as the specific

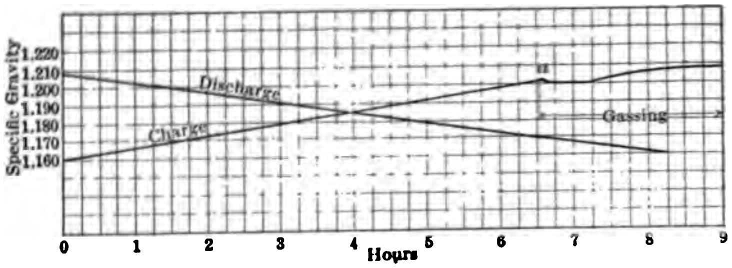


FIG. 98.—Variation of specific gravity in a stationary battery.

gravity of the electrolyte is an accurate indication of the condition of charge of the battery.

98. Specific Gravity.—When the battery is charged, hydrogen is given off at the negative plate and oxygen is given to the positive plate to convert it into the peroxide. The electrolyte gives up water, which means that the solution becomes more and more concentrated. The specific gravity will rise from the complete discharge value of 1.160 to 1.210 when fully charged as

shown in Fig. 98. Point *a* is called the gassing point because it is the point at which hydrogen gas is given off rapidly. Here the specific gravity drops off slightly due to the presence of the hydrogen bubbles in the electrolyte. After the charging has ceased the specific gravity continues to rise for some time. This is due to the very concentrated acid in the pores of the active material working out into the solution and also to the fact that the hydrogen bubbles have escaped from the solution. The discharge curve shown in Fig. 98 is very similar to the charge curve. The specific gravity will be found to drop even after the battery has ceased to deliver current. This is now due to the dilute acid in the pores of the active material passing out into the solution. The specific gravity is such a good indicator of the state of charge of the battery that the hydrometer reading is generally used to determine how nearly charged or discharged the battery may be.

As the hydrogen and the oxygen gas which escape from the battery during the charging and the discharging periods are only dissociated water, the battery loses nothing but the equivalent of water. Ordinarily, therefore, nothing but water need be added to replace the electrolyte. A small amount of the acid is carried away as a spray by the gas bubbles, but this loss is rarely of appreciable magnitude. Acid need only be added when an actual loss of electrolyte takes place, such as occurs with a leaky tank. Distilled water is used, as a rule, to replace the evaporation of the electrolyte. If any doubt exists as to the suitability of local water, the battery companies upon receipt of a sample will analyze the water and report upon the matter without charge.

99. Installing and Removing from Service.—The plates, tanks, electrolyte and containers of stationary batteries are packed separately when shipped. When received the separators should be placed immediately where they may be kept wet. The jars should be set in sand trays which rest on insulators as shown in Fig. 100. The plates should be handled carefully and placed in the jars in the manner shown in Fig. 99. The separators should be carefully slid into position as shown in Fig. 100. As the active material on the plates is more or less converted into lead salts during exposure to the atmosphere, these salts must

be reduced electrically before the battery is ready for service. Therefore the battery should be given an initial charge at the normal charging rate for about 40 hours or more.

If the battery stands over a long period without being used, the active material becomes more or less converted into inactive lead sulphate, which is a non-conductor, and so is difficult to reduce electrically. Therefore a battery if idle should be charged occasionally. If the battery is to remain idle over a long period

Fig. 99.—Lowering plates into position.

Fig. 100.—Stationary battery in position.

and it is impracticable to charge it periodically the following procedure is necessary to prevent sulphation. Give the battery a full charge, then siphon off the electrolyte, which may be saved and again used. Fill the cells with water and allow them to stand 12 to 15 hours. Siphon off the water and the cells will stand indefinitely without injury to the plates. To put back in service, fill the battery with the electrolyte having a specific gravity of 1.210 and charge for 35 hours or more at the normal rate or its equivalent.

100. Vehicle Batteries.—In the design of batteries for propelling vehicles and for automobile starting it is necessary to

obtain a very high discharge rate with minimum weight and size. Therefore pasted plates are used for both positives and negatives. These are made extremely thin and are insulated from one another by very thin wooden separators. They are then packed tightly into a hard rubber jar as shown in Fig. 101. This jar is sealed in with an asphaltum compound to prevent the liquid splashing out. There is a hole in the top of the jar

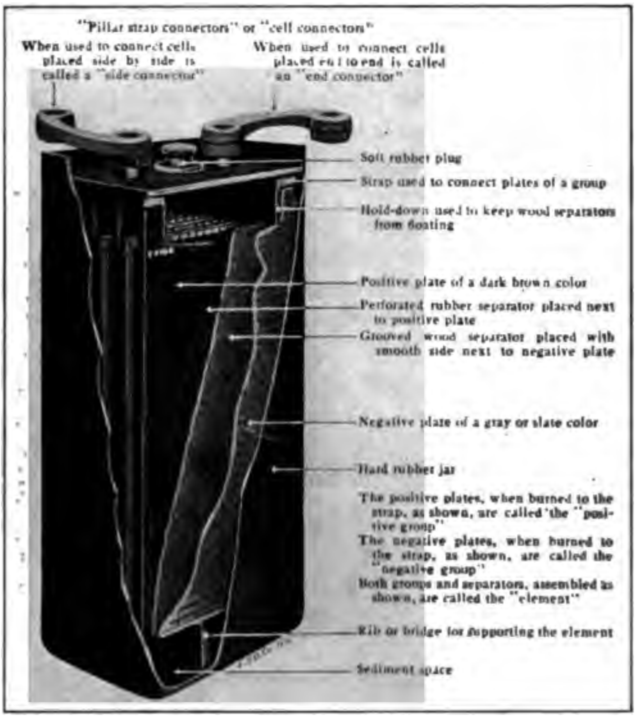


FIG. 101.—Assembly of an Exide vehicle cell.

which is closed with a cap. This permits the replenishing of the electrolyte and a vent in the cap allows the gases to escape. Because of the high discharge rates which occur where this type of battery starts a gasoline engine, and because of the necessity for a high ampere capacity for the weight, the specific gravity of the electrolyte is as high as 1.280 and 1.300. Further, the amount of electrolyte is very small and therefore it is necessary to work it between wide limits, the lower limit being 1.185 and the upper 1.280 and 1.300.

The individual cells are mounted beside one another in boxes or crates and are connected together on top by lead connectors which may be burend or held by lead nuts. The number of cells in such a unit depends upon the voltage which is desired.

Vehicle batteries are usually shipped assembled, charged and complete with the electrolyte so that they are ready for use when received. However, a preliminary charge is advisable.

Because of its ruggedness, the "Iron-clad Exide" (see par. 93) is used to a large extent in electric vehicles.

As the space for the electrolyte is very limited in vehicle batteries, the level of the electrolyte falls quite rapidly, so that frequent additions of water are necessary.

101. Rating of Batteries.—Practically all batteries have a nominal rating based on the 8-hour rate of discharge. Thus, if a Planté battery can deliver a current of 40 amp. continuously for 8 hours, the battery will have a rating of $40 \times 8 = 320$ ampere-hours. The normal charging rate of such a battery would be 40 amp. Although the above battery is just capable of delivering 40 amp. for 8 hours, it would not be able to deliver 64 amp. for 5 hours (= 320 ampere hours) but only 88 per cent. of this or 56.4 amp. for 5 hours. 56.4 amp. is called the 5-hour rate.

Below is given a table showing the percetnage capacity with various discharge rates.

Discharge rate, hours	8	5	3	1
Percentage of capacity at 8-hour rate:				
Planté type	100	88	75	50
Pasted type	100	93	83	60

This falling off in capacity with higher rates of discharge is due to the inability of the free solution to penetrate the pores of the active material. Consequently it is not possible to reduce all the active material during the short periods of discharge. After such a battery has stood a short time it will be found to have recovered to some extent and is therefore capable of delivering more current, after apparently having become exhausted. This is due to the free solution finally penetrating the pores of the active material.

Batteries are able to discharge at enormous rates for very short intervals. For instance, a starting battery having an 8-hour

rating of 10 amp. is often called upon to supply 450 amp. when doing starting duty.

102. Charging.—There are two general methods of charging a battery, the constant current method and the constant potential method. In the constant current method the current is kept at its nominal 5-hour or 8-hour value until the gassing period begins. (See Fig. 98.) If the plates are of the pasted type the current should be reduced about one-half when gassing begins, for gassing represents a waste of energy because a considerable portion of the charging energy is used in merely breaking up the water into hydrogen and oxygen. In addition, gassing causes the battery to become heated, the acid is carried out in a fine spray by the bubbles and active material may be carried from the plates by the mechanical agitation of the bubbles.

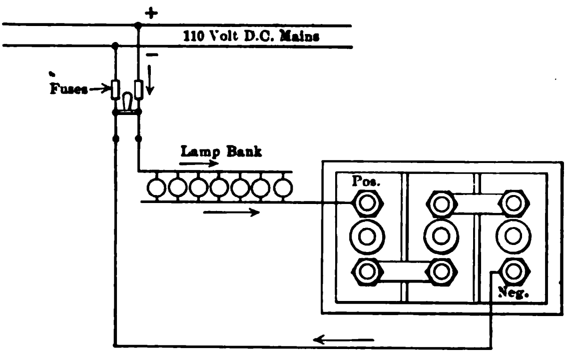


FIG. 102.—Charging a starting battery from 110-volt mains.

The charging rate with Planté plates is much in excess of the above. The charge may be started at the 3-hour rate and ended at not less than the 8-hour rate.

A common example of the constant current rate is the charging of low voltage batteries from 110-volt mains. This is illustrated by Fig. 102, which shows the charging of a 6-volt starting battery. It should be definitely determined that the mains supply *direct* current and it is also necessary to know which main is positive. If doubt exists as to the polarity and a voltmeter is not available, dip the two ends of the wires which connect the mains to the battery into a glass of slightly acidulated water or in salt water. Bubbles form about the *negative* wire. When using

the constant current method of charging one must reduce the charging rate as the battery approaches the fully charged condition.

The constant potential method of charging is to be preferred as the charging current automatically tapers off due to the rise in the cell electromotive force as the cell approaches the charged condition. The source of potential should be about 2.3 volts per cell when there is no series resistance in the circuit.

When a battery *floats* on constant potential bus-bars, ready to take load as occasion demands, it is necessary to use a series booster for raising the charging potential to a value sufficiently

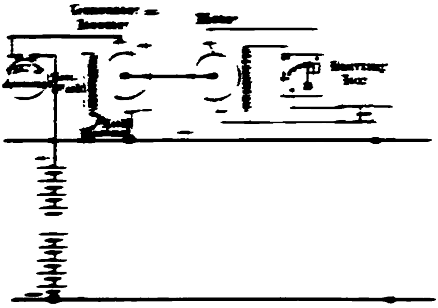


Fig. 103.—Booster method of charging a storage battery.

high to force current into the battery. The booster ordinarily consists of a low voltage, separately excited shunt generator driven by a shunt motor. Fig. 103 shows the connections of the set when the battery is being charged. The booster raises the voltage just enough to send the necessary current into the battery.

As an example, consider a 110-volt installation with a floating battery. As the average cell voltage is about 2 volts, 55 cells are necessary. Assume that the battery has a 320-ampere-hour rating. The charging current will be 320/8 or 40 amp. (the nominal 8-hour rating). The voltage of each cell should be boosted to 2.3 volts on charge. Therefore the total voltage necessary will be $2.3 \times 55 = 126.5$ volts. Of this 126.5 volts, the bus-bars can supply 110 volts. The booster supplies the remaining 16.5 volts and its rating will be

$$\frac{16.5 \times 40}{1{,}000} = 0.66 \text{ kw.}$$

The *total* power utilized in charging the battery is, however,

$$\frac{126.5 \times 40}{1,000} = 5.06 \text{ kw.}$$

The terminal voltage of a cell rises on being charged, as is shown in Fig. 104. The terminal voltage is about 2 volts at the beginning of charge and rises slowly to about 2.4 volts, after which it rises very rapidly to 2.6 volts. This last rise occurs in the gassing period. This final rise of voltage also indicates

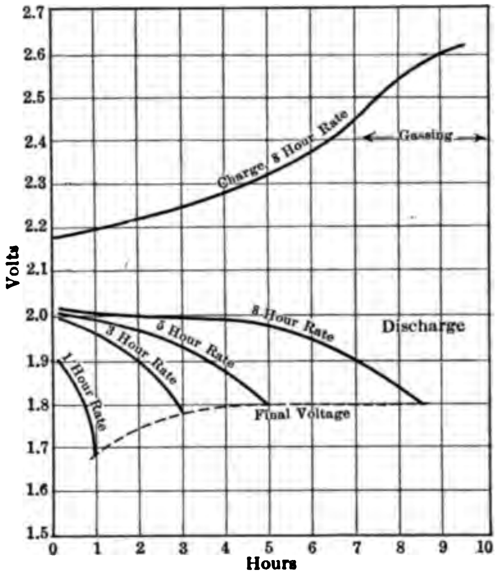


FIG. 104.—Voltage curves on charge and discharge for lead cell.

that the cell is nearing the completion of charge. It is this rise of voltage which automatically cuts down the charging rate when the constant potential method is used. The voltage does not rise so rapidly when the charging rate is reduced toward the end of charge, because of the lesser *IR* drop in the cell itself. The *drop* of voltage at various rates of discharge is shown in Fig. 104. It will be noted that the battery voltage curve at the 8-hour discharge rate is fairly flat, which is a very distinct advantage if the battery is used to supply incandescent lamps.

103. Battery Installations.—Batteries should be installed in dry, well-ventilated rooms. Small glass jars may be mounted on wooden racks painted with asphaltum paint. (See Fig. 100.) The jars are set in glass trays containing sand, which are in turn set on glass insulators. The larger battery jars should be set on porcelain pedestals 6 in. or so above the floor. The floor should be of acid-resisting tile or vitrified brick. All wooden surfaces should be covered with asphaltum paint. The room should be well ventilated, as the spray which is carried out of the jars on charge settles on horizontal surfaces and attracts other moisture. Therefore it is desirable to have a stream of air sweeping along the floor. As hydrogen gas is given off, no flame should be allowed in the room and no switches should be installed in the room. In addition to the danger due to the arcing at the switch contacts, the acid in the air will corrode the copper.

104. Temperature.—Below is given the relation between the freezing point of the electrolyte and its specific gravity. It will be noted that the freezing point is very considerably reduced with increasing values of the specific gravity, so that if a battery is well charged there is no danger of freezing in the temperate zone.

Specific gravity	Freezing temp. F.
1.180	− 6°
1.200	− 16°
1.240	− 51°
1.280	− 90°

At the higher temperatures the rate of diffusion of the acid throughout the pores of the active material is increased so that the rating of a battery increases very appreciably with increasing temperature. Above 70° this increase is of the order of from 0.5 to 1.0 per cent. per degree Fahrenheit.

105. Capacities and Weights of Lead Cells.—Below are given the relations of weights to kilowatt capacity for the various types of cells which have just been described.

KILOWATT CAPACITY (AS RELATED TO WEIGHT) OF REPRESENTATIVE CELLS
Manufactured by

THE ELECTRIC STORAGE BATTERY CO.

Size of plates	H	G	MV
No. of plates per cell	81	41	13
Type of plates	"Exide"	"Chloride Accumulator"	"Iron-clad"
Weight of plates in cell, lb.	1,988	896	26.8
*Weight of cell, lb.	3,790	1,841	40.0
Kilowatts—per cell:			
For 1 hour	10.98	2.82	0.237
For 4 hours	4.16	1.22	0.0904
For 8 hours	2.42	0.764	0.0533
Kilowatts—per lb. of plates:			
For 1 hour	0.00552	0.00314	0.00883
For 4 hours	0.00209	0.00136	0.00337
For 8 hours	0.00122	0.000853	0.00199
Kilowatts—per lb. of cell:			
For 1 hour	0.0029	0.00152	0.00592
For 4 hours	0.001096	0.000663	0.00225
For 8 hours	0.000638	0.000379	0.00138

*The necessary insulating supports for the "H" and "G" cells and tray for the "MV" cell are not included—these add approximately 2 per cent. and 10 per cent. respectively to the cell weights.

Type H "Exide" batteries used in central station stand-by service. Largest battery 150 cells, 169 plates per cell, capacity 3,460 kw. for 1 hour; two other batteries 3,420 kw. each.

Type G "Chloride Accumulator" used in power plants for peak, regulating and exciter bus service, also in telephone exchanges, large isolated plants, etc. Largest battery 288 cells, 85 plates per cell, capacity 1,700 kw. for 1 hour; two other batteries same size and capacity.

Type MV "Iron-clad Exide" used in electric vehicles, locomotives, industrial trucks and tractors, and for yacht lighting, etc.

THE EDISON BATTERY

106. The Nickel-iron-alkaline Battery.—Instead of using acid as an electrolyte, the Edison cell uses an alkali, consisting of a 21 per cent. potassium hydrate solution. The positive plate consists of nickel pencils about ¼ in. diameter and 4½ in. long, filled with green nickel oxide. As the nickel oxide is a very poor electrical conductor, very fine metallic nickel flakes are mixed with it to produce sufficient conductivity. The negative plate consists of flat perforated nickel-plated steel

stampings, containing iron in a very finely divided form. These flat pockets are mounted on a nickel-plated steel frame for support. Both the positive and the negative plates are shown in Fig. 105.

The chemical reaction in the cell is complex, but its nature is indicated by the following chemical equation:

Positive Plate *Negative Plate*

$$8KOH + 6NiO_2 + 3Fe = Fe_3O_4 + 2Ni_2O_4 + 8KOH$$

The above read from left to right indicates discharge, and read from right to left indicates charge. It is to be noted in the above

FIG. 105. — Positive and negative plates of an Edison storage cell.

FIG. 106.—Assembly, Edison battery plates removed from container.

reaction that the same quantity of potassium hydrate solution (KOH) appears on both sides of the equatio.n This indicates that ultimately all the reaction occurs between the electrodes themselves, and also that no water is formed. Therefore the specific gravity of the solution does not change during charge or discharge.

The plates all have a perforated lug by which they are fastened together with a steel bolt and to a binding post. The bolt is threaded and steel nuts clamp the plates together. Steel washers between the plates act as spacers. The positive and negative plates are insulated from one another by hard rubber grids. An Edison cell assembly is shown in Fig. 106. The positive and negative assembly is placed in a corrugated, nickel-plated,

welded steel tank. The top is then welded to the rest of the container. The binding posts are insulated from the cover by hard rubber bushings. In the top is a valve which allows the gases to escape during charging and through which water may be

FIG. 107.—Five Edison storage cells mounted in a tray.

added to the electrolyte. This valve should never be allowed to become so encrusted with a potash deposit that it sticks, because the internal pressure may become sufficient to cause the sides of the container to bulge.

The individual cells are usually mounted in wooden racks, as shown in Fig. 107, the cells being connected together by nickel-plated steel connectors.

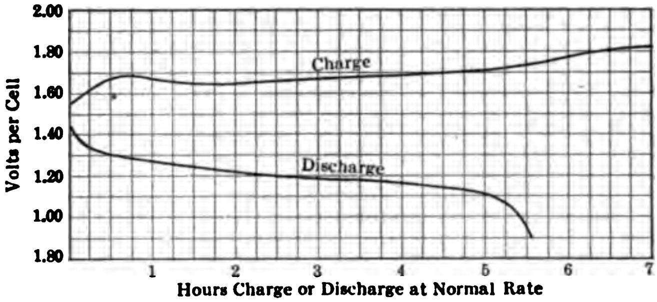


FIG. 108.—Voltage changes during the charge and discharge of an Edison cell.

107. Charging and Discharging.—The Edison cell is rated on the basis of a 5-hour charging rate. Fig. 108 shows typical charge and discharge curves for the Edison battery. It will be

noted that the average voltage on discharge is about 1.2 volts per cell. The specific gravity of the electrolyte changes but slightly so that it cannot be used to indicate the condition of charge, as with the lead cell. Moreover, there is no sharp voltage rise near the completion of charge. If doubt exists as to the condition of charge, it is advisable to give an overcharge in order to be on the safe side. The overcharge does not injure the cell although it may slightly reduce the efficiency.

The electrolyte in an Edison cell evaporates rapidly and frequent additions of water are necessary. As the electrolyte is changed to potassium carbonate very readily, only freshly distilled water should be used in replacing the electrolyte, as tap water usually contains carbonates in solution. In spite of the usual precautions, the electrolyte is slowly converted into potassium carbonate by contact with the air, and it should be replaced by fresh electrolyte every 250 complete cycles of charge and discharge.

The Edison cell has many advantages. It is light, rugged, and can stand for a long time in a discharged condition without chemical deterioration. The plates do not buckle and the active material does not "flake" or drop from the plates.

108. Applications.—Edison cells are used for vehicle lighting and ignition, and are also much used in motor boats. They are also used in various types of electric trucks and for battery street cars. In automobiles they are not generally used for starting, as their comparatively large internal resistance does not permit a sufficiently high discharge rate.

Below is given the relation between the battery weight and capacity.

The figures are based upon the capacity obtainable on normal charge:

Discharge rate, hours	Watt-hours per pound of cell	Watt-hours per pound of plates
1	9.75	18.35
4	14.95	28.15
8	16.33	30.80

109. Efficiency of Storage Batteries.—The efficiency of a storage battery is the ratio of the watt-hour output to the watt-hour input.

For example, a normally discharged cell is charged at a uni-

form rate of 40 amp. for 6 hours at an average voltage of 2.3 volts. The cell is then completely discharged at a uniform rate of 38 amp. for 6 hours, the average voltage being 1.95 volts. What is the efficiency of this cell?

$$\text{Watt-hours output} = 38 \times 1.95 \times 6 = 445$$
$$\text{Watt-hours input} = 40 \times 2.3 \times 6 = 552$$

$$\text{Efficiency} = \frac{445}{552} \text{ or } 80.7 \text{ per cent.}$$

One often hears of the ampere-hour efficiency of a storage battery. As amperes do not represent energy, the ampere-hour efficiency is not a measure of a battery's ability to store energy. In the above example the ampere-hour efficiency may be found as follows:

$$\text{Ampere-hours output} = 38 \times 6 = 228$$
$$\text{Ampere-hours input} = 40 \times 6 = 240$$

$$\text{Ampere-hour efficiency} = \frac{228}{240} \text{ or } 95 \text{ per. cent.}$$

The much lower watt-hour efficiency is due to the great difference between the voltage of charge and that of discharge, as shown in Figs. 104 and 108.

The efficiency of a storage battery varies with the rate, both of charge and discharge, and somewhat with the temperature. As high charge and discharge rates produce relatively high I^2R and polarization losses, the efficiency is lowered under these conditions. Further, a cell may be charged at the 8-hour rate and discharged at the 3-hour rate and have an apparent efficiency of 60 per cent. This does not represent the true efficiency as the cell actually will not be completely discharged, even though it appears to be. Owing to the inability of the free acid to permeate the active material, much of the active material has not been reduced, and after a short time the cell will be found to have recuperated to a considerable extent and to be able to deliver more energy.

The ampere-hour efficiency of a storage battery is of the order of magnitude of 95 per cent. For a complete cycle the watt-hour efficiency of a stationary battery of moderate size is about 80 per cent. at the 8-hour charge and discharge rates. The watt-

hour efficiency of a large stationary battery is about 85 per cent. under the same conditions. Where a battery merely "floats" and the cycle of charge and discharge is a matter of minutes or perhaps of seconds even, the watt-hour efficiency may be as high as 95 or 96 per cent.

The ampere-hour and the watt-hour efficiency for the Edison cell are less than for the lead cell. This is due partly to the fact that the Edison cell has a lower electromotive force and the *IR* drop is proportionately greater. For the Edison cell the ampere-hour efficiency is about 82 per cent. and the watt-hour efficiency about 60 per cent.

In selecting a battery, the efficiency is but one of the factors to be considered. The first costs and the maintenance of batteries are high so that these factors, as well as the efficiency, should be given due consideration.

Note.—The uses of storage batteries in the generation and distribution of power are considered in Chap. XIV.

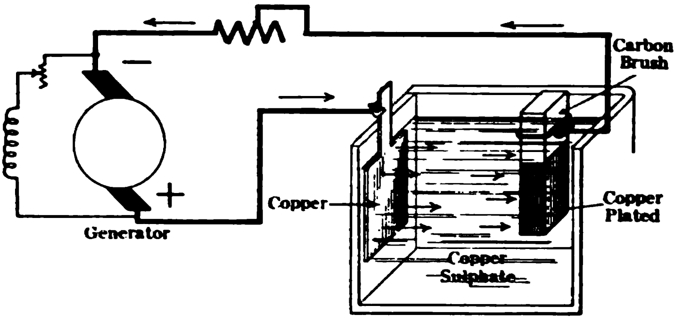


Fig. 109.—Copper plating bath.

110. Electroplating.[1]—Electroplating is a very important electrical industry and is closely related to the subject of batteries. The principle is very simple. Assume that it is desired to copper plate a carbon dynamo brush. The portions of the brush to be plated are immersed in a solution of copper sulphate as shown in Fig. 109. A copper strip is also immersed in the solution and is connected to the + terminal of a dynamo or some other source of direct current supply. The article to be plated is

[1] See "Standard Handbook," Fourth Edition, Section 19, Pars. 186 to 206, for a more complete discussion.

connected to the negative terminal of this supply. Under these conditions the current will carry copper from the solution and deposit it on the carbon brush. This copper which leaves the solution is replaced by copper which is carried from the copper strip (the anode) into the solution so that there is no change in the solution itself. The current should be such that the density is about 0.02 amp. per sq. in. of the surface to be plated.

It is not necessary that the anode be of the metal which it is desired to deposit. Other metals may be used. Under these conditions, however, the solution in time becomes contaminated by the going into solution of the anode. If an inert substance such as carbon is used, as anode, acid is formed in the solution.

The *only opposing* electromotive force in the bath just described is the IR drop in the solution. This may be reduced by bringing the electrodes close together, but if the electrodes are too close together the deposit will not be uniform. The amount of metal deposited per second is proportional to the current. Because of the nature of electroplating baths, they are naturally low voltage devices. When practicable, several are connected in series. A low voltage and high current generator is generally used for plating purposes. In practice there are many refinements to be observed.

Acid is added to the solution to prevent impurities from depositing. A cyanide solution of copper is found to give better results than the sulphate. Nickel, tin, zinc, silver, gold, etc., may be deposited by the use of suitable baths and electrodes.

A gravity cell is an example of electroplating in which the source of current is derived from the bath itself. The current flows from the zinc to the copper within the solution, zinc is carried into the solution as sulphate and copper is deposited or plated from its sulphate on the positive electrode.

Electrotyping is another common example of electroplating. An impression is made in wax with the type or object to be reproduced. The surface of the wax is made conducting by applying a thin coat of graphite. Copper is then plated on this surface. It is later backed by type metal to give it the necessary mechanical strength.

CHAPTER VII

ELECTRICAL INSTRUMENTS AND ELECTRICAL MEASUREMENTS

111. Principle of Direct Current Instruments.—If a coil like that shown in Fig. 110 carries a current, a magnetic field results (Chap. II) with a north and a south pole at opposite ends of the coil. If the coil carrying current be placed in a magentic field, the coil will tend to turn in such a direction that:

The resulting magnetic field due to both the main field and that of the coil will be a maximum (see Par. 17, Chap. I), and the north

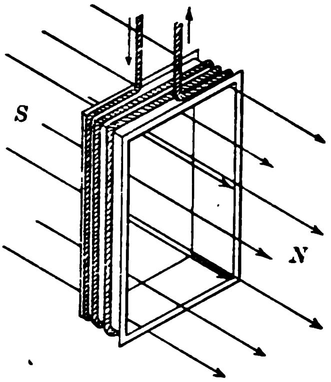


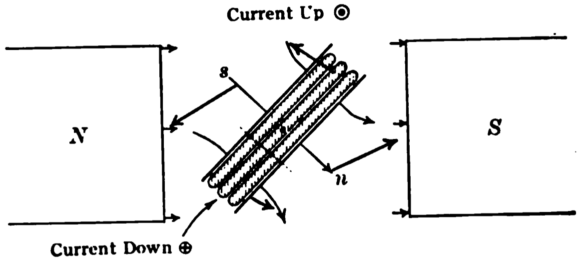
Fig. 110.—Magnetic field produced by an instrument coil.

pole of the coil will be attracted toward the south pole of the magnetic field and the south pole of the coil will be attracted to the north pole of the magnetic field.

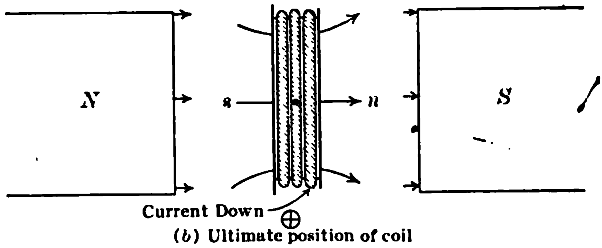
This tendency of the coil to turn is shown in Fig. 111 (*a*) where the coil attempts to turn in the direction indicated by the arrows. If the coil is pivoted and free to turn it will reach the position shown in Fig. 111 (*b*). Under these conditions the coil has placed itself in such a position that its flux is acting in the same

direction as that of the main field. Also the unlike poles are as near each other as possible and the like poles are as far away from each other as possible.

This behavior of a coil carrying a current and placed in a magnetic field should be thoroughly understood, for it is the underlying principle of most current measuring instruments and is in addition the principle upon which all electric motors operate.



(*a*) Coil tending to turn in a magnetic field



(*b*) Ultimate position of coil

Fig. 111.—Turning moment of an instrument coil.

112. The D'Arsonval Galvanometer.—A galvanometer is a sensitive instrument used for measuring and detecting small electric currents. The D'Arsonval galvanometer, which is based on the principle of a coil turning in a magnetic field, is the most common type of galvanometer. Due to its simplicity it has superseded practically all other types. In addition it is comparatively rugged and is not appreciably affected by stray magnetic fields. Fig. 112 shows the principle of its construction. A coil of very fine wire is suspended between the poles of a permanent magnet by means of a filament, usually a flat strip

of phosphor-bronze. The coil may be wound with or without a bobbin. The bobbin is usually of fiber, or of aluminum. The advantage of an aluminum bobbin will be considered later.

Between the poles of a magnet a soft iron core is usually placed (Fig. 112 and Fig. 113). The addition of this core results in two distinct advantages. The length of the air path is reduced so that the amount of flux linking the coil is increased, thus making the galvanometer more sensitive; the flux tends to enter the core radially. This last effect makes the deflections of the galvanometer almost directly proportional to the current flowing in the galvanometer coil.

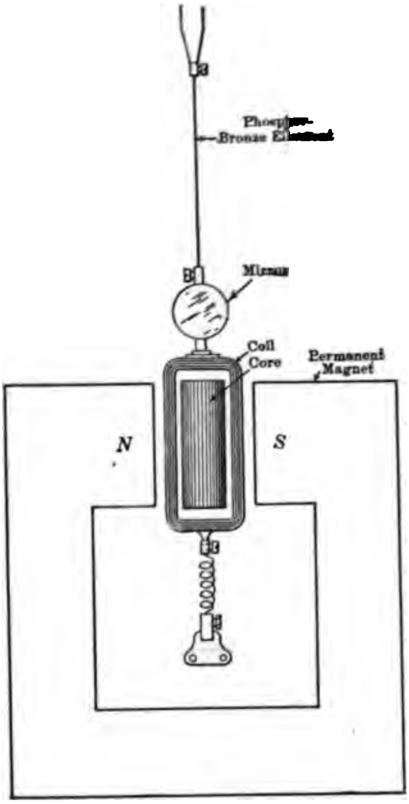


Fig. 112.—Principle of the D'Arsonval galvanometer.

The coil is usually suspended by a phosphor-bronze filament. Any turning of the coil produces torsion in the filament which opposes the turning of the coil and is called the restoring force. When the moment of the restoring force and the turning moment due to the current are equal, the galvanometer assumes a steady deflection. For all practical purposes the galvanometer deflection is proportional to the current. This phosphor-bronze filament usually serves as one of the leading in wires carrying current to the coil. The other leading in wire consists of a very flexible spiral filament fastened to the bottom of the coil, as shown in Fig. 112.

There are two common methods of reading the deflection of a galvanometer. A plane mirror is mounted on the coil system

and a scale and telescope are mounted about ½ m. from the galvanometer. The reflection of the scale in the mirror can be seen with the telescope (Fig. 114). When the mirror turns, the reflection of the scale in the mirror deflects. The

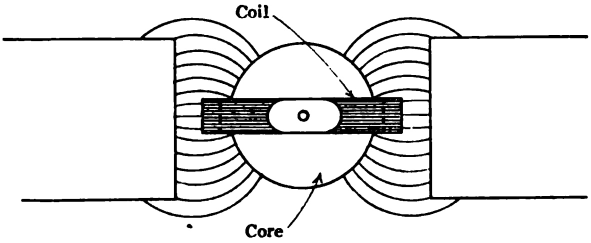


FIG. 113.—Effect of core upon the magnetic field of a galvanometer.

value of this deflection is determined by means of a cross hair in the telescope.

Another method is to use a concave mirror on the galvanometer moving system. A lamp filament is placed some distance from the mirror and its image focused on a ground glass to

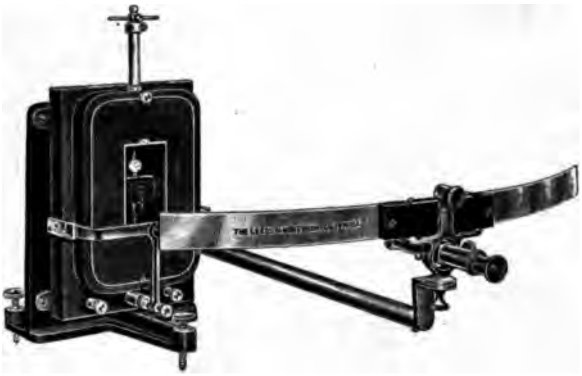


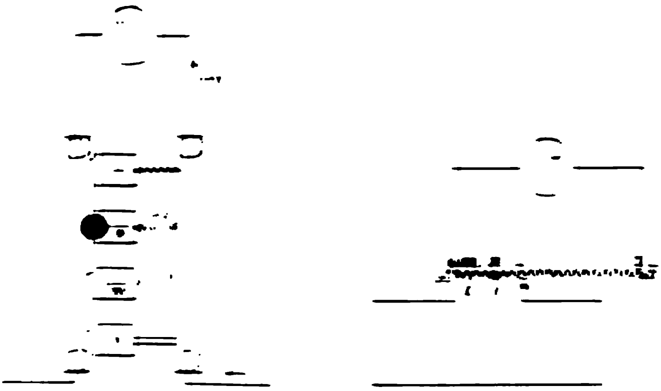
FIG. 114.—Telescope and scale method of reading a galvanometer.

which a scale graduated in centimeters is fastened. As the mirror deflects the beam of light travels across the scale.

Damping.—If a galvanometer coil, which is hung freely, starts to swing, it will continue swinging for some time unless it is in some way retarded or damped. One method of damping is to attach an air vane to the coil. This air vane is enclosed so

that it swings in a restricted space and damps any swinging movement of the coil. The most satisfactory method is electrical damping. If the coil is wound on an aluminum bobbin, the motion of the bobbin through the magnetic field will induce currents within itself, and these will be in such a direction as to put an electric load on the moving coil as in an electric generator. This opposes the motion of the coil. The same result may be obtained by winding short-circuited copper coils on the main coil, by shunting the galvanometer externally with a resistance (see [illegible] shunt) or even by short-circuiting it.

223. Galvanometer Shunts.—When galvanometers are used to detect small currents as in null methods (see Wheatstone



(a) Galvanometer shunt (b) Ayrton shunt

Fig. [illegible].—Types of galvanometer shunt.

Bridge) the apparatus may be so far out of adjustment that a [illegible] large current [illegible] through the galvanometer. This [illegible] and may result in injury to the galvanometer. In certain other measurements the current [illegible] with the galvanometer [illegible] beyond the scale.

[illegible] protected by the [illegible] certain known [illegible] There are [illegible] Fig. 115 a, [illegible]

It consists of three or four separate resistances which are plugged across the galvanometer one at a time. These are so adjusted in value that with a given current to be measured the successive galvanometer currents are in the ratio of 10 to 1. For instance, if the galvanometer is to measure 1/10 the external current, the top resistance, Fig. 115 (*a*), is of such a value that it shunts 9/10 of the current away from the galvanometer when it is plugged across the galvanometer, etc.

To determine the values of these resistances proceed as follows:

Let R_g = galvanometer resistance.
I_g = galvanometer current for full scale deflection.
I = circuit current.
I_s = shunt current.
R_s = shunt resistance.

To reduce the galvanometer deflection to one-tenth the value which it would have if all of the current I passed through the galvanometer, I_g must be one-tenth of I. That is,

$$\frac{I_g}{I} = \frac{1}{10} \tag{1}$$

The shunt current

$$I_s = I - I_g \tag{2}$$

But the shunt current and the galvanometer current are inversely as their respective resistances. Hence:

$$\frac{R_g}{R_s} = \frac{I_s}{I_g} = \frac{I - I_g}{I_g} \tag{3}$$

as $$I_g = \frac{I}{10} \quad \text{from (1)}$$

$$\frac{R_g}{R_s} = \frac{I - I/10}{I/10} = 9$$

$$R_s = \frac{1}{9} R_g$$

For a reduction of 100 to 1

$$\frac{R_g}{R_s} = \frac{I - I/100}{I/100} = 99$$

$$R_s = \frac{1}{99} R_g$$

Example.—A galvanometer has a resistance of 600 ohms. What resistances should shunt it in order to reduce its deflections in the ratio of 10 to 1 and 100 to 1?

$$R_s = \frac{600}{9} = 66.7 \text{ ohms. } Ans.$$

$$R_s = \frac{600}{99} = 6.06 \text{ ohms. } Ans.$$

Ayrton Shunt.—The *Ayrton shunt* is shown in Fig. 115 (*b*). A permanent resistance *AB* is connected across the galvanometer terminals. One line terminal is permanently connected to one end of this resistance, and the other line terminal, *C*, is movable and can be connected to various points along *AB*. With a fixed line current the maximum deflection is obtained when *C* is at *B*. If point *C* be moved to *a*, where resistance *Aa* is 1/1000 the total resistance *AB*, the galvanometer deflection will be 1/1000 of its maximum value. If *C* be moved to *b*, where *Ab* is 1/100 of the resistance *AB*, the galvanometer deflection will be 1/100 of its maximum value, etc.

The advantages of the Ayrton shunt are:

(1) A shunt is applicable to any galvanometer, regardless of the galvanometer resistance.

(2) A fixed resistance is shunted across the galvanometer, which gives a constant value of damping in open circuit ballistic measurements. (See par. 159.) When the shunt is adjusted to give the maximum galvanometer deflection, this deflection for the same value of external current is less than it would be were the shunt not used. That is, the maximum sensitivity of the galvanometer is reduced by the addition of the shunt. If the shunt has a resistance of only 5 times that of the galvanometer the sensitivity will be reduced only in the ratio of 6 to 5, which is not usually objectionable.

114. Ammeters.—An ammeter is an electrical instrument which measures the current flowing in an electric circuit.

There were many early types of ammeters, most of which depended for their operation upon the pull exerted by a solenoid on some type of iron plunger. The amount of pull is dependent upon the current strength in the solenoid, so by restraining the motion of the plunger by gravity or by means of a spring, the deflection of a pointer attached to the plunger might be made to read amperes. Fig. 116 shows a typical instrument of

this class. Such an instrument is inaccurate, due: (1) To magnetic hysteresis or lag, which for a given current results in a higher reading for decreasing values of current than for increasing values. (2) The weight of the plunger makes it impossible to mount the moving system so that the friction error is negligible. (3) The isntrument is not damped, and fluctuates violently no slightly fluctuating loads.

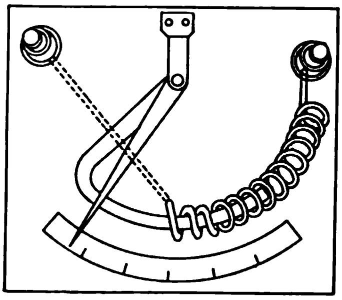


FIG. 116.—Early type of plunger ammeter.

For direct-current measurements, the Weston instrument, developed by Edward Weston, has come into almost universal use. The instrument is based on the principle of the D'Arsonval galvanometer, but it is so constructed that it is easily portable and it is provided with a pointer and scale for indicating the deflections of the moving coil.

The essential parts of the instrument are shown in Fig. 117. As in the D'Arsonval galvanometer, a permanent magnet is necessary, being made in horseshoe form. Two soft-iron pole pieces are fitted to the magnet poles and a cylindrical core is held between these pole pieces by a strip of brass. This gives a uniform air gap and a radial field. The length of the air gap is very much shorter than is usual with D'Arsonval galvanometers. The moving coil is made of very fine silk-covered copper wire wound on an aluminum bobbin. The aluminum bobbin, besides supporting the coil mechanically, also makes the instrument highly damped. This damping is due to the currents set up in the aluminum because of its cutting the magnetic field.

FIG. 117.—Movement of a Weston instrument.

Instead of suspending the coil by a filament, it is supported at the top and bottom by hardened steel pivots turning in cup-shaped jewels, usually sapphire. This method of supporting the moving coil is almost frictionless and makes the instrument portable, whereas the D'Arsonval galvanometer is not so. The current is led in and out of the coil by two flat spiral springs, one at the top of the coil and the other at the bottom. These springs also serve as the controlling device for the coil. That is, any tendency of the coil to turn is opposed by these two springs. The top and the bottom springs are coiled in opposite directions so that the effect of change of temperature, which causes a spiral spring to coil or uncoil, will not cause the needle to leave its zero position. A very light and delicate aluminum pointer is attached to the moving element to indicate the deflection of the coil. This is carefully balanced by very small counter-weights so that the whole moving element holds its zero position very closely, even if the instrument is not level. The pointer moves over a graduated scale, which may be marked in volts or in amperes as the case may be. Because of the radial field, the deflection of the moving coil in this type of instrument is practically proportional to the current in the moving coil, so that the scale of the instrument has substantially uniform graduations, which is desirable. The internal connections of a Weston instrument are shown in Fig. 118.

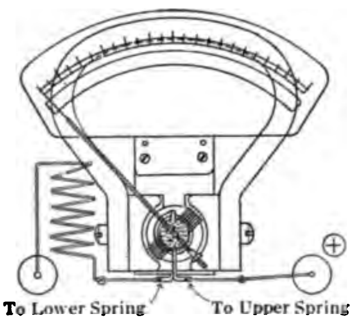


FIG. 118.—A typical Weston direct-current milli-voltmeter.


FIG. 119.—Weston portable galvanometer.

Instruments of this construction having very weak springs are often used for portable galvanometers. Although lacking the

extreme sensitivity of the suspended type, they can be made sufficiently sensitive for certain classes of work and their ruggedness and portability make them very useful. Such a galvanometer is shown in Fig. 119.

The moving coil of Weston portable instruments deflects to the full scale value with about 0.01 amp. in the coil. Therefore, to measure currents greater than this, the larger portion of the current must be diverted from the moving coil by a *shunt*. The shunt is merely a low resistance, usually made of manganin strip (M) brazed to comparatively heavy copper blocks (c) as

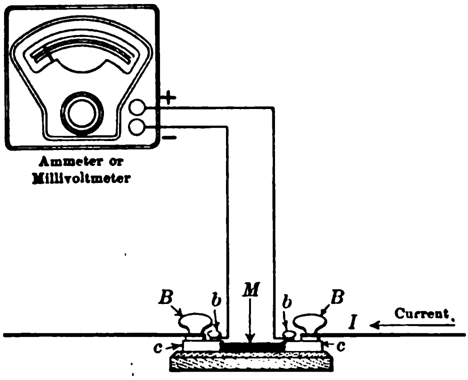


FIG. 120.—Ammeter with an external shunt.

shown in Fig. 120. Two sets of binding nuts are fastened to the copper blocks. The heavy wing nuts (BB) are for carrying the main current through the shunt. The small posts (bb) are used to connect the ammeter leads. The copper blocks serve two purposes. They are an excellent conductor of heat, so carry the heat away from the manganin strip, and their low resistance keeps all parts of each copper block at very nearly the same potential. The ammeter is in reality a voltmeter reading the voltage drop across a resistance. A complete set of shunts, with their current ratings, is shown in Fig. 121. The heavy copper terminals for connection to bus-bars should be noted.

The voltage drop across the shunt is

$$V_{sh} = I_{sh} R_{sh},$$

where I_s and R_s are the shunt current and the shunt resistance respectively. If R_s is constant, the voltage drop across the shunt is proportional to the current in the shunt, so that the instrument readings are proportional to the current in the shunt. For this reason the ammeter itself (Fig. 118) is often marked "Millivoltmeter." For full scale deflection the drop across a shunt is about 50 millivolts. The current taken by the instrument itself is usually about 0.01 amp. so that it is almost always

FIG. 121.—Ammeter shunts.
No. 1 from 25 to 200 amperes.
No. 9 from 4500 to 6000 amperes.

negligible as compared with the main current. Therefore, in most cases the line current equals the shunt current, practically.

An ammeter and its shunt may also be considered as a divided circuit. In Fig. 122 let R_s and I_s be the shunt resistance and the shunt current respectively, and let R_m and I_m be the instrument resistance and the instrument current respectively. By the law of divided circuits:

$$\frac{I_s}{I_m} = \frac{R_m}{R_s}$$

That is, the current divides between the instrument and the shunt inversely as their resistances.

Example.—Assume that an instrument has a resistance of 4 ohms, the shunt a resistance of 0.0005 ohm, and that the line current is 90 amp. What is the value of the instrument current?

As the current in the line differs from the shunt current by a very small amount, the two may be assumed equal. Then,

$$\frac{90}{I_m} = \frac{4}{0.0005}$$
$$I_m = 0.0113 \text{ amp.}$$

For accuracy, the current must always divide between the instrument and the shunt in a fixed ratio. This means either that the resistance of the shunt and the resistance of the instrument must not change at all or that both must change in the same ratio. As the shunt operates at a higher temperature than the instrument, it should be made of a metal whose resistance does not change appreciably with the temperature, such as manganin. The resistance of the instrument circuit should also remain constant. The resistance of the leads connecting the shunt to the instrument should remain constant and the leads with which the instrument is calibrated should always be used to connect the shunt to the instrument. The lugs and binding post contacts should be kept clean from oxide and dirt. A low adjustable resistance (the spiral, Fig. 118) is connected inside the instrument. By varying this resistance the instrument is adjusted to its shunt.

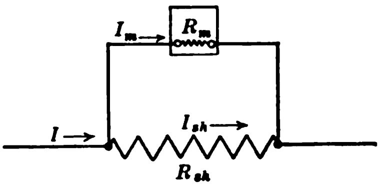


Fig. 122.—Division of current between an ammeter and its shunt.

An ammeter with an external shunt may be made to have a large number of scales or ranges. Assume in the example just given that the instrument gives full scale deflection when the instrument current is 0.0125 amp. The volts across the instrument terminals are 0.0125 × 4 = 0.050 volt or 50 millivolts. Dividing this voltage by the shunt resistance, the shunt current is

$$I = \frac{0.05}{0.0005} = 100 \text{ amp.}$$

The instrument then deflects full scale with 100 amp. in the line.

If a shunt having a resistance of 0.005 ohm be substituted, the 50 millivolts drop across the shunt may be obtained with 10 amp. ($10 \times 0.005 = 0.050$). Therefore a 10 scale ammeter results. By the choice of suitable shunts the same instrument may be made to give full scale deflection with 1 amp., and with 5,000 amp. For instance, all the shunts shown in Fig. 121 could be used with the same instrument and as many different scales thereby obtained.

In the smaller sizes of instruments up to 50 amp. and where only one scale is desired, the shunt is usually placed within the instrument. For ranges between 50 and 100 amp. the use of an internal or an external shunt is optional. Above 100 amp. it is usual to have the shunt external to the instrument on account of its size and its heating loss.

An ammeter can usually be distinguished from a voltmeter by the fact that its binding posts are heavy and are of bare metal, except in the case of an instrument having an external shunt. The posts of millivoltmeters and voltmeters are of much lighter construction and the metal posts are covered with hard rubber, mostly for insulation purposes.

Any instrument when connected in a circuit should disturb the circuit conditons as little as possible. An ammeter shunt, as it goes in series with the line, should have as low a resistance as is practicable, so that when it is connected, very little additional resistance is introduced into the circuit. To protect ammeters from heavy currents, etc., provision may be made for *short-circuiting* them when readings are not being taken.

115. Voltmeters.—The construction of a voltmeter does not differ materially from that of an ammeter in so far as the movement and magnet are concerned. (See Fig. 117.) The moving coil of the voltmeter is usually wound with more turns and of finer wire than that of the ammeter and so has a higher resistance. The principal difference, however, lies in the manner of connecting the instrument to the circuit. As a voltmeter is connected directly across the line to measure the voltage, it is desirable that the voltmeter take as little current as is practicable. Because of its comparatively low resistance, the moving coil of

the voltmeter cannot be connected directly across the line, as it would ordinarily take an excessive current and might be burnt out. Therefore it is necessary to connect a high resistance in series with the moving coil. This is shown in Fig. 123. By Ohm's Law the current through the instrument is proportional to the voltage, so that the instrument scale can be graduated in volts. The resistance required is easily determined. Assume that an instrument gives full scale deflection with 0.01 amp. in

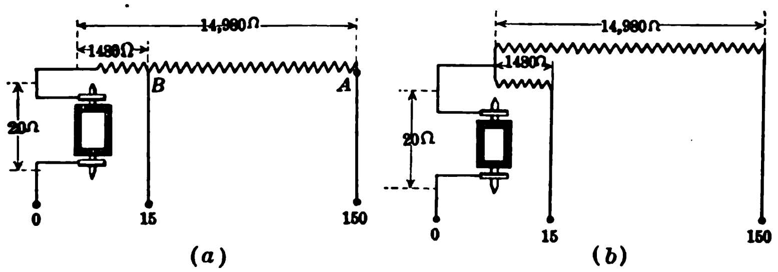


FIG. 123.—Methods of connecting resistance in a voltmeter.

the moving coil, and that the coil resistance is 20 ohms. If it is desired that the instrument indicate 150 volts, full scale, then the total resistance of the instrument circuit must be

$$R = \frac{V}{I} = \frac{150}{0.01} = 15{,}000 \text{ ohms.}$$

As the instrument has a resistance of 20 ohms, this means that 14,980 ohms additional are necessary.

If it be desired that this same instrument also have a full scale deflection with 15 volts, the resistance of 14,980 ohms may be tapped so that the resistance OB (Fig. 123 (a)) $= \frac{15}{0.01} = 1{,}500$ ohms, and this tap can be brought to a binding post. Another method of securing the same result is shown in Fig. 123 (b). Wind another resistance equal to $1{,}500 - 20 = 1{,}480$ ohms and connect it from a binding post to the junction of the resistance and the moving coil. This last method is advantageous as it permits independent adjustment of each resistance; also injury or repair in one resistance does not affect the other.

116. Multipliers or Extension Coils.—The range of a voltmeter having its resistance incorporated within the instrument, may be

increased by the use of external resistance connected in series with the instrument.

Example.—A 150 scale voltmeter has a resistance of 17,000 ohms. What external resistance should be connected in series with it so that its range is (*a*) 300 volts? (*b*) 600 volts?

(*a*) In order to maintain the same current through the instrument at 300 volts as flows at 150 volts, the resistance of the circuit must be doubled.

Therefore, $17{,}000 \times 2 = 34{,}000$ ohms are necessary.

As the instrument already has 17,000 ohms, the added resistance will be,

$$34{,}000 - 17{,}000 = 17{,}000 \text{ ohms. } \textit{Ans.}$$

(*b*) The total resistance must now be,

$$\frac{600}{150} \times 17{,}000 = 68{,}000 \text{ ohms.}$$

As 17,000 ohms is already within the instrument, 68,000 − 17,000 = 51-, 000 ohms must be added external to the instrument. *Ans.*

External resistances used in this manner are called ***multipliers***, or sometimes ***extension coils***. They are usually placed within a perforated box and the terminals brought out to binding posts. The multiplying power of the multiplier is marked near a terminal.

The equation giving the relation between the resistance of the multiplier R_x, the resistance of the instrument R_m, and the multiplying power M is as follows:

$$M = \frac{R_x + R_m}{R_m} \tag{49}$$

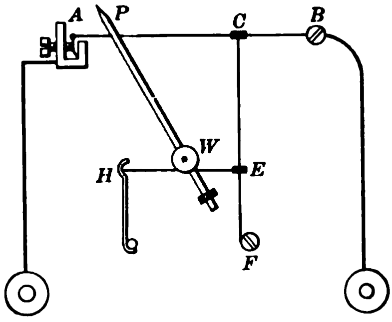


FIG. 124.—Principle of Hartmann and Braun hot-wire instruments.

Example.—In the above problem (*b*) the multiplying power of the multiplier is as follows:

$$M = \frac{51{,}000 + 17{,}000}{17{,}000} = 4$$

117. Hot Wire Instruments. In the instruments heretofore considered, the action of the instrument depends on the electromagnetic action of a current. There is another type of instrument which depends for its indications upon the heating action of the current. A diagram of this instrument is shown in Fig. 124. *AB* is a fine wire of platinum-silver through which the current passes. At *C*, a wire *CF* is attached to *AB*.

At *E*, on *CF*, a silk fiber *EH* is attached. This passes around the pulley *W*, and is held in tension by the spring *H*. When a current flows through *AB*, the heat expands the wire *AB*, reducing the tension in the wire *CF*, and allowing the spring *H* to pull the silk fiber to the left. This fiber, acting on the pulley *W*, moves the pointer *P* over the scale.

When used as an ammeter, a shunt is necessary unless the current is very small. When used as a voltmeter, a high resistance is connected in series with the wire *AB*.

This type of instrument is "dead beat," that is, it is very sluggish in its behavior and only reaches its ultimate deflecton after the lapse of considerable time. This is an advantage in the measurement of fluctuating currents as the needle follows the fluctuations very slowly so can be accurately read. Another advantage of the hot wire type of instrument is that it can be used for alternating as well as for direct currents. It is often used as a transfer instrument to measure alternating currents in terms of direct current. This type of instrument is particularly useful for the measurement of high frequency alternating currents, as its indications are independent of the frequency if a shunt is not used. For this reason this type is very useful in radio telegraphy. Such instruments are affected by temperature and do not hold their calibration for very long periods. Therefore, for accurate work they should be calibrated at the time of using.

ELECTRICAL MEASUREMENTS

Measurement of Resistance

118. Voltmeter-ammeter Method.—The resistance of any portion of an electric circuit is, by Ohm's Law,

$$R = \frac{V}{I}$$

where V is the voltage across that portion of the circuit and I is the steady current flowing in that portion of the circuit. Obviously, the voltage V may be measured with a voltmeter, the current I measured with an ammeter, and the resistance R computed.

Let it be required to determine the resistance R in the circuit shown in Fig. 125. The source of power is the 110-volt supply.

The resistance R is comparatively small and if connected directly across 110 volts would take an excessive current. Therefore, it is necessary to insert a resistance R' in series with R to limit the current. The voltmeter, however, must be connected directly across R as it is desired to know the resistance of this portion of the circuit only.

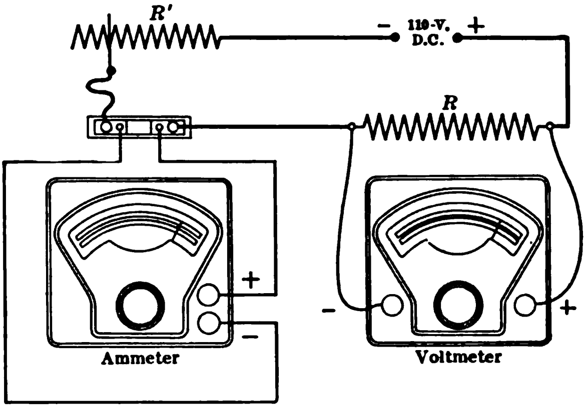


Fig. 125.—Voltmeter-ammeter method of measuring resistance.

Example.—The voltmeter (Fig. 125) reads 19 volts when the ammeter reads 24 amp. What is the value of the resistance R?

The resistance:

$$R = \frac{19}{24} = 0.792 \text{ ohm}.$$

As a matter of interest let it be required to determine the resistance of R'. The voltmeter terminals are transferred from across R to across R'. Under these conditions the voltmeter reads 91 volts and the ammeter still reads 24 amp. Therefore:

$$R' = \frac{91}{24} = 3.79 \text{ ohms}.$$

It is sometimes desired to measure resistances of such low value that, if a voltmeter were connected directly across their terminals, the contact resistance, which may be comparatively large, would introduce considerable error and might even exceed in magnitude the resistance which it is desired to measure. To eliminate this error due to contact resistance, the voltmeter terminals are connected well inside the terminals BB (Fig. 126) through which the current is led to the specimen. As the voltmeter takes but a very small current, small sharp pointed con-

tacts CC may be used. As the resistance of the voltmeter is comparatively high, it is only necessary that the contact resistances at CC be negligible compared to the resistance of the instrument. This condition is easily met. As these contacts are small and sharp the points of contact on the specimen can be determined very accurately.

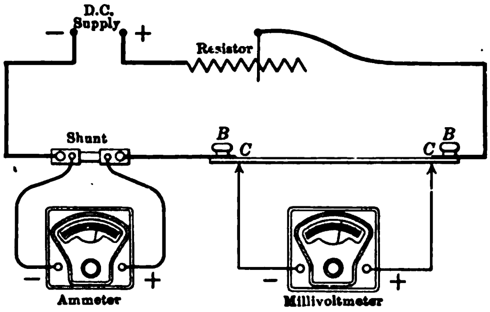


FIG. 126.—Measuring the resistance of a metal rod.

Example.—When the ammeter (Fig. 126) reads 50 amp., the millivoltmeter indicates 40 millivolts. The contacts CC are 23 in. apart. What is the resistance per inch length of the rod?

The resistance for 23 in. is:

$$R = \frac{0.040}{50} = 0.00080 \text{ ohm.}$$

The resistance per inch:

$$R = \frac{0.00080}{23} = 0.0000348 \text{ ohm.} \quad \textit{Ans.}$$

119. The Voltmeter Method.—It is possible to measure a resistance by means of a voltmeter alone provided the resistance to be measured is comparable with that of the voltmeter. In Fig. 127 (*a*) let it be required to measure the resistance R. The voltmeter is first connected across the source of supply and a reading V_1 taken. It is then transferred so that the resistance R is in series with it across the source of supply and the voltmeter reading is again taken. Let this reading be V_2.

As V_1 is the total circuit voltage and V_2 is the voltage across the instrument, the voltage across the unknown resistance R is obviously $V_1 - V_2$. When the voltmeter is in series with R,

the same current i must flow through each so that the voltages are as follows:

$$V_2 = iR_v \tag{1}$$
$$V_1 - V_2 = iR \tag{2}$$

where R_v is the resistance of the voltmeter.

Dividing (2) by (1) and solving for R,

$$R = R_v \frac{V_1 - V_2}{V_2} \tag{50}$$

This method of measuring resistance is particularly useful in determining insulation resistance of dynamo windings, cables

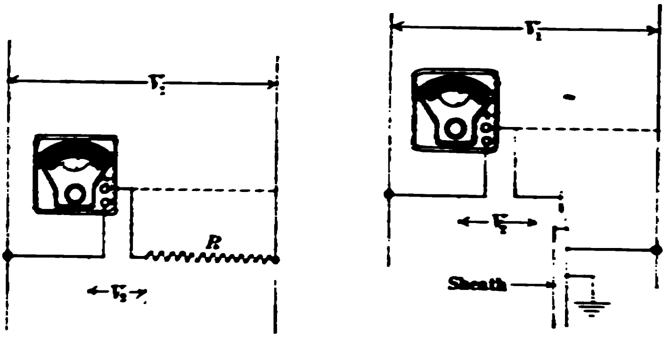


FIG. 127. Measurement of resistance by the voltmeter method.

etc. As such resistances are very high they are usually expressed in megohms (1 megohm = 1,000,000 ohms). It will be seen from equation (50) that the greater the value of R_v, the greater the resistance that can be measured by this method. For this reason special 150 scale voltmeters, having resistances of 100,000 ohms (one-tenth of a megohm) are available. These give a sensitivity about six times as great as can be obtained with the ordinary 150 scale voltmeter.

Fig. 127 (b) shows the application of this method to the measurement of the insulation resistance of a cable.

Example.—When a 100,000-ohm voltmeter is connected across a direct current line it reads 120 volts. One terminal of the voltmeter is then connected to the core of a lead-covered cable and the sheath of the cable is connected to the other side of the line as in Fig. 127 (b). The voltmeter now reads 10 volts. What is the insulation resistance of the cable?

$$X = 0.1 \frac{120 - 10}{10} = 1.1 \text{ megohms.} \qquad \textit{Ans.}$$

120. The Wheatstone Bridge.—In distinction to the foregoing methods of measuring resistance, the Wheatstone Bridge method is one in which the unknown resistance is balanced against other known resistances. The bridge in its simplest form is shown in Fig. 128. Three known resistances M, N, P, and the unknown resistance X are connected to form a diamond. Current from a battery B feeds the two opposite corners o and c of the diamond. Across the other two corners a and b, is connected a galvanometer.

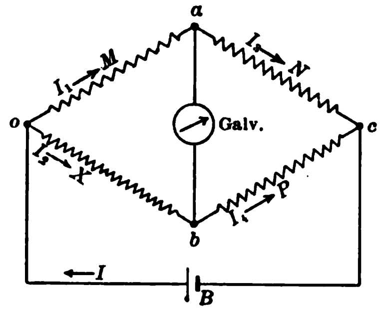


FIG. 128.—Elementary Wheatstone bridge.

To make a measurement, the two arms M and N are each set at some fixed value of resistance, usually 1, 10, 100, 1,000 ohms, etc. The arm P is then adjusted until the galvanometer does not deflect. If the galvanometer does not deflect, no currnet flows through it and therefore the two points a and b must be at the *same potential*. Also the currents $I_1 = I_3$ and $I_2 = I_4$, as no current passes through the galvanometer.

If the points a and b are at the same potential, the voltage drop $oa = ob$ and:

$$I_1M = I_2X \tag{1}$$

Also the voltage drop $ac = bc$ and

$$I_3N = I_4P$$

And since $I_1 = I_3$ and $I_2 = I_4$

$$I_1N = I_2P \tag{2}$$

Dividing (1) by (2)

$$\frac{I_1M}{I_1N} = \frac{I_2X}{I_2P} \text{ or } \frac{M}{N} = \frac{X}{P}$$

$$X = \frac{M}{N}P \tag{51}$$

which is the equation of the Wheatstone Bridge. M and N are called the ratio arms and P the balance or rheostat arm. Obviously the battery and the galvanometer may be interchanged without affecting the relation given in equation (51).

The many types of Wheatstone Bridge found in practice do not differ in principle from that shown in Fig. 128. The differences lie in the positions of the arms M, N, and P on the bridge as well as in the manner in which the coils in these arms are cut in and out of circuit.

A common plug type of bridge is shown in Fig. 129. M consists of three resistances of 1,000, 100, and 1 ohms respectively,

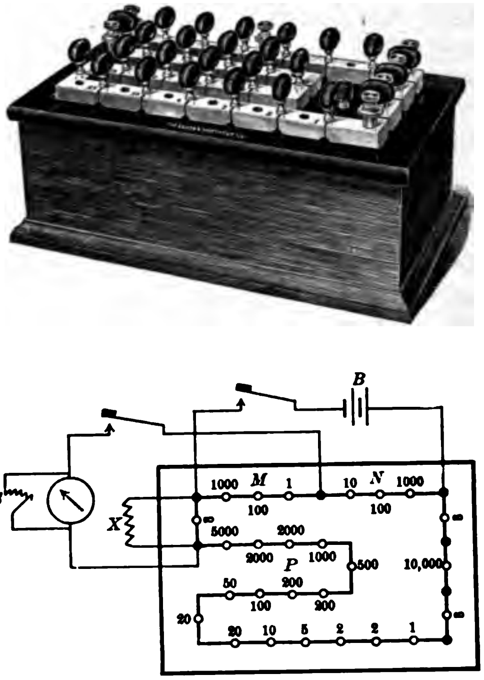


FIG. 129.—Massachusetts Institute of Technology pattern of Wheatstone bridge.

and N consists of three of 10, 100, and 1,000 ohms respectively. P consists of a number of resistances ranging from 5,000 ohms to 1 ohm and of such values that with the proper combinations P may be made equal to any whole number between 1 and 11,110 ohms. Between the outer ends of N and P are two infinity plugs (∞) and a 10,000-ohm coil. The infinite plugs mean that the bridge can be open circuited at these points and by their position the 10,000-ohm coil may be made a part of N or a part of P.

The unknown resistance X may be connected across any one of the infinite resistances, if it is foud andvisable to do so. Between M and P is another infinite resistance, across which the unknown resistance may also be connected, the infinite plug being removed.

In this type of bridge the resistance coils are connected across gaps cut in heavy brass or composition bars. When it is desired to insert a resistance the plug is removed, and when it is desired to remove a resistance it is short-circuited by the plug. These plugs have hard rubber tops and are tapered. As the principle source of error in this type of bridge lies in the contact resistance of these plugs, they should be made to fit tightly when used. This is accomplished by exerting a slight pressure and simultaneously twisting them, thus giving a wiping contact. As dirt and oxide are a frequent source of error the plugs should be kept clean.

In using the bridge, much time may be saved if a systematic procedure is followed in obtaining a balance. Assume that it is desired to measure a certain unknown resistance. Connect the bridge as shown in Fig. 129, placing keys in the battery and in the galvanometer circuits and a shunt around the galvanometer to protect it from deflecting violently when the bridge is considerably out of balance. Make the ratio arms M and N each 1,000 ohms, a 1 to 1 ratio. With the galvanometer well shunted and all the plugs in P (Res. = 0), depress first the battery and then the galvanometer key. The galvanometer is observed to deflect to the left. Now remove the 5,000-ohm plug and the galvanometer deflects to the right. From these observations, two facts are determined. The unknown resistance is less than 5,000 ohms and when the galvanometer deflects to the left, the value of resistance in P is too small, and when it deflects to the right the value of P is too large. By inserting the 5,000-ohm plug and removing the 1,000-ohm plug the galvanometer still deflects to the right, indicating that 1,000 ohms in P is too large. This is repeated with 500 ohms, 200 ohms, etc. By proceeding in this manner, it is found that the galvanometer does not reverse until a 2-ohm plug is removed. This means that the unknown resistance lies between 2 and 5 ohms. By removing the two 2-ohm plugs and then a 1 and a 2 the unknown resistance is narrowed down to between 2 and 3 ohms. To get a more precise value the ratio arms must be changed. M is now made 1 ohm and 2,000 ohms unplugged in P. By successive trials, all the time reducing the shunt S, a balance is obtained at 2,761 ohms in P. Then:

$$X = \frac{M}{N} P = \frac{1}{1{,}000} 2{,}761 = 2.761 \text{ ohms}.$$

In obtaining a balance the battery key should always be depressed before the galvanometer key, so that the current in the bridge has time to reach

a constant value. Otherwise the electromotive force of self-induction may introduce an error.

A more convenient arrangement of the resistance units of the rheostat arm R is shown in Fig. 130. The resistances are arranged in groups of equal resistances, one group consisting of ten 1-ohm coils, the next of ten 10-ohm coils, the next of ten 100-ohm coils, etc. Each group is called a *decade*. Only one plug per decade is necessary. This arrangement has the advantage that the plugs are always in service and are not so likely to be mislaid or to become dirty. There is less probability of error in reading; it is a simple matter to see that the few plugs used are fitting tightly, and

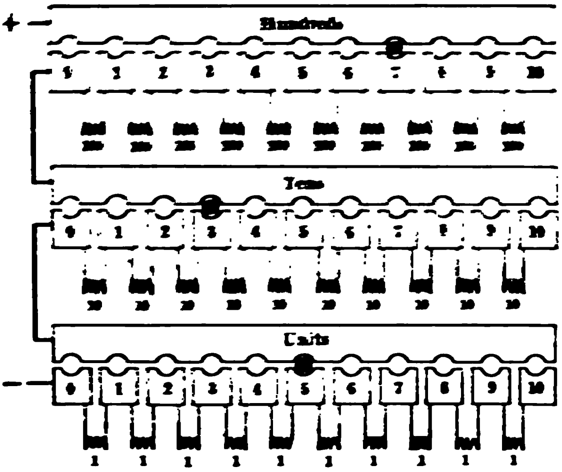


FIG. 130.—Arrangement of rheostat arm resistances in a decade bridge.

a balance can be quickly obtained. It is obvious that nine coils per decade are sufficient for obtaining any desired resistance, although ten coils per decade are often used.

The decade principle has been extended to an even more convenient type of bridge, the dial bridge. Instead of using plugs, a dial arm similar to the type used in rheostats is employed to select the required resistances. Because of its ease of manipulation this type has come into extensive use. Care should be taken to keep the dials and contacts free from dirt and oxides. Fig. 131 shows a dial bridge of the Leeds & Northrup type.

121. The Slide Wire Bridge.—The slide wire is a simplified Wheatstone Bridge, in which the balance is obtained by means of a slider which moves over a German silver or manganin resistance wire. A typical slide wire bridge is shown in Fig. 132. The resistance wire AB, 100 cm. long, is stretched tightly between two heavy copper blocks CD, 100 cm. apart. A meter scale is

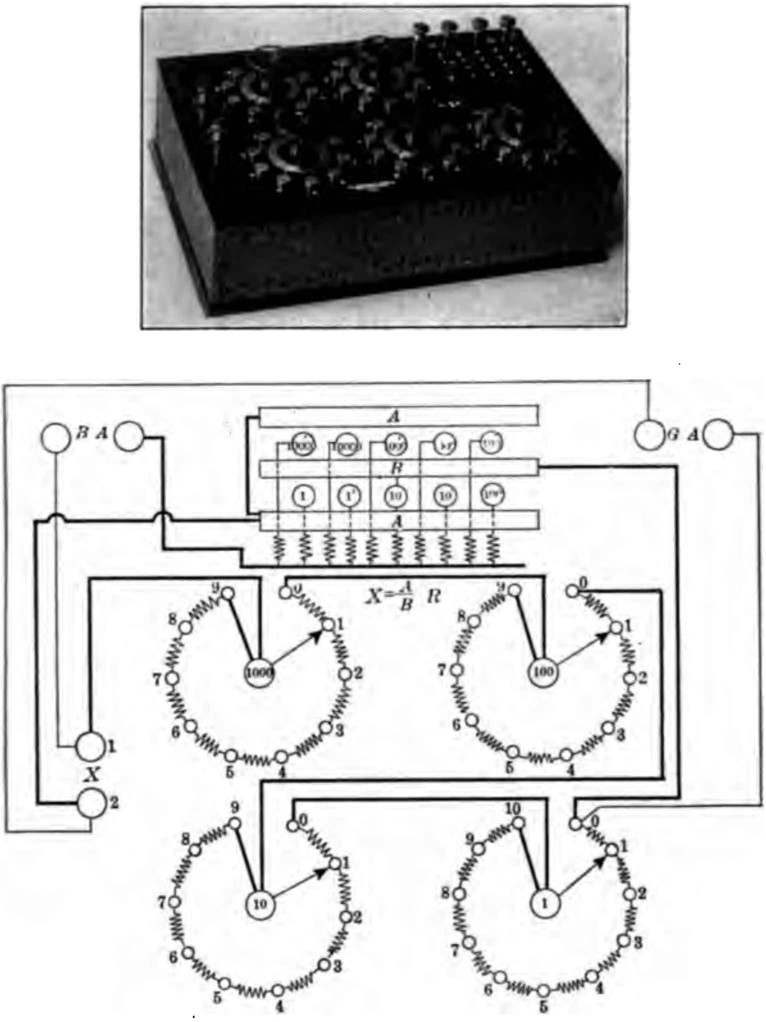


FIG. 131.—Leeds & Northrup dial bridge.

placed along this wire. A contact key K' is movable along the scale and when the key K' is pressed a knife edge makes contact with the wire. The rest of the bridge consists of a heavy copper bar E, a known resistance R, and the unknown resistance X. R is connected between D and E and X between C and E, although the positions of R and X are interchangeable.

The galvanometer is connected between the key K' and E and the battery terminals are connected to C and D. A balance is obtained by moving K' along the wire until the galvanometer shows no deflection.

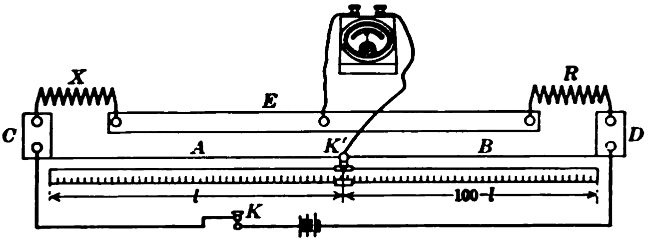


Fig. 132.—Slide-wire bridge.

Let l be the distance in centimeters from one end of the scale to K' when a balance is obtained. Then $100 - l$ is the distance from K' to the other end of the scale. Let r be the resistance per unit length of the wire. Then the resistance of l is lr and that of the remainder of the wire is $(100 - l)r$.

By the law of the Wheatstone Bridge:

$$\frac{X}{lr} = \frac{R}{(100 - l)r} \tag{52}$$

r cancels out and (52) becomes:

$$X = l\frac{R}{(100 - l)} \tag{53}$$

(52) may also be written

$$\frac{X}{R} = \frac{l}{100 - l} \tag{54}$$

This is equivalent to stating that when a balance is obtained the slide wire is divided into two parts which are to each other as X is to R.

The slide wire is not as accurate as the coil bridge, because the

slide wire may not be uniform; the solder at the points of contact at C and D makes the length of the wire uncertain; the slide wire cannot be read as accurately as the resistance units of a bridge can be adjusted.

Example.—Assume that R (Fig. 132) equals 10 ohms and that a balance is obtained at 74.6 cm. from the left-hand end of the scale. Find the unknown resistance X.

From equation (53)

$$X = 74.6 \frac{10}{100 - 74.6} = 74.6 \frac{10}{25.4} = 29.37 \text{ ohms.} \quad \textit{Ans.}$$

CABLE TESTING

122. The Murray Loop.—The slide wire bridge offers a very convenient method of locating grounds in cables and wires. Fig. 133 shows a cable AB which has become grounded at the

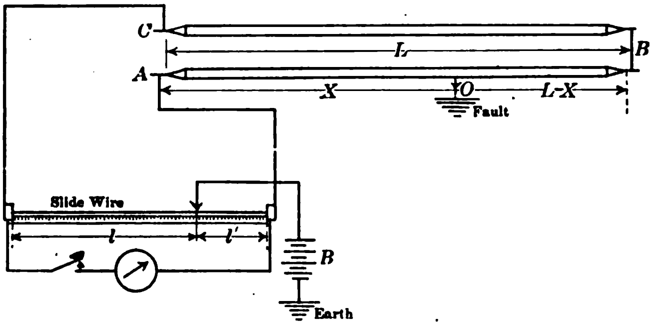


FIG. 133.—Murray-loop test.

point O, owing to a defect in the insulation. CB is the return conductor and is similar to AB except that it has no ground or "fault." The two conductors are looped together at B, the far end of the two conductors, which may be at some power station, telephone exchange, etc.

The slide wire is then connected to the home ends of the cable as shown. It will be noted that the battery and the galvanometer are not in the positions shown in Fig. 132, but have been interchanged. This is done in order that earth currents shall not disturb the galvanometer readings. Also if the ground has a high resistance, the electromotive force of the battery B may be increased until sufficient current to operate the bridge is sent through

this resistance. The resistance of the fault to ground does not produce any error in the measurement so long as the conductor is not broken. If the conductor is broken with both ends lying on the ground, the resistance of the conductor is increased and a false location of the fault may result.

In Fig. 133, the distance X to the fault may be found as follows:

$$\frac{X}{l'} = \frac{L + (L - X)}{l} \tag{55}$$

where L is the length of one cable.

The slide wire is split into two sections which are to each other as the two lengths of cable on each side of the fault.

Solving (55) for X,

$$X = \frac{2Ll'}{l + l'} \tag{56}$$

This assumes that the resistance per foot of both conductors is the same, and is uniform. The jumper tieing the cable ends together at B should make good connection, as contact resistance at this point may introduce an appreciable error. A ratio and rheostat arm of a bridge box may obviously be used instead of the slide wire.

Example.—A cable 2,000 ft. long consists of two conductors. One conductor is grounded at some point between stations. A Murray loop test, with a 100-cm. slide wire bridge, is connected as in Fig. 133 to locate the fault.

A balance is obtained at 85 cm. How far from the station is the ground? From equation (56):

$$L = 2{,}000 \quad l' = 15 \quad l = 85$$

$X = \dfrac{4{,}000 \times 15}{100} = 600$ ft. from the station at which the measurement is made.

123. The Varley Loop.—The Varley loop is also used to locate cable faults. It is similar in principle to the Murray loop, a bridge box being necessary, however. The connections are shown in Fig. 134. M and N are the two ratio arms of a bridge and P is the rheostat arm. It is necessary that the battery and the galvanometer occupy the positions shown, in order to avoid disturbances in the galvanometer due to earth currents. A balance is first obtained by means of P, with the switch S at a. Let r be the resistance per foot length per conductor, assumed uniform.

(It will be noted that P and X form one arm of the bridge.)

$$\frac{M}{N} = \frac{r(L + L - X)}{P + rX} \tag{57}$$

Before X can be found it is necessary to know r. To obtain this, the switch S is thrown to position b. This throws both lengths

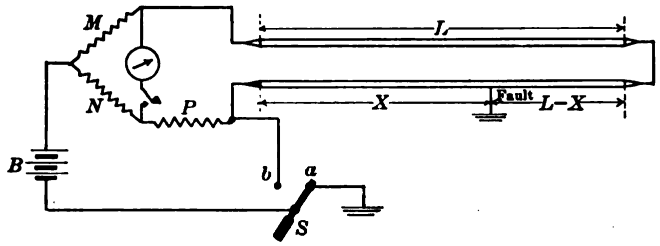


FIG. 134.—Varley-loop test.

of cable in series and makes them the fourth arm of a bridge. A simple bridge measurement is then made of the total loop resistance. Call this resistance R. Then the resistance per foot of cable

$$r = \frac{R}{2L}$$

(This measurement is not necessary if the resistance per foot or the total resistance of the cable is already known.)

Substituting this value of R in (57)

$$\frac{M}{N} = \frac{R/2L(2L - X)}{P + RX/2L}$$

Solving for X,

$$X = \frac{2L}{R}\left(\frac{NR - MP}{M + N}\right) \tag{58}$$

This equation gives the distance in feet to the fault. The equation is frequently given as follows:

$$R_x = \frac{NR - MP}{M + N} \tag{59}$$

In this case R_x is not the *distance* to the fault, but rather the *resistance* along the grounded conductor to the fault.

If $M = N$ in equation (58)

$$X = \frac{L}{R}(R - P) \tag{60}$$

which is simpler in form than (58).

Example.—In locating a fault by the Varley loop test, the connections shown in Fig. 134 were used. Each conductor is 2,800 ft. long. With the switch at (*a*) and $M = 10$, $N = 1{,}000$. P was found to be 137 when a balance was obtained. Switch S is then thrown over to (*b*). Under these conditions, a balance was obtained when $M = 10$, $N = 1{,}000$, $P = 221$, making $R = 2.21$.

By equation (58) the distance in feet to the fault

$$X = \frac{2 \times 2{,}800}{2.21}\left(\frac{1{,}000 \times 2.21 - 10 \times 137}{1{,}010}\right) = 2{,}100 \text{ ft.} \quad \textit{Ans.}$$

124. Insulation Testing.—In practice it is necessary to measure the resistance of the insulation of cables, both at the factory and after the cable is installed. A low value of insulation resistance may indicate that the insulation is of an inferior grade.

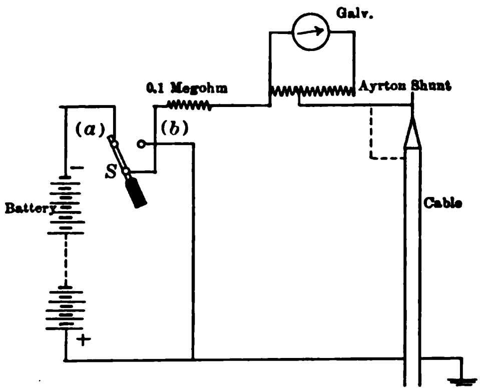


Fig. 135.—Measurement of the insulation resistance of a cable.

A low insulation resistance after installation may indicate improper handling or faulty installation. The voltmeter method described in Par. 119 is applicable in many cases, but where the insulation resistance is high even a high resistance voltmeter is not sufficiently sensitive.

To make the measurement, a sensitive galvanometer is utilized. A considerable source of potential, from 100 to 500 volts, is also usually necessary. Such potential may be secured from direct current mains, although dry cells, silver chloride cells, and test-tube batteries connected in series are more satisfactory. A simple diagram of connections is shown in Fig. 135.

The method is one of substitution. A known resistance, usu-

ally 0.1 megohm (100,000 ohms), is first connected in the circuit and the galvanometer deflection noted. The unknown resistance X is then substituted and the galvanometer reading again noted. As the currents in the two cases are inversely proportional to the circuit resistances, the unknown resistance can be determined, the galvanometer deflections being used rather than actual values of current. Let D_1 be the deflection with the 0.1 megohm and D_2 be the deflection with the unknown resistance.

$$\frac{X}{0.1} = \frac{D_1}{D_2}$$

$$X = 0.1\frac{D_1}{D_2} \tag{61}$$

Under ordinary circumstances it would not be possible to obtain accurate results under these conditions alone, because the unknown resistance may be in the hundreds of megohms and the known resistance is but 0.1 megohm. This would make the deflection D_2 so many times smaller than D_1 that it would not be readable.

This difficulty is overcome by the use of the Ayrton shunt described in Par. 113. When the 0.1 megohm only is in circuit, the galvanometer sensitivity ordinarily is such that it would deflect off the scale unless the galvanometer were shunted.

Therefore the shunt is adjusted to some low value as 0.0001. Call this reading of the shunt S_1 and the galvanometer deflection D_1. The multiplying power of the shunt equals $M_1 = 1/S_1$. The cable is now introduced into the circuit and the shunt adjusted until a reasonable deflection is obtained. Call this reading D_2 and the value of the shunt S_2. Its multiplying power is now $M_2 = 1/S_2$.

The current in the circuit in the two cases

$$\frac{I_1}{I_2} = \frac{M_1D_1}{M_2D_2}$$

Therefore the unknown resistance, from (61), is

$$X = 0.1\frac{I_1}{I_2} = 0.1\frac{M_1D_1}{M_2D_2} \tag{62}$$

In practice, instead of substituting the cable for the 0.1 megohm, the cable is first short-circuited by the wire shown dotted, Fig. 135, and the constant determined. This wire is then removed, placing the cable in circuit. The 0.1 megohm is left permanently

in circuit to protect the galvanometer in case of accidental short-circuit of the cable. Its resistance is usually not appreciable compared to that of the cable, so that no correction is ordinarily necessary for it.

A switch or key S is ordinarily provided. When in position (a) the circuit is closed through the cable. When thrown over to (b), the cable, which is charged electrostatically, discharges through the galvanometer.

When the switch (a) is first closed there is a rush of current which charges the cable electrostatically. (See par. 153, Chap. IX.) It takes time to charge the cable, so for some time this

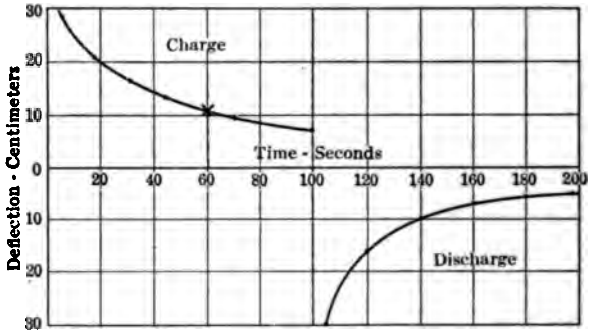


Fig. 136.—Charge and discharge curves of a cable.

charging current flows, decreasing continuously. This is shown in Fig. 136, giving the relation of the galvanometer deflection to the time. As it is often inconvenient to wait for the galvanometer to reach a steady deflection, it has been arbitrarily agreed to take the deflection at the end of one minute as the value to be used in determining insulation resistance.

When the switch S is thrown to (b) the electrostatic charge in the cable rushes out through the galvanometer in the reverse direction. Due to absorption it requires considerable time for the cable to become totally discharged. This is also shown in Fig. 136.

In making insulation resistance measurements, precautions must be taken to insulate thoroughly the apparatus itself. Hard rubber posts should be used for supports and, wherever possible,

the leads should be carried through the air rather than be allowed to rest on the ground. The insulation resistance varies enormously with temperature, so the temperature at which the measurements are made should be carefully determined and stated.

Example.—The cable whose insulation curves are shown in Fig. 136 was tested for insulation. The deflection with 0.1 megohm only in circuit was 20 cm. and the shunt read 0.0001. When the curve shown in Fig. 136 was obtained the shunt read 0.1. The cable was 2,200 ft. long.

(*a*) What is its insulation resistance?

(*b*) What is its insulation resistance per mile?

$$M_1 = 1/0.0001 = 10{,}000$$
$$M_2 = 1/0.1 = 10$$
$$D_2 \text{ (from curve)} = 11 \text{ cm.}$$
$$(a)\quad X = 0.1\,\frac{10{,}000 \times 20}{10 \times 11} = 182 \text{ megohms. } \textit{Ans.}$$

(*b*) The resistance per mile will be *less* than that of the 2,200-ft. length because the amount of leakage current is directly proportional to the length of the cable. Therefore the resistance of this leakage path is inversely proportional to the length of cable. The cross-sectional area of the leakage path for the mile length is greater than it is for the 2,200-ft. length. Therefore the resistance per mile

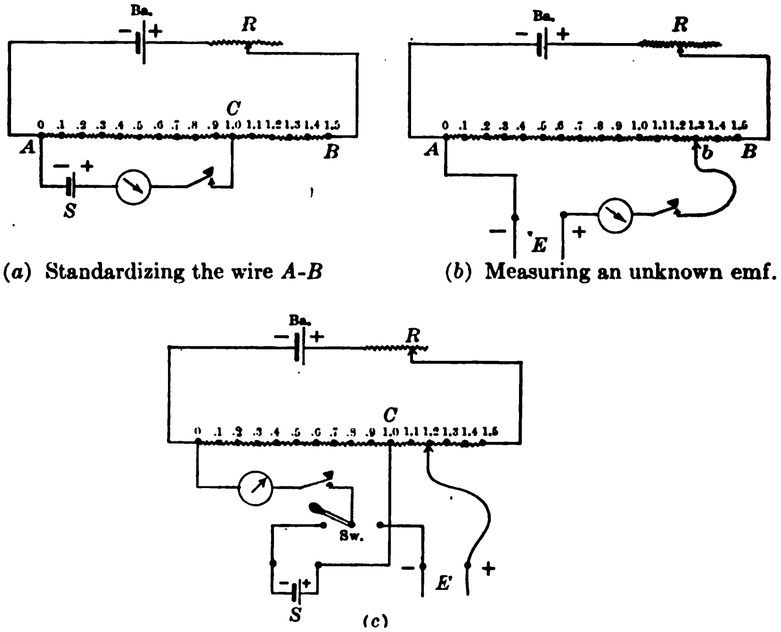
$$R = \frac{2{,}200}{5{,}280}\,182 = 75.9 \text{ megohms. } \textit{Ans.}$$

POTENTIOMETERS

125. The Potentiometer.—The potentiometer is an instrument for making accurate measurements of voltage. Its standardization depends primarily upon the Weston standard cell. (See Par. 89, Chap. VI.) The principle is as follows:

Assume in Fig. 137(*a*) that a standard cell *S* has an electromotive force of exactly 1 volt. Let a storage cell *Ba* supply current to a wire *AB* through a rheostat *R*. Let the wire *AB* be divided into 15 divisions each of 1 ohm resistance, making the total resistance of *AB* equal to 15 ohms. The standard cell is connected with its negative terminal to the negative terminal of the storage cell and its positive terminal is connected to the tenth 1-ohm coil *C* through a key and galvanometer. If 0.1 amp. flows through the wire *AB*, the voltage drop through each resistance will be 0.1 volt and the voltage drop across *AC* will be 1.0 volt.

If the key be depressed no current will flow through the galvanometer, as the standard cell emf. is in exact opposition to this 1-volt drop. If, however, the current in AB is not exactly 0.1 amp., current will flow through the standard cell circuit due to the voltage AC being either greater or less than 1 volt. If the current is less than 0.1 amp. the galvanometer deflects in one direction, and if it is greater than 0.1 amp. the galvanometer



(a) Standardizing the wire A-B (b) Measuring an unknown emf.

(c)

Fig. 137.—Simple potentiometer.

deflects in the reverse direction. Obviously it is possible to so adjust the current in AB that the galvanometer deflection is zero. Under these conditions the current in AB is exactly 0.1 amp. and the potential drop across each resistance in AB is 0.1 volt. Therefore AB may be marked in volts as shown.

Let it be required to measure some unknown electromotive force E whose value is known to be less than 1.5 volts. Its negative terminal is connected to the end A of the wire AB, Fig. 137(b). The positive terminal of the electromotive force is connected through the galvanometer and key to a movable

contact *b*. It is assumed that the current in *AB* has been adjusted to exactly 0.1 amp. as just described. Contact *b* is moved along *AB* until the galvanometer deflection is zero. This means that the electromotive force *E* is just balanced against an equal drop in the wire *AB*. As *AB* is calibrated in volts, the value of *E* may be read directly on *AB*. This method of measuring voltage is the *Poggendorf Method* and is the fundamental principle of the potentiometer.

The two diagrams (*a*) and (*b*) (Fig. 137) may be combined into one by the use of the single-pole, double-throw (S.-P.D.-T.) switch S_W. When the switch is in its left-hand position the standard cell is in circuit for calibration as in (*a*). When it is in its right-hand position the unknown emf. is in contact with the wire *AB* so that its value may be determined.

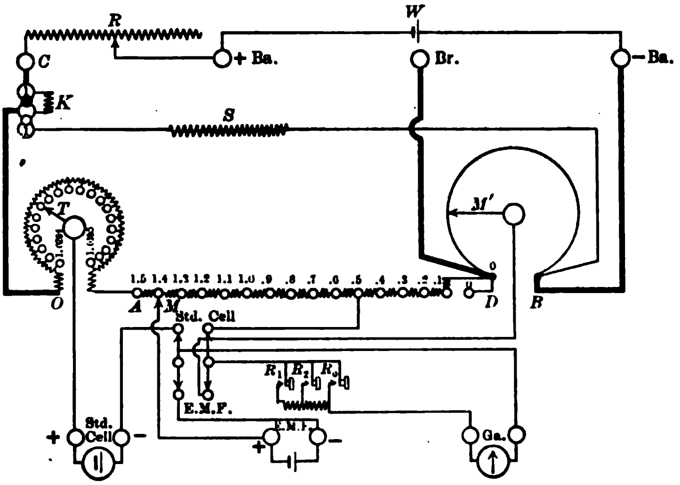


FIG. 138.—Connections of Leeds & Northrup low-resistance potentiometer.

126. The Leeds & Northrup Low Resistance Potentiometer.—Fig. 138 shows the Leeds & Northrup low resistance potentiometer. In most respects it is similar to the simple potentiometer shown in Fig. 137. The battery *W* supplies current through the rheostat *R* and then through the low resistance *OA*, to the potentiometer wire *AB*. *AB* consists of two parts, fifteen 5-ohm coils and a slide wire *DB* of 5.5 ohms. As each contact represents 0.1 volt, the working current is 0.1/5 = 1/50

amp. As the slide wire DB is 5.5 ohms, the voltage drop across it when in adjustment is 1.1 volts. This slide wire consists of 11 turns of resistance wire mounted on a marble cylinder. Each turn represents 0.01 volt and the entire wire is divided into 1,100 divisions.

The standard cell has a voltage slightly in excess of 1.0 volt so that instead of connecting the standard cell exactly as in Fig. 137 (*a*), an added resistance AO is necessary to allow for this small excess voltage. A contact T is movable on AO so that the setting can be made to correspond with the electromotive force of the standard cell used. M and M' are the movable contacts, which are adjusted to balance the unknown emf.

Fig. 139.—Leeds & Northrup potentiometer without accessories.

M moves over the 15 contacts, each corresponding to 0.1 volt, and M' moves over the slide wire. A double-pole, double-throw (D.-P.D.-T.) switch (corresponding to S_W, Fig. 137(*c*)) changes the connection of the galvanometer from the standard cell to the unknown emf. There are three galvanometer keys, R_1, R_2, and R_0. R_1 should first be depressed as it inserts a high resistance in series with the galvanometer and prevents a violent deflection if there is considerable unbalancing. R_2 inserts less resistance and there is no resistance in series with R_0 which is depressed when the final balance is obtained.

A resistance S shunts 0.9 of the current from OB, when the plug at K is changed. The resistance K is automatically put in circuit, keeping the total potentiometer resistance, and therefore the load on the battery, constant. By this arrangement,

the readings on the potentiometer are all made one-tenth their previous values.

An external view of this potentiometer is shown in Fig. 139.

127. Voltage Measurements with the Potentiometer.—Potentiometers are designed to measure potentials up to 1.6 volts only. For the measurement of potentials in excess of this value a *volt box* is necessary. A volt box is merely a very high resistance from which suitable taps are brought. This is illustrated by the resistance *AD*, Fig. 140. Assume *AD* to have a resistance

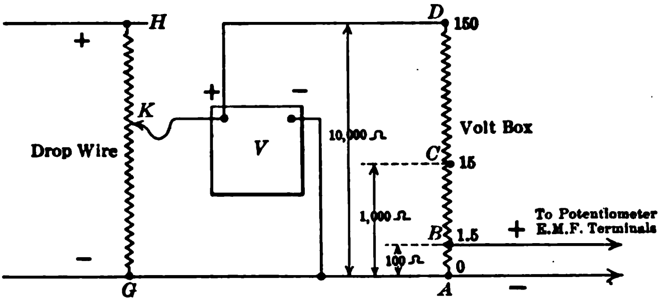


FIG. 140.—Volt-box and drop-wire connections.

of 10,000 ohms and *AB* a resistance of 100 ohms. If no current leaves the wire at *B*, the voltage drop across *AB* will be $^{100}/_{10,000}$ = $^{1}/_{100}$ that across *AD*. If leads be carried from *AB* to the potentiometer, the potentiometer will measure $^{1}/_{100}$ the voltage across *AD*, since the potentiometer principle is an opposition method so that no current is taken from *B*. Therefore, if a voltmeter *V* is being calibrated it should be connected in parallel with *AD*. If the voltmeter reads 119.0 volts and the potentiometer reads 1.184 volts, the true line voltage across the voltmeter will be 1.184 × 100 = 118.4. Therefore the correction to the voltmeter is − 0.6 volt.

In a similar manner, voltages from 1.5 to 15 volts are connected across *AC*, the multiplying factor in this case being 10.

The Drop Wire.—*GH* is a resistance connected directly across the line. One voltmeter terminal and one terminal of the volt box are connected to the end *G* of this wire. The other terminal of the voltmeter and the remaining terminal of the volt box are connected to a movable contact *K*. By sliding *K* along *GH* any

desired voltage may be obtained. When used in this manner, *GH* is called a *drop wire.* It is not necessary to the operation of the volt box, but is merely a convenient means for adjusting the voltage.

128. The Measurement of Current with Potentiometer.—As has just been pointed out, a potentiometer is designed to measure *voltage.* It may also be used to measure current by merely applying Ohm's Law. Let an unknown current I flow through a known resistance R. If E, the voltage drop across R, be measured, the current I is immediately determined, since for this part of the circuit both the voltage and the resistance are known. Therefore:

$$I = \frac{E}{R}$$

The method of making the measurement is shown in Fig. 141. It is desired to know the exact current passing through the ammeter, in order to determine its errors, if any exist.

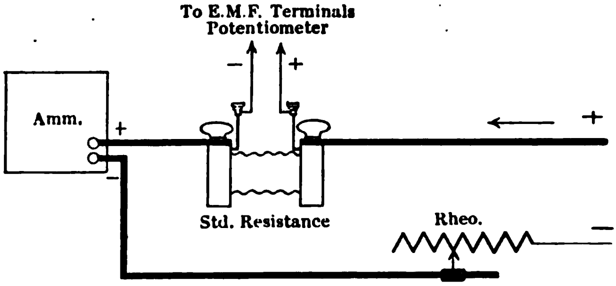


FIG. 141.—Calibration of an ammeter with a potentiometer.

The ammeter is connected in series with the standard resistance, and also with a rheostat to control the current. Standard resistances are provided with four terminals as a rule, two heavy ones for current and two smaller binding posts for potential. The two potential binding posts are connected to the potentiometer, the proper polarity being observed. The voltage across the standard resistance is then measured by means of the potentiometer.

Standard resistances are usually adjusted to even decimal values such as 10, 1, 0.1, 0.01, etc., ohms. They are ordinarily rated to carry a current that will give 1.0 volt drop. Thus

the 1 ohm can carry 1 amp., the 0.001 ohm, 1,000 amp., etc. To keep the resistances cool they are often immersed in oil. The type shown in Fig. 142 (*a*) is set in a water-jacketed oil bath provided with a motor-driven stirrer. The type shown in (*b*) is

(*a*) 0.01 ohm. (*b*) Self-contained 0.001 ohm.
FIG. 142.—Standard resistances.

rated for larger currents, 1,000 amp. and more. The water jacket, the stirrer, etc., are included within the unit itself.

Knowing that the potentiometer is limited to 1.5 volts, it is easy to select the proper standard resistance. An instrument

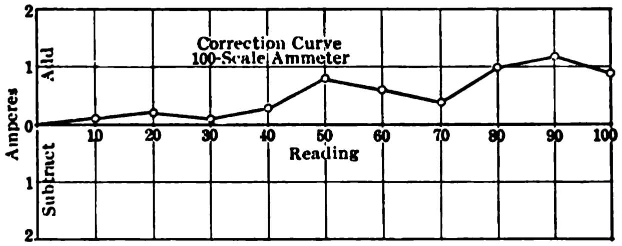


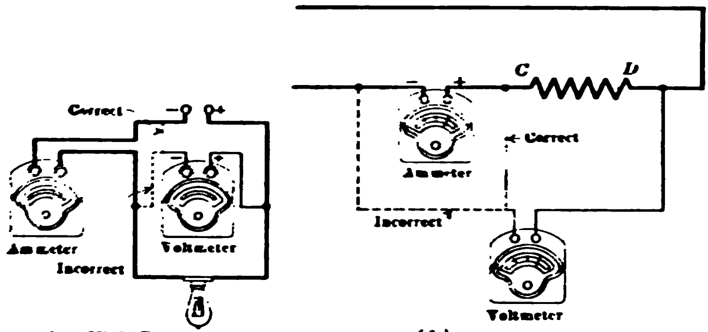
FIG. 143.—Ammeter calibration curve.

having a range of 100 amp., would require $1.5/100 = 0.015$ ohm. 0.01 ohm would be used. Likewise a 15-scale instrument would require $1.5/15 = 0.1$ ohm.

When instruments are calibrated, they should be checked at ten or fifteen points on the scale and the corresponding corrections at each point are plotted as ordinates. (The instrument read-

ings are plotted as abscissas. As an instrument scale is subject to scale errors, etc., it is customary to connect successive points by straight lines as shown in Fig. 143. For instance, (Fig. 143), the correct current when the instrument reads 50 amp. is 50 + 0.8 = 50.8 amp.

129. Measurement of Power.—Direct current power is usually measured by means of a voltmeter and an ammeter. Since the power is the product of the volts and the amperes ($P = EI$), it is merely necessary to multiply the volts by the amperes to obtain the power in watts. Certain precautions may be necessary in measuring the power, however.



(a) High Resistance (b) Low Resistance

FIG. 144.—Correct and incorrect methods of connecting voltmeters and ammeters in power measurements.

Assume that it is desired to measure the power taken by an incandescent lamp. If the voltmeter is connected as shown by the dotted line in Fig. 144 (a), the current taken by the voltmeter is being registered by the ammeter. In other words, the voltmeter is a load connected in parallel with the lamp. As the current taken by the lamp is small, this voltmeter current, although of itself small, may introduce a very appreciable error into the measurement. That is, the power taken by the voltmeter will be included in the measurement. There are three methods of eliminating this error. The voltmeter power may be calculated, knowing the voltmeter resistance, and proper correction made. The voltmeter may be open-circuited when the ammeter is being read if it is certain that this will not alter the voltage across the lamp. The voltmeter lead may be connected as shown by the

solid line so that the voltmeter current does not pass through the ammeter. In this last case the voltmeter is not reading the true voltage across the lamps, but its reading is too high by the drop through the ammeter. As the resistance of the lamp is high and that of the ammeter low, this last error is usually negligible.

However, if a low resistance CD is being measured, Fig. 144(b), the drop across the resistance is necessarily low, and if the voltmeter in this case is connected outside the ammeter, a very appreciable error may be introduced, as the voltmeter reading includes the voltage drop in the ammeter. The voltmeter should now be connected *inside* the ammeter. This will not introduce an appreciable error, for presumably a large current is required for the measurement of the low resistance, and the addition of the very small voltmeter current to the ammeter reading is negligible.

The above precautions should be observed also in making resistance measurements.

Example.—It is desired to measure the power taken by a 40-watt tungsten lamp. A 0.5 scale ammeter having a resistance of 0.15 ohm and a 150 scale voltmeter having a resistance of 16,000 ohms are used for the measurement. When the voltmeter is connected inside the ammeter it reads 120 volts and the ammeter reads 0.35 amp. What is the true power taken by the lamp and what is the apparent power if the voltmeter loss is neglected?

Apparent power $= 120 \times 0.35 = 42$ watts.

Power taken by voltmeter $= \dfrac{(120)^2}{16,000} = 0.9$ watt.

True power to lamp $= 41.1$ watts.

The voltmeter introduces a 2 per cent. error in this case.

If connected outside the ammeter, the ammeter will now read:

$$0.35 - \frac{120}{16,000} = 0.3425 \text{ amp.}$$

The voltmeter will now read:

$$120 + (0.15 \times 0.3425) = 120.05$$

and the apparent power $= 120.05 \times 34.25 = 41.12$, an error of 0.05 per cent., which is negligible.

130. The Wattmeter.—The wattmeter measures power directly. It consists of fixed coils FF and a pivoted coil M, free to turn within the magnetic field produced by coils FF as shown in Fig. 145. The coils FF are wound with comparatively few turns of wire which are capable of carrying the entire current of the circuit.

The moving coil M is wound with very fine wire and the current is led into it through two control springs in the same manner that current is led into the coil of a Weston instrument. The fixed coil is connected in series with the load in the same manner as an ammeter is connected. The moving coil is connected across the line in series with a high resistance R in the same manner as a voltmeter coil is ordinarily connected.

The field of the coils FF is proportional to the current and the current in the coil M is proportional to the voltage. Therefore the turning moment is proportional to the power of the circuit and it also depends on the angular position of M with respect to FF, which is taken into consideration when the scale is marked.

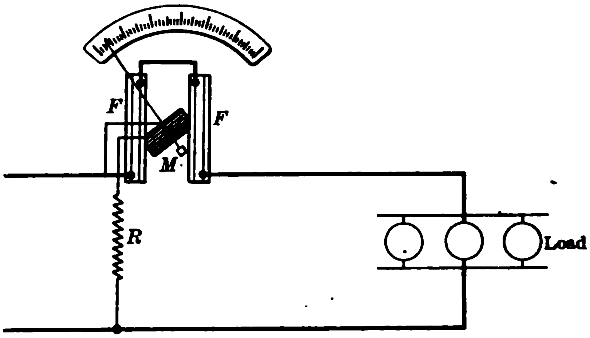


FIG. 145.—The indicating wattmeter.

Owing to the high degree of accuracy obtainable by the use of the voltmeter and ammeter, the wattmeter is seldom used for direct current measurements. As it is subject to stray fields, reversed readings should be taken, that is, both the current and voltage should be reversed and the average of the two readings used. The wattmeter is used more extensively for alternating current than for direct current. A more complete description together with its uses is found in Chap. III, Vol. II.

131. The Watt-hour Meter.—The watt-hour meter is a device for measuring *energy*. (See Par. 63.) As energy is the product of power and time, the watt-hour meter must take into consideration both of these factors. As power is usually sold on an energy basis, many dollars may depend upon the accuracy of such a

meter. Therefore a proper understanding of its mechanism and the method of adjustment is very often essential.

In principle the watt-hour meter is a small motor whose instantaneous speed is proportional to the power passing through it, and whose total revolutions in a given time are proportional to the total energy or watt-hours delivered during that time.

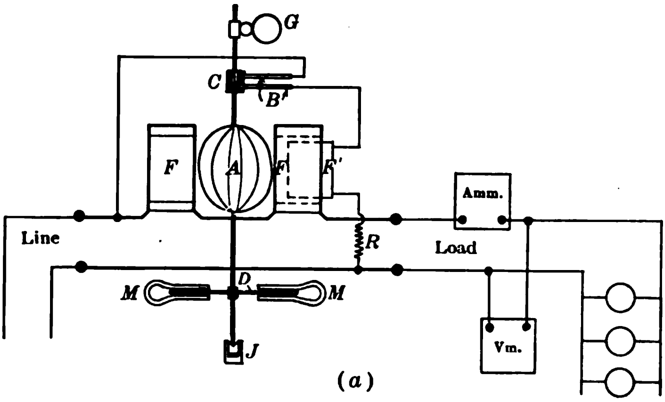


FIG. 146(*a*).—Connections of the watt-hour meter.

FIG. 146(*b*).—Interior of Thomson watt-hour meter.

Referring to Fig. 146 (*a*), the line is connected to two terminals on the left-hand side of the meter. The upper terminal is connected to two coils *FF* in series, wound with wire sufficiently heavy to carry the maximum current taken by the load, which should

not greatly exceed the rated current of the meter. This line terminates at the upper binding post on the right-hand side of the meter. These coils *FF* are wound so that they aid each other and they supply the field in which the armature rotates. The other line wire runs straight through the meter to the load. A shunt circuit is tapped to the upper line on the left-hand side. It runs first to the armature, through the silver brushes *B*, which rest on the small commutator *C*. From the brushes the line passes through coil *F'*, and through a resistance *R* to the lower line wire. This resistance *R* is omitted in certain types of meters.

As the load current passes through *FF*, and there is no iron in circuit, the magnetic field produced by these coils is proportional to the load *current*. As the armature, in series with resistance, is connected directly across the line, the current in the meter armature is proportional to the line *voltage*. Neglecting the small voltage drop in *FF*, the torque acting on the armature must then be proportional to the product of the load current and the load voltage or, in other words, it is proportional to the power passing through the meter to the load.

It can be proved that if the meter is to register correctly, there must be a retarding torque acting on the moving element which is proportional to its speed of rotation. To meet this condition an aluminum disc *D* is pressed on the motor shaft. This disc rotates between the poles of two permanent magnets *MM*. In cutting the field produced by these magnets, eddy currents are set up in the disc, retarding its motion. As the strength of these currents is proportional to the velocity of the disc and they are acting in conjunction with a magnetic field of constant strength, their retarding effect is proportional to the speed of rotation, so that the condition for correct registration is fulfilled.

Friction cannot be entirely eliminated in the rotating element, even with the most careful construction. Near the rated load of the meter the effect of friction is practically negligible. but at light loads the effect of friction, which is constant, is a much greater proportion of the load. As the ordinary meter may operate at light loads during a considerable portion of the time, it is desirable that the error due to friction be eliminated. This is accomplished by means of coil *F'* connected in series with the armature. *F'* is so connected that its field acts in the same direc-

tion as that due to coils *FF*. Therefore it assists the armature *A* to rotate. Being connected in the shunt circuit, it is acting continuously. The coil is movable so that its position can be so adjusted that the friction error is just compensated.

To reduce friction and wear, the rotating element of the meter is made as light as possible. The element rests on a jewel bearing *J*, which is a sapphire in the smaller sizes and a diamond in the heavier types. The jewel is supported on a spring. A hardened steel pivot rests in the jewel. In time the pivot becomes dulled and the jewel roughened, which increases friction and causes the meter to run more slowly unless *F′* is readjusted. The moving element turns the clock work of the meter dials through a worm and the gears *G*.

Fig. 146 (*b*) shows the interior view of a Thomson watt-hour meter.

132. Adjustment of the Watt-hour Meter.—Even if the initial adjustment be accurate the registration of a watt-hour meter may, in time, become incorrect. This is due to many causes, such as pitting of the commutator, roughening of the jewel, wear on the pivot, change in the strength of the retarding magnets, etc. As the cost of energy to consumers is largely based on the registration of such meters, it is important that they be kept in adjustment, as a small error in the larger sizes may ultimately mean a difference of many dollars one way or the other.

To adjust the meter it may be loaded as shown in Fig. 146(*a*). The power taken by the load is measured by a calibrated voltmeter and ammeter. The revolutions of the disc *D* are counted over a period of time which is measured with a stop watch. The relation between watt-hours and the revolutions of the *disc*, in most meters, is as follows:

$$W \times H = K \times N \tag{63}$$

where W is in watts

H is in hours

K is the meter "constant" usually found on the disc

N is the revolutions of the disc.

This equation means that the meter constant multiplied by the revolutions of the disc gives the watt-hours registered by the meter. The gear ratios and clockwork take care of the dial registration.

When checking a meter, the time is usually measured in seconds.

Equation (63) then becomes

$$\frac{W \times t}{3{,}600} = K \times N \qquad (64)$$

where t is the time in seconds.

When the meter is tested, the voltmeter and ammeter are read intermittently while the revolutions of the disc are being counted. A run ot about a minute gives good results.

Let the average watts determined from the corrected voltmeter and ammeter readings be W_1.

The average watts as indicated by the meter during the same period are, from (64),

$$W = \frac{K \times N \times 3{,}600}{t} \qquad (65)$$

The per cent. accuracy of the meter is

$$100\ W/W_1$$

Example.—In the test of a 10-amp. watt-hour meter having a constant of 0.4, the disc makes 40 revolutions in 53.6 seconds. The average volts and amperes during this period are 116 volts and 9.4 amp. What is the per cent. accuracy of the meter at this load?

Average standard watts $W_1 = 116 \times 9.4 = 1{,}090$.

Average meter watts from (65)

$$W = \frac{0.4 \times 40 \times 3{,}600}{53.6} = 1{,}074$$

$$\text{Per cent. accuracy} = \frac{100 \times 1{,}074}{1{,}090} = 98.5 \quad \textit{Ans.}$$

This means that the meter is 1.5 per cent. slow and should be speeded up slightly. With calibrated indicating instruments and careful adjustment, a meter may easily be brought within 0.5 per cent. of accurate registration.

There are two adjustments to be made. Near full load the magnets are moved. If the meter is running slow the magnets are moved nearer the center of the disc where the effect of the retarding currents is reduced, and if the meter is running fast the. magnets are moved farther from the center. If the meter has been correctly adjusted near full load, and is found to be in error near light load, the error is obviously due to friction. The light load adjustment (made at from 5 to 10 per cent. rated load) is effected by moving the friction compensating coil F'. If the meter is slow the coil F' is moved in nearer the armature, and

if the meter is fast it is pulled out further from the armature. This adjustment of F' may affect the full load adjustment slightly

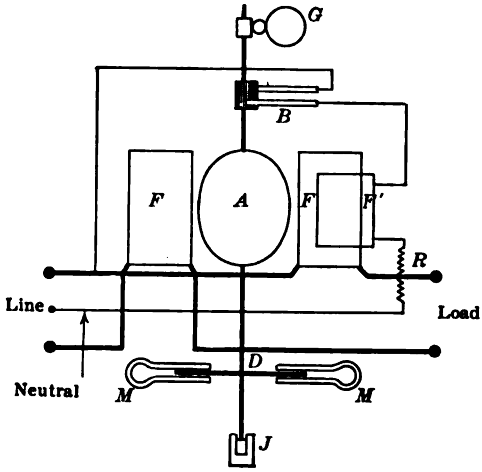


Fig. 147.—Diagram of a 3-wire watt-hour meter.

so that the meter should be re-checked at full load and then again at light load.

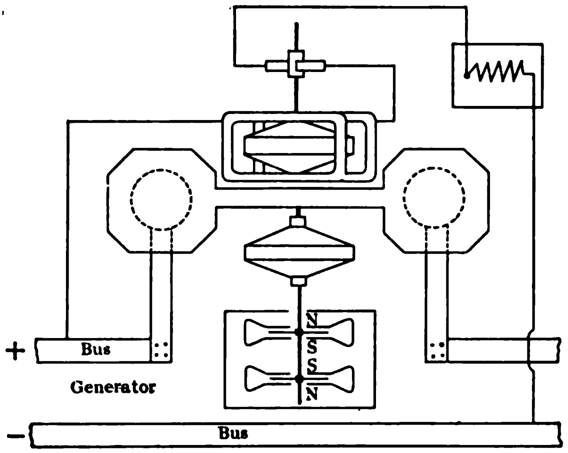


Fig. 148.—Astatic heavy current watt-hour meter.

Other Types of Watt-hour Meters.—The three-wire meter is designed to register energy upon a three-wire system. It does

not differ materially from the meter shown in Fig. 146 except that the two coils *FF* are connected in opposite sides of the line as shown in Fig. 147. The armature circuit may be connected to the neutral as shown or it may be connected across the outer wires. If this latter connection is used the neutral connection to the meter is omitted. In the former case the meter does not register accurately unless the voltages between the two outer lines and neutral are equal. This error is usually small.

The meters already described should not be installed near bus-bars carrying heavy currents because the strength of the meter field and of the retarding magnets may be affected by the stray fields. To eliminate the effect of stray fields an astatic type of meter is used, Fig. 148. There are two armatures on the spindle which rotate in the magnetic field created by a single heavy conductor. One armature is above and the other is below the conductor. Any stray field will presumably strengthen the field in which one armature rotates as much as it will weaken the field in which the other armature rotates so that the resulting effect will be nil. There are two sets of retarding magnets. These magnets are so placed that if the strength of one is increased the strength of the other is decreased. For further protection these magnets are surrounded by an iron box.

CHAPTER VIII

THE MAGNETIC CIRCUIT

133. The Magnetic Circuit.—Although the general nature and characteristics of magnetism were discussed in both Chapters I and II, no quantitative relations were considered. If the magnetic properties of a circuit and the ampere-turns linked with this circuit be known, the magnetic flux can be calculated in the same manner that the current in the electric circuit may be calculated if the resistance and voltage be known. In this respect the two circuits are similar. The magnetic circuit differs from the electric circuit in three respects, which makes it difficult to attain the same degree of precision in magnetic calculations as are obtained in electrical calculations.

The electric current has been considered as confined to a known path, for example, a wire. The surrounding air and the insulating supports for the wire have an extremely high resistance, so that any leakage current which escapes from the wire is negligible compared with the current flowing in the wire or conductor. In the magnetic circuit there is no known insulator for magnetic flux. In fact, the air itself is a fairly good magnetic conductor. Therefore it is impossible to restrict magnetic lines to definite paths in the same way that electric currents are restricted. This is illustrated by the fact that even in the best designed dynamos from 15 to 20 per cent. of the total flux produced leaks across air paths where it cannot be utilized. The presence of this leakage flux may be detected with a compass, and its intensity is often sufficient to magnetize watches even when they are several feet distant from the machine.

Magnetic paths are usually short and have large cross-sections in proportion to their length. They are often so complicated in their geometry that only approximations to their magnetic resistance can be obtained. This often causes errors of considerable magnitude in magnetic calculations.

Under ordinary conditions of use the resistance of most electric conductors is substantially constant, although temperature changes may cause variations of several per cent. Correction for the effect of temperature changes can be accurately made. The magnetic resistance of materials, however, is not constant but varies over wide ranges. This resistance depends to a large extent on the magnetic history of the material. The magnetic resistance of iron may easily increase fifty times when the flux alters from a low to a high magnetic density.

MAGNETIC UNITS

134. Ampere-turns (IN).—The ampere-turns acting on a circuit are given by the product of the turns linked with the circuit and the amperes flowing through these turns. For instance, 10 amp. flowing through 150 turns give 1,500 ampere-turns. The same result is produced by 15 amp. flowing through 100 turns. If any ampere-turns act in opposition, they must be subtracted.

Magnetomotive Force (mmf. also F).—Magnetomotive force tends to drive the flux through the circuit and corresponds to emf. in the electric circuit. It is directly proportional to the ampere-turns of the circuit and only differs from the value of the ampere-turns by the constant factor $0.4\pi = 1.257$. That is, $F = 0.4\pi IN = 1.257\ IN$.

The magnetomotive force of a circuit is measured by the work done in carrying a unit north pole through the entire circuit.

The unit of magnetomotive force is the *gilbert*, but the name gilbert is seldom used in commercial work. The gilberts acting on a circuit are obtained by multiplying the ampere-turns by 0.4π or 1.257.

Reluctance ($\mathcal{R}$).—Reluctance is resistance to the passage of magnetic flux and corresponds to resistance in the electric circuit. The unit of reluctance is that of a centimeter-cube of air. This unit is called the *oersted*. The name oersted is seldom used in commercial work.

Permeance ($\mathcal{P}$).—The permeance of a circuit is the reciprocal of the reluctance ($\mathcal{P} = \frac{1}{\mathcal{R}}$) and may be defined as that property of the circuit which permits the passage of the magnetic flux

or of the lines of induction. It corresponds to conductance in the electric circuit.

Permeability (μ).—The permeability of a material is the ratio of the flux or of the number of lines of induction existing in that material to the flux or the number of lines of induction which would exist if that material were replaced by air, the mmf. acting on this part of the circuit remaining unchanged. The permeability of air is taken as unity and with the exception of iron, steel, nickel, liquid oxygen, and certain iron oxides, all materials may be considered as having a permeability of unity. The permeability of commercial iron and steel ranges from 50 and even lower to about 2,000. In special investigations, vacuum-treated iron has attained a permeability of 5,000 and even greater.

Example.—In a ring solenoid wound on a core similar to that of Fig. 13*a*, page 13, the magnetic flux is found to be 4,000 lines or maxwells. When the iron core is removed the flux in air is but 20 lines. What is the permeability of the iron?

Removing the iron core does not change the ampere-turns and the flux path does not change appreciably. Therefore

$$\mu = \frac{4{,}000}{20} = 200. \quad \textit{Ans.}$$

Flux (ϕ).—The magnetic flux is equal to the total number of lines of induction existing in the circuit and corresponds to current in the electric circuit. The unit of flux is the maxwell, but "line of induction" or simply "line" is more often used.

Flux Density (B).—The flux density is the number of maxwells or of lines of induction per unit area, the area being taken at right angles to the direction of the flux. The unit of flux density in the C. G. S. system is one line per sq. cm. and is called the *gauss*. Flux density is usually expressed in "lines per square centimeter" or "lines per square inch."

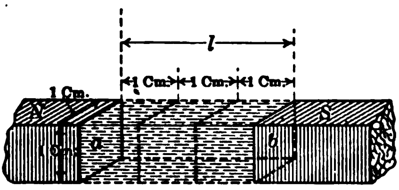
$$B = \phi/A$$

where A is the area and ϕ the flux through and normal to this area.

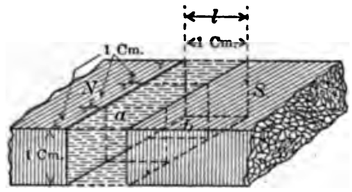
135. Reluctance of the Magnetic Circuit.—The unit reluctance is defined as that of a centimeter-cube of air. If a portion of a magnetic circuit between pole faces a and b, Fig. 149 (a),[1] con-

[1] The actual flux path between pole faces would not exist as shown in Fig. 149 (a), but the flux would "fringe" as shown in Figs. 13b and 14, page 13.

sists of a path in air having a length of 3 cm. and a cross-section of 1 sq. cm. as shown in the figure, this path is equivalent to three centimeter-cubes placed in series. As the total flux must pass successively through each cube, it is evident that the total reluctance is 3 units (oersteds). The reluctance is proportional to the length of the flux path.



(a) Path whose reluctance is 3 oersteds.



(b) Path whose reluctance is 1/3 oersted.

FIG. 149.—Reluctance of simple magnetic paths.

On the other hand, if the path has a length of 1 cm. and a cross-section of 3 sq. cm., as shown in Fig. 149 (b), the reluctance of the path through which the flux passes is one-third that of one cube alone, or 1/3 oersted. The reluctance is inversely proportional to the cross-section of the path.

Moreover, if these paths were in iron, having a permeability μ, the flux would be μ times its value in air, provided the same mmf. were maintained between the two poles faces. This would, of course, mean a lower reluctance. The reluctance of any portion of a magnetic circuit is proportional to its length, inversely proportional to its cross-section and inversely proportional to the permeability of the material. The constant of proportionality is unity, since the reluctance of a path in air 1 cm. long and 1 sq. cm. cross-section is one oersted. Hence,

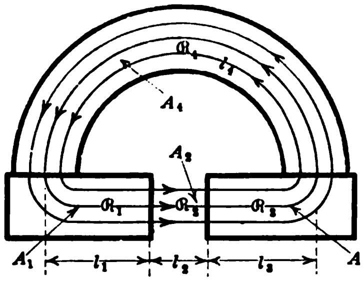


FIG. 150.—Reluctances in series.

$$\mathcal{R}_1 = \frac{l_1}{A_1\mu_1}$$

where l_1 = length in cm. of that part of the circuit under consideration; A_1 = the uniform cross-section in sq. cm. of

that portion of the circuit; and μ_1 = the permeability of that portion of the circuit.

If a magnetic circuit consists of several parts in series as shown in Fig. 150, the total reluctance is:

$$\begin{aligned}\mathcal{R} &= \mathcal{R}_1 + \mathcal{R}_2 + \mathcal{R}_3 + \mathcal{R}_4 \\ &= l_1/A_1\mu_1 + l_2/A_2\mu_2 + l_3/A_3\mu_3 + l_4/A_4\mu_4. \qquad (66)\end{aligned}$$

Permeances in parallel are added together to find the total permeance just as conductances in parallel are added together to find the total conductance.

The total permeance

$$\mathcal{P} = \mathcal{P}_1 + \mathcal{P}_2 + \mathcal{P}_3 + \mathcal{P}_4$$

and reluctances in parallel combine just as resistances in parallel.

$$1/\mathcal{R} = 1/\mathcal{R}_1 + 1/\mathcal{R}_2 + 1/\mathcal{R}_3 + 1/\mathcal{R}_4$$

136. Permeability of Iron and Steel.—The permeability of iron or steel depends on the quality of the material, the flux density and the previous magnetic history.

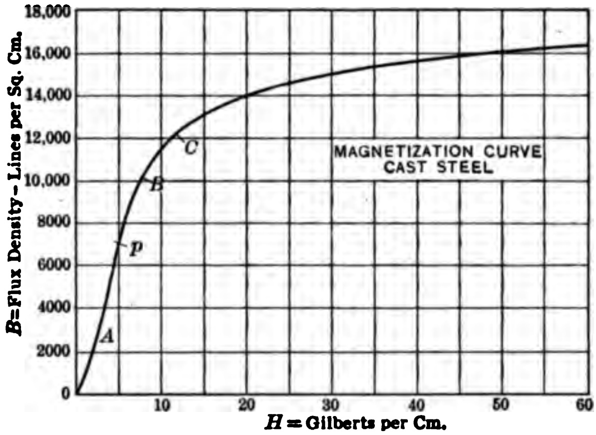


FIG. 151.—Magnetization curve for cast steel.

The relation of the flux in iron or steel to the magnetomotive force cannot be expressed in simple form. It is necessary to show this relation by a curve called the "magnetization curve." Such a curve for one grade of cast steel is shown in Fig. 151. Abscissas are magnetomotive force in gilberts per centimeter (H), and ordinates are the corresponding flux densities (B).

From A to B the curve is practically a straight line. Beyond B the flux density increases much less rapidly for a given increase in magnetomotive force and the iron approaches saturation. The point C, where the bend in the curve is very decided, is the "knee of the curve." Beyond C the flux can be increased but slightly even with a very great increase in the magnetomotive force. The iron is then said to be *saturated.* The type of curve shown in Fig. 151 is called the *normal* saturation or induction curve. Fig. 154 shows normal induction curves for other commercial grades of iron.

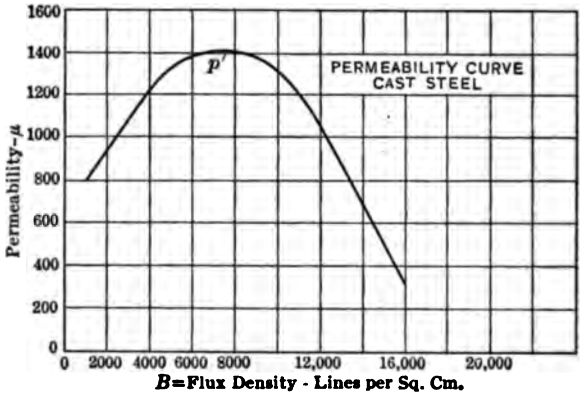


FIG. 152.—Permeability curve for cast steel.

Fig. 152 shows the permeability curve for this same steel. Each ordinate is obtained by dividing B by H[1] for each point of the curve in Fig. 151. It will be noted that the permeability varies over a wide range. It begins at a comparatively low value, increases to a maximum at the point p, and then decreases to about one-fifth its maximum value.

137. Law of the Magnetic Circuit.—The relation between flux, magnetomotive force, and reluctance, for the magnetic circuit, is identical with the relation between current, emf., and resistance for the electric circuit.

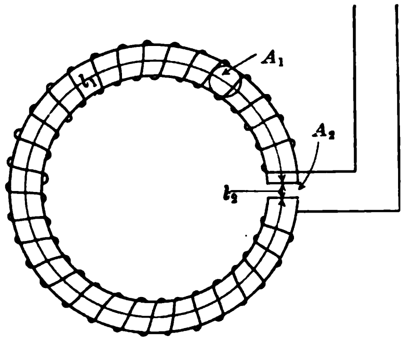
$$\phi = \frac{F}{\mathcal{R}} \tag{67}$$

[1] H, the gilberts per cm., is also equal to the lines per sq. cm. in air, since in air $\phi = H/\mathcal{R}$. $\mathcal{R}$ is unity, being a centimeter-cube, so $\phi = H$ and $\phi = B$, since the cross-section of the cube is 1 sq. cm.

The flux is proportional to the magnetomotive force and inversely proportional to the reluctance of the circuit.

If the magnetic circuit consists of several distinct parts having reluctances $\mathcal{R}_1$, $\mathcal{R}_2$, etc., in series and magnetomotive forces F_1, F_2, from equation (66)

$$\phi = \frac{F_1 + F_2 + F_3 + \ldots}{\dfrac{l_1}{A_1\mu_1} + \dfrac{l_2}{A_2\mu_2} + \dfrac{l_3}{A_3\mu_3} + \ldots} = \frac{0.4\,\pi IN}{\dfrac{l_1}{A_1\mu_1} + \dfrac{l_2}{A_2\mu_2} + \dfrac{l_3}{A_3\mu_3} + \ldots} \qquad (68)$$



$l_1 = 18$ in. $A_1 = A_2 = 0.2$ sq. in.
$l_2 = \frac{3}{16}$ in.

FIG. 153.—Ring-type electromagnet.

Example.—The ring magnet, Fig. 153, is wound with 250 turns of wire, through which a current of 1.5 amp. flows. Assume the permeability of the iron to be 800. Neglecting fringing, determine the flux in the ring, and also the flux density.

$F = 0.4\pi \times 1.5 \times 250 = 471$
$l_1 = 18$ in. $= 18 \times 2.54 = 45.7$ cm.
$l_2 = \frac{3}{16}$ in. $= \frac{3}{16} \times 2.54 = 0.476$ cm.
$A_1 = A_2 = 0.2$ sq. in. $= 0.2 \times 2.54 \times 2.54 = 1.29$ sq. cm.

From equation (68)

$$\phi = \frac{471}{\dfrac{45.7}{1.29 \times 800} + \dfrac{0.476}{1.29 \times 1.0}} = \frac{471}{0.0443 + 0.369} =$$

1,140 lines (maxwells). *Ans.*

The flux density:

$$B = \frac{1{,}140}{1.29} = 884 \text{ lines per sq. cm. (gausses)}$$
$$= 5{,}700 \text{ lines per sq. in.}$$

138. Method of Trial and Error.—Magnetic problems cannot be solved readily by the method used in Par. 137. This is due

to the fact that the permeability (which is a variable but is given in the problem as a constant value of 800) is not ordinarily known until the flux density is known and curves similar to those of Figs. 151 and 152 consulted. Therefore the permeability is not known until the answer has been determined. As the answer in turn depends upon the permeability, it is usually necessary to resort to trial and error.

Example.—The iron ring of Fig. 153 and Par. 137 is made of cast steel whose permeability curve is given in Fig. 152. The air gap is reduced to $\frac{1}{16}$ in. Determine the flux and the flux density.

Assume that the permeability is 800.

$$\mathcal{R}_1 = \frac{18.13 \times 2.54}{1.29 \times 800} = 0.0446$$

$$\mathcal{R}_2 = \frac{\frac{1}{16} \times 2.54}{1.29} = 0.123$$

$$\phi = \frac{471}{0.0446 + 0.123} = 2{,}810 \text{ maxwells}$$

$$B = \frac{2{,}810}{1.29} = 2{,}180 \text{ gausses.}$$

From Fig. 152 the permeability at this density is 980. Therefore $\mathcal{R}_1$ must be recalculated using the new value of permeability.

$$\mathcal{R}_1 = \frac{18.13 \times 2.54}{1.29 \times 980} = 0.0365$$

$$\phi = \frac{471}{0.0365 + 0.123} = 2{,}950 \text{ maxwells.}$$

The new value of B = 2,290 gausses.

As the value of μ corresponding to this flux density is 990 or sufficiently close to the value 980 just used, the last two values of flux and flux density are substantially correct.

139. Determination of Ampere-turns.—It was shown in Par. 68, Chap. IV, that the voltage drop per unit length of a conductor is independent of the total current but depends only upon the *current density* and the resistivity of the conductor. In a similar manner the magnetomotive force per unit length depends only upon the *flux density* and the reluctivity of the material. This is proved as follows:

Writing equation (68) for one portion of the circuit,

$$\phi = \frac{F}{\frac{l}{A\mu}}$$

Since $\phi = BA$, where A is the cross-section of the path,

$$BA = \frac{F}{\dfrac{l}{A\mu}}$$

$$\frac{F}{l} = \frac{B}{\mu}$$

$$F = \frac{Bl}{\mu} \tag{69}$$

The magnetomotive force is equal to the product of the ***flux density*** and the length of the magnetic path, divided by the permeability of the material. To determine the magnetomotive

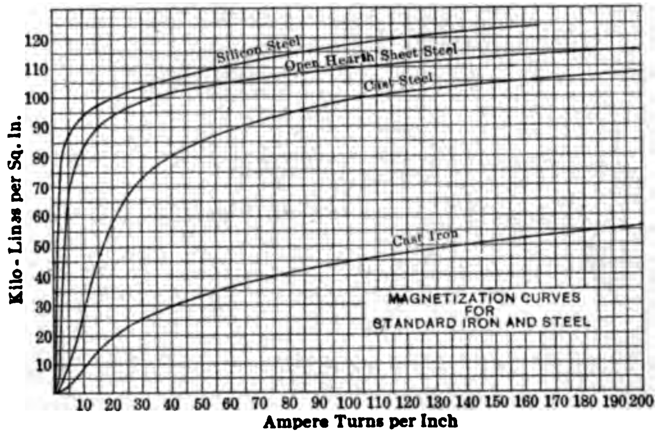


Fig. 154.—Typical magnetization curves.

force for a unit length of a circuit it is only necessary to know the flux density and the permeability. Instead of plotting the permeability against flux density the magnetization curve is usually plotted with ampere-turns per unit length as abscissas and the corresponding flux density as ordinates. This is more convenient and avoids using 0.4π and also the permeability. Such curves are shown in Fig. 154 for various commercial steels used in the manufacture of electrical machinery.

In problems where flux and the cross-section of the magnetic paths are known, and it is desired to find the requisite ampere-turns to produce this flux, the curves just referred to, enable the solution to be readily obtained.

140. Use of the Magnetization Curves.—To illustrate the use of the magnetization curves the following problem is given.

Example.—Determine the ampere-turns necessary to produce an air-gap flux of 750,000 lines in the electromagnet of Fig. 155. The cores are cast iron and the yoke and pole pieces are cast steel. Neglect fringing and leakage.

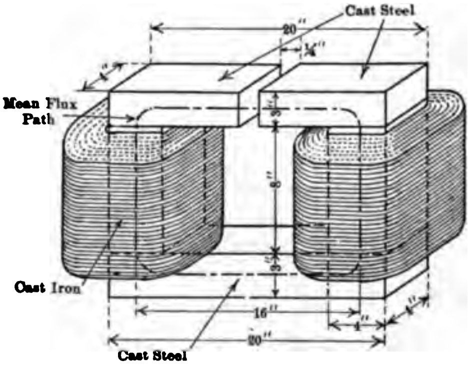


FIG. 155.—Typical electromagnet.

The flux density in the lower yoke:

$$B_1 = \frac{750{,}000}{3 \times 4} = 62{,}500$$

The ampere-turns per inch for a density of 62,500, from Fig. 154 (cast steel), is 23.

The mean length of flux path is (approximately) 16 in.

$$I_1N_1 = 16 \times 23 = 368$$

or 368 ampere-turns is required to produce a flux of 750,000 lines in the lower yoke.

The density in the cores is

$$B_2 = \frac{750{,}000}{4 \times 4} = 46{,}900$$

From the curve (cast iron) the ampere-turns per inch = 118.

As there are two cores, the total length will be 16 in.

$$I_2N_2 = 16 \times 118 = 1{,}890$$

The pole pieces are in every way identical with the yoke, except that the path is 0.25 in. shorter. This small difference will not make any appreciable error, so the amperes-turns for the two poles pieces are:

$$I_3N_3 = 368$$

For the air gap, the mmf. $= 0.4\pi IN = \dfrac{Bl}{\mu}$

where B is in lines per sq. cm. and l is in cm.

$$IN = \frac{Bl}{0.4\pi} = 0.796\,Bl \qquad (70)$$

as $\mu = 1$ for the air gap.

In inch units

$$IN = 0.796\,\frac{Bl}{(2.54)^2}\,(2.54)$$

$$IN = 0.313B'l' \qquad (71)$$

where B' = lines per sq. in. and l' the length of the path in inches.

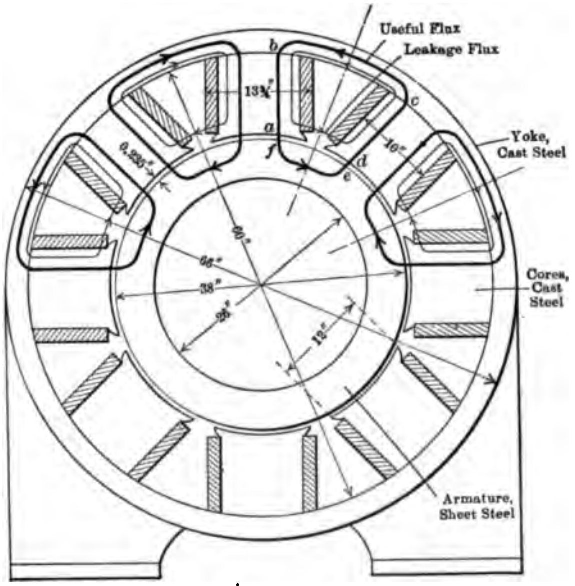
The ampere-turns for the air gap then become

$$I_4N_4 = 0.313 \times 62{,}500 \times 0.25 = 4{,}900 \text{ (from equation 71).}$$

As all the various parts are in series the total ampere-turns =

$$368 + 1{,}890 + 368 + 4{,}900 = 7{,}526. \quad \textit{Ans.}$$

141. Magnetic Calculations in Dynamos.—The magnetic circuits of dynamos have already been discussed in Chap. II.



Axial length of armature stampings and pole-faces = 16 in.

FIG. 156.—8-pole 100 R.P.M. 250-volt D.C. generator.

The calculation of the exciting ampere-turns is somewhat complicated by the irregular nature of the air gap, due to the arma-

ture teeth, air ducts, fringing, etc. The amount of leakage flux between poles introduces another factor which must be considered.

As a simple example of such calculations, consider the dynamo shown in Fig. 156. It is desired to send a flux of 7,500,000 lines from each pole into the armature. The air gap has an effective length of 0.235 in., after correction has been made for armature teeth, fringing, etc. The leakage coefficient (ratio of core flux to armature flux) is equal to 1.15.

The paths of the fluxes from the various poles, including the leakage flux, are shown in the figure. The lengths of path are easily determined. Consider the flux path *abcdef*.

The length $ab = \frac{60'' - 38''}{2} - 0.235 = 10.8$ in. (approximately)

bc (approximately one-eighth the mean circumference of the yoke, less 5 in.) $= \frac{\pi 63''}{8} - 5'' = 24.7'' - 5'' = 19.7$ in.

$$fe = \frac{\pi 32}{8} = 12.6 \text{ in.}$$

The flux densities are as follows:

Flux in cores $= 7{,}500{,}000 \times 1.15 = 8{,}630{,}000$ as the flux in the core is equal to the armature flux plus the leakage flux.

Flux density in cores $= \frac{8{,}630{,}000}{16 \times 10} = 54{,}000.$

Flux density in yoke $= \frac{8{,}630{,}000}{2(16 \times 3)} = 90{,}000$ as the pole flux divides, one-half going each way in the yoke.

Flux density in armature $= \frac{7{,}500{,}000}{2(6 \times 16)} = 39{,}000.$

This must be increased about 25 per cent. to allow for the air duct space and the spaces between laminations.

This makes the density in the armature:

$$39{,}000 \times 1.25 = 48{,}800$$

The air-gap density $= \frac{7{,}500{,}000}{16 \times 12} = 39{,}000$

Knowing the above factors, and utilizing the magnetization curves of Fig. 154, it is a comparatively simple matter to determine the total ampere-turns per pole.

For 54,000 lines per sq. in., 19 ampere-turns per inch are necessary for cast steel (Fig. 154). Therefore for *ab*:

Core *ab* I_1N_1	$= 19 \times 10.8$	$= 205$ (cast steel).	
Yoke *bc* I_2N_2	$= 64 \times 19.7$	$= 1{,}260$ cast steel).	
Core *cd* I_3N_3	$= I_1N_1$	$= 205$ (cast steel).	
Gap *de* I_4N_4	$= 0.313 \times 39{,}000 \times 0.235$	$= 2{,}870$ (air).	(See equation 71)
Arm. *ef* I_5N_5	$= 3 \times 12.6$	$= 38$ (O. H. sheet steel).	
Gap *fa* I_6N_6	$= I_4N_4$	$= 2{,}870$ (air).	
	Total	$= 7{,}448$ ampere-turns.	

As two poles in series supply the excitation for this flux the ampere-turns per pole are

$$IN = 7{,}448/2 = 3{,}724. \quad \textit{Ans.}$$

As this machine is symmetrical, each complete magnetic circuit requires this same number of ampere-turns per pole. The design of the exciting coils themselves is not a difficult matter.

142. Hysteresis.—If the magnetomotive force acting on an iron sample begins at zero and increases, the relation between mmf. and the flux (or flux density) will be similar to that shown by curve Oa (Fig. 157). This curve is called the *normal saturation* or magnetization curve and has already been discussed.

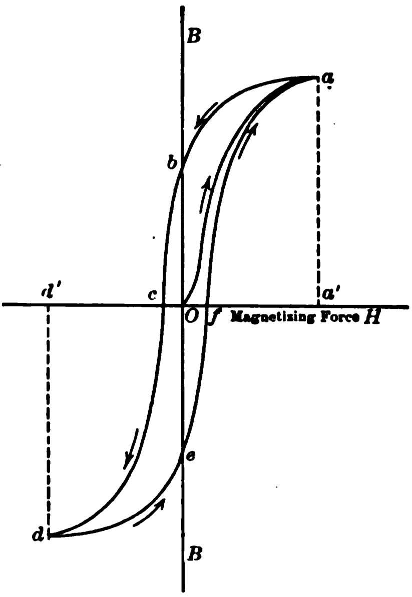


Fig. 157.—Hysteresis loop.

If the magnetomotive force now decreases, the flux will *not* decrease along the line aO, but will decrease less rapidly along ab. When point b is reached, the mmf. is zero but the magnetic induction has not reached zero. The flux density Ob is called the *remanence.* Before the flux density can be reduced to zero, the magnetizing force must be reversed in direction. That is, it requires a negative magnetizing force Oc to reduce the flux density to zero. The magnetizing force Oc is called the *coercive force.*

If now the magnetizing force be increased in the negative direction to d' where $Od' = Oa'$, the flux density will be carried to a negative maximum d. The negative maximum flux density $d'd$ is equal to $a'a$. If the magnetizing force is now increased toward zero, the curve will pass through point e when the magnetizing force is again zero and the negative remanence $Oe = Ob$. A positive coercive force $Of = Oc$ is necessary to bring the flux density again to zero. When the magnetizing force again becomes Oa'

the flux density will return to its original value at a, closing the loop.

This is one complete cycle of magnetization, and the curve is called a *hysteresis loop*. Such a loop shows that the magnetization in iron lags behind the magnetomotive force per centimeter or the magnetizing force, and that an expenditure of energy is required to carry the iron through a cycle of magnetization.

If several loops are taken, each having different maximum flux densities, they will have the appearance of the three loops shown in Fig. 158. The maximum points a, a_1, a_2 all lie along the normal saturation curve Oa_2.

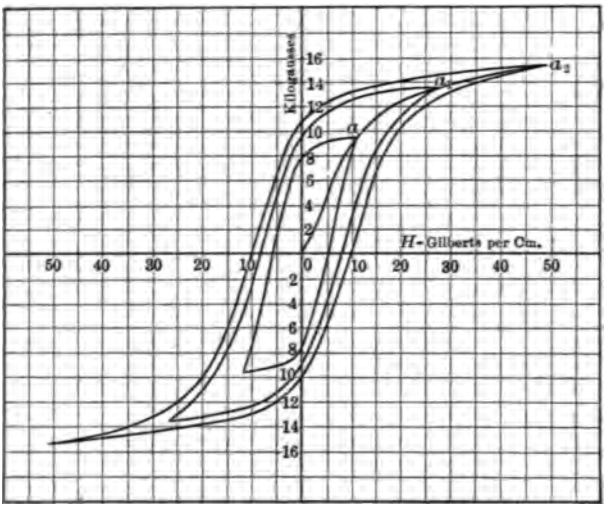


Fig. 158.—Hysteresis loops for three maximum flux densities.

143. Hysteresis Loss.—The hysteresis loss is proportional to the area of the hysteresis loop, Figs. 157 and 158. In fact the hysteresis loss may be obtained by finding the area of the loop to scale, and dividing by 4π. This gives the loss in ergs per cycle.

For example, let the area of the smallest loop, Fig. 158, be A sq. in. The scale is such that 1 in. on the abscissa scale represents 10 gilberts per cm., and 1 in. on the ordinate scale represents 4 kilogausses. The ergs loss per cycle is:

$$W_h = \frac{A \times 10 \times 4{,}000}{4\pi} \text{ ergs.}$$

To get this energy loss into joules or watt-seconds divide by 10^7.

The hysteresis loss per cycle depends upon two factors, the magnetic material and the maximum flux density. The loss within certain limits may be expressed by the Steinmetz Law as follows:

$$W_h = \eta B^{1.6} \tag{72}$$

W_h is the hysteresis loss per cu. cm. in ergs per cycle, η is a constant depending on the material, and B is the maximum flux density in gausses.

Below are given a few typical values of η:

Hard cast steel	0.025	Sheet iron	0.004
Forged steel	0.020	Silicon sheet steel	0.0010
Cast iron	0.013	Silicon steel	0.0009

Example.—What will be the ergs loss per cycle in a core of sheet iron having a volume of 40 cu. cm., in which the maximum flux density is 8,000 gausses?

$$W_h = 0.004 \times 8{,}000^{1.6}$$

$$\log 8{,}000 = 3.9031$$
$$1.6 \times 3.9031 = 6.2449$$
$$\log 1{,}757{,}000 = 6.2449$$

$W_h = 0.004 \times 1{,}757{,}000 = 7{,}028$ ergs per cu. cm. per cycle.
Total loss $W = 7{,}028 \times 40 = 281{,}000$ ergs per cycle. *Ans.*

INDUCTANCE

144. Linkages.—If a current flows in a conductor, a magnetic flux is set up about the conductor. This magnetic flux completely encircles the conductor and the current in the conductor completely encircles the flux. Some familiar examples of this are given in Fig. 159, where the currents and related fluxes are shown. As a current and the resulting flux always completely encircle each other they are said to *link* with each other. This is shown particularly well in Fig. 159 (*c*), where a conductor carrying a current is linked with an anchor ring.

The product of the turns of conductor and the number of lines of flux linking these turns is called the *linkages* of the circuit.

Example.—A certain solenoid has 800 turns. A current of 5 amp. flowing in the winding produces a flux of 2,500,000 lines. What are the linkages?

$$800 \times 2{,}500{,}000 = 20 \times 10^8 \text{ linkages.}$$

The number of *these linkages per unit current* in a circuit is called the *inductance* of the circuit and is represented by the symbol "*L*," implying linkages. The unit of inductance is the ***henry***.

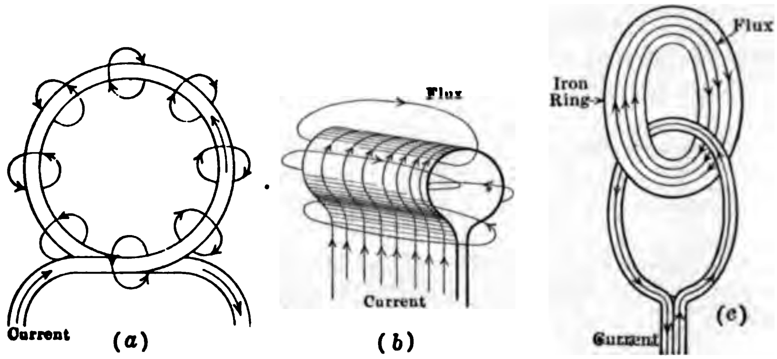


FIG. 159.—Illustrations of flux-current linkages.

Inductance from definition:

$$L = \frac{N\phi}{I \times 10^8} \tag{73}$$

where L is the inductance in henrys, ϕ is the flux in maxwells, and I is the current in amperes.

Note.—It is necessary to divide by 10^8 because 10^8 magnetic lines are equal to *one* line in the practical system of volts, amperes, etc.

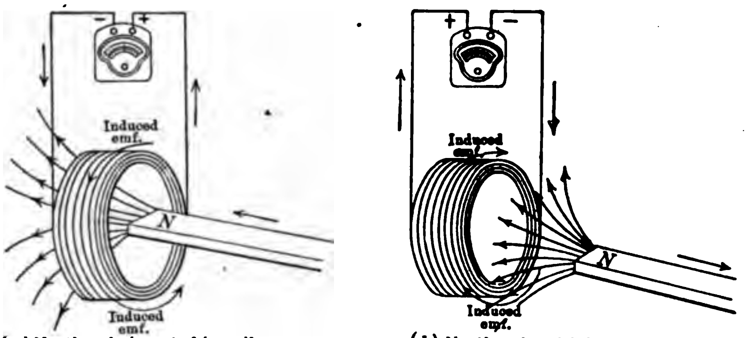
Example.—What is the inductance of the above circuit?

$$L = \frac{20 \times 10^8}{5 \times 10^8} = 4.0 \text{ henrys.}$$

145. Induced Electromotive Force.—If the terminals of an insulated coil, Fig. 160 (*a*), be connected to a galvanometer, and a magnetic field be set up through this coil, either by thrusting a bar magnet into the coil or by some other means, the galvanometer will be observed to deflect momentarily and then to return

to rest. This shows that an emf. has been temporarily induced in the coil. When the flux through the coil has ceased to change, this electromotive force also ceases. If investigation be made, it will be found that the direction of this induced electromotive force is that shown in the figure and that this direction is such that if the emf. be allowed to produce a current, this current will tend to push the bar magnet *out* of the coil, or what is the same thing, will oppose its entering the coil.

If the magnet be withdrawn from the coil, Fig. 160(b), the galvanometer will be observed to deflect again, momentarily as before, but the deflection is opposite to its direction in the first



(a) North pole inserted in coil (b) North pole withdrawn

FIG. 160.—Induced electromotive force.

case. The direction of the induced electromotive force is now such that if the emf. produces a current, this current will tend to prevent the magnet from being withdrawn from the coil. The electromotive force in each case is transient and ceases when the *change* of flux through the coil ceases.

If careful measurements be made, the value of this electromotive force will be found to depend upon: (1) the number of turns in the coil, (2) the rate at which the flux linked with the coil changes.

The average electromotive force in volts is given by

$$e = -\frac{N\phi\, 10^{-8}}{t} \tag{74}$$

where N is the number of turns in the coil, ϕ is the total change of flux in lines linked with the coil, and t is the time in seconds re-

quired to insert or withdraw the flux from the coil. [illegible] reduces the flux ϕ to practical units so that e becomes volts. The minus sign indicates that the induced emf. is in opposition to the force which produces it.

$\frac{\phi}{t}$ is the average rate of change of flux, so that the induced electromotive force may be said to be proportional to the *number of turns and the rate of change of flux.*

Example. A flux of 1,500,000 lines links a coil having 350 turns. This flux through the coil is decreased at a uniform rate to zero in 0.2 second. What is the induced electromotive force during the time of withdrawal?

$$e = 350 \frac{1{,}500{,}000}{0.2} 10^{-8}$$
$$= 26.25 \text{ volts. } Ans.$$

The fact that the currents produced by induction oppose the motion producing them should be carefully noted, for this principle is manifest in practically all types of electric machinery. The principle was first formulated by Lenz in a form known as *Lenz's Law* which says:

"*In all cases of electromagnetic induction, the induced currents have such a direction that their reaction tends to stop the motion which produces them.*"

This law is also based upon the law of the conservation of energy. That is, the induced currents, which represent energy, are produced at the expense of the mechanical energy required to push the magnet in the coil against their opposition, or the energy required to withdraw the magnet against the opposition of the induced currents, which try to prevent this withdrawal.

146. Electromotive Force of Self Induction.—If a coil be connected to a battery and a switch *S* closed (Fig. 161), current will begin to flow in the coil. This current produces a flux linking the coil. As this flux increases it must induce an emf. in the coil, the magnitude of which depends on the number of turns in the coil and the rate at which the flux increases. By Lenz's Law, and also from a consideration of Fig. 160 (*a*), the electromotive force thus induced must have such a direction as to oppose the increase in the flux linking the coil and hence must *oppose* any increase of current. Therefore this current cannot reach its

maximum value at once, but is retarded in its rise by the opposing electromotive force.

In Fig. 162 is shown the rise of current in a circuit containing resistance only, the impressed voltage being 10 volts and the re-

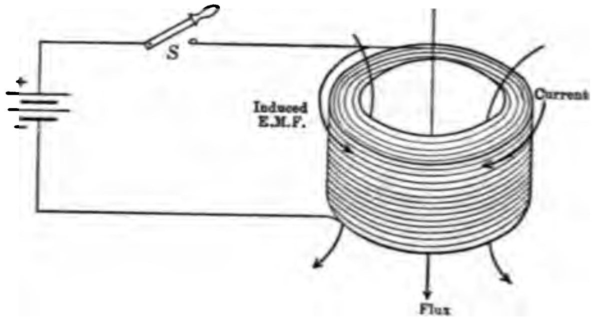


FIG. 161.—Relation of emf. of self-induction to current.

sistance 20 ohms. When the switch S is closed the current reaches its maximum or Ohm's Law value of 0.5 amp. at once.

In the case of the inductive circuit, the current approaches its Ohm's Law valuc more or less slowly. To be exact, it takes an infinite time for the current to reach its Ohm's Law value, al-

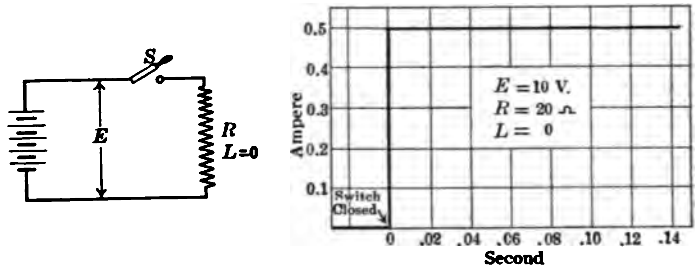


FIG. 162.—Rise of current in a non-inductive circuit.

though in a comparatively short time it reaches substantially this value. An idea of the time required to build up a current in an inductive circuit may be obtained from the inductance and the resistance of the circuit. The ratio of the inductance in *henrys* to the resistance in ohms, L/R, is called the *time constant* of the circuit. This is the time in seconds required for the current to reach 63.2 per cent. of its final value. It is a measure of the

rapidity with which the current in a circuit rises to its ultimate value. Fig. 183 shows the rise of current in a circuit whose impressed voltage is 10 volts, resistance 20 ohms and inductance 0.6 henry. The time constant of this circuit is 0.6/20 = 0.03 seconds and is shown in the diagram. This curve should be compared with Fig. 182 in which the circuit has the same impressed voltage and the same resistance but has no inductance.

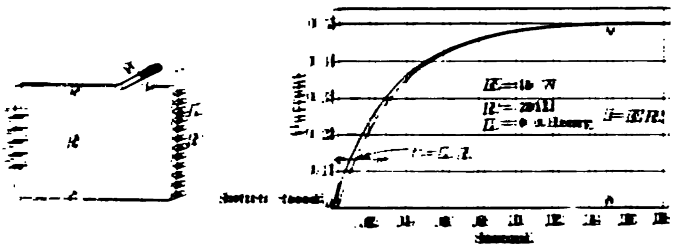


Fig. 183.—Rise of current in an inductive circuit.

Example.—A relay having a resistance of 400 ohms and an inductance of 0.6 henry is connected across a 110-volt circuit. What is the time constant of the relay? To what value does the current in the relay rise in this time?

$$\text{The time constant} = \frac{0.6}{400} = 0.0015 \text{ second.}$$

$$i = 0.632 \frac{110}{400} = 0.1738 \text{ amp.}$$

This delayed rise of current in a circuit due to self inductance should be carefully kept in mind, since it accounts for some of the time lag observed in relays, trip coils, etc. When a short-circuit takes place there may be considerable delay between the time at which the short-circuit occurs and the opening of the breaker or switch controlled by the relay. The effect of inductance is also one of the controlling factors in the initial current-rush on short-circuit.

If an inductive circuit carrying current be short-circuited, the current does not cease immediately, as it does in a non-inductive

The equation of the curve showing the rise of current is $i = \frac{E}{R}\left(1 - \epsilon^{-\frac{Rt}{L}}\right)$ where E is the impressed voltage, i = current at time t seconds after closing switch, and ϵ the Naperian logarithmic base. The current increases at a rate of E/L amp. per second at the instant when the switch is closed.

circuit under similar conditions, but continues to flow and does not become zero until an appreciable time after the instant of the short-circuit. This is due to the electromotive force of self induction. The flux linking the coil is due to the current, and when the current decreases, this flux also decreases. In decreasing, the flux induces an electromotive force in the coil. In the same way that the current due to the induced electromotive force tended to prevent the flux being withdrawn in Fig. 160(*b*), so now the electromotive force of self-induction tends to prevent the decrease of the current.

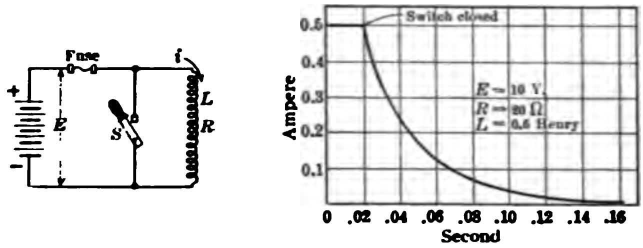


FIG. 164.—Decay of current in an inductive circuit.

A curve[1] showing the decrease of the current with time is given in Fig. 164. The circuit has the same constants as the circuit shown in Fig. 163. It is usually advisable to fuse the battery so that it will not be injured, since short-circuiting the inductive circuit also short-circuits the battery, as is shown in Fig. 164.

It thus appears that the effect of inductance is always to oppose any change in circuit conditions. If the current tends to increase, inductance opposes it; if it tends to decrease, inductance tends to oppose this decrease. Inductance corresponds to inertia in mechanics. A body having inertia opposes any force tending to set it in motion when the body is at rest, and if the body is in motion, inertia opposes any force tending to bring the body to rest.

[1] The equation of this curve,

$$i = I_0 \epsilon^{-\frac{Rt}{L}}$$

where i is the value of the current at a time, t seconds after the closing of the switch, and I_0 is the initial value of current.

If, after having established the current in the circuit of Fig. 163, the switch S be opened, a noticeable arc will appear at the switch blades. This arc will be much greater in magnitude than that formed at the contacts of the switch in the circuit of Fig. 162, with resistance only in the circuit, although the current and circuit voltage are the same in each case. This arc is due to the electromotive force of self induction and in some circuits may have such a value as to cause severe arcing at the switch contacts. In fact this voltage has been known to reach such values in alternator fields as to puncture their insulation when the field

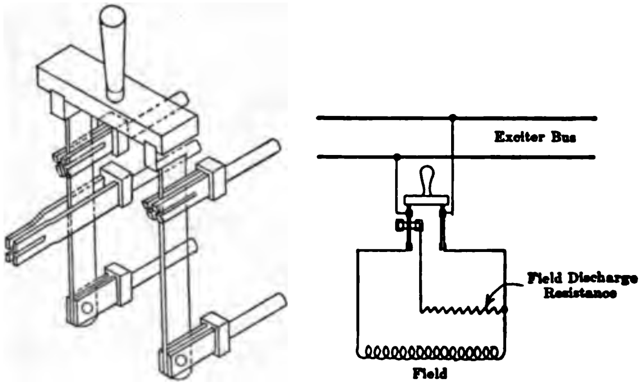


Fig. 165.—Field-discharge switch with connections.

circuit is opened. To protect the field from puncture, a field discharge switch shown in Fig. 165 is often used. At the instant of opening the switch the field (and the line temporarily) is paralleled by the field discharge resistance. The energy of the field is dissipated partly in this resistance rather than at the switch contacts. Contact with switches opening inductive circuits, even in the case of very low voltages, should be carefully avoided. Not only is there the danger of being burned by the arc, but of being injured from the high induced voltages as well.

Calculation of the Electromotive Force of Self Induction.—From equation (74) page 185, the electromotive force induced in a coil due to a change in the flux linking the coil is

$$e = -N\frac{\phi}{t}10^{-8}$$

where N is the number of turns, and ϕ/t the rate at which the flux changes.

Remembering that

$$L = \frac{N\phi}{I} 10^{-8} \text{ or } N\phi\, 10^{-8} = LI \text{ (equation 73, page 184),}$$

and also that the electromotive force of self induction opposes the change in current, its value may be written:

$$e = -\frac{N\phi 10^{-8}}{t} = -L\frac{I}{t} \qquad (75)$$

The electromotive force of self induction is proportional to the product of the inductance and the rate of change of current with respect to time. The minus sign indicates that this electromotive force *opposes* the change of current.

If the inductance varies as well as the flux, equation (75) may be written:

$$e = -\left(L\frac{I}{t} \pm I\frac{L}{t}\right) \qquad (76)$$

the additional term accounting for the electromotive force due to any change in the inductance.

Example.—The field circuit of a generator has an inductance of 6 henrys. If the field current of 12 amp. is interrupted in 0.05 second, what is the average induced electromotive force in the field winding?

$$e = 6\,\frac{12}{0.05} = 1{,}440 \text{ volts.} \quad \textit{Ans.}$$

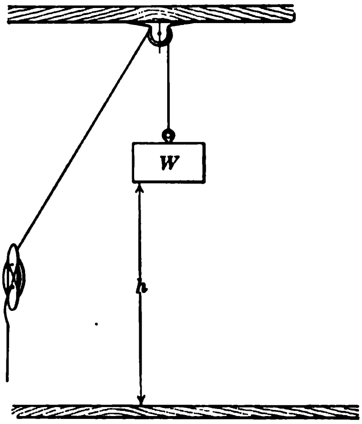


Fig. 166.—Energy of a suspended weight.

147. Energy of the Magnetic Field.—To *establish* a magnetic field energy must be expended. To maintain a constant field does not require an expending of energy even in electromagnets. The energy lost in the exciting coils of electromagnets is accounted for as heat in the copper and is not concerned with the energy of the magnetic field itself. The energy of the magnetic field is stored or potential energy and is similar to the energy of a raised weight, Fig. 166. Work is performed in raising the

weight to its position, but no expenditure of energy is required to maintain the weight in this position. The energy of the weight due to its position is Wh foot-pounds, where W is the weight in pounds and h the height in feet through which the weight has been raised. This energy is available and can be utilized in many ways.

In the same way the energy stored in the magnetic field is available and may make itself manifest in many ways, as, for example, the arc at the switch contacts. In an alternating current circuit this energy may all be returned to the circuit.

The energy of the field in joules, or watt-seconds, is

$$W = 1/2LI^2 \qquad (77)$$

where L is the circuit inductance in henrys and I the current flowing.

Example.—In a circuit having an inductance of 4 henrys, the current is 10 amp. What is the energy of the magnetic field? If this circuit is interrupted in 0.2 second, what is the average value of the power expended by the magnetic field during this time?

$$W = 1/2 \times 4 \times 10^2 = 200 \text{ watt-seconds. } \textit{Ans.}$$

$$P = \frac{200}{0.2} = 1{,}000 \text{ watts} = 1 \text{ kilowatt. } \textit{Ans.}$$

Equation (77) shows that the energy of the magnetic field is proportional to the *square* of the current. Therefore if the current can be reduced by a suitable resistance to one-half its initial value before opening a highly inductive circuit, the energy of the arc at the switch contacts can be reduced to one-fourth of its initial value. This fact should be remembered when opening the field circuit of a dynamo.

A very common use of the electromotive force of self induction occurs in the so-called spark coil used for gas lighting. This coil consists of a considerable number of turns of wire wound on a laminated iron core. The core is usually made of iron wires as shown in Fig. 167. This coil is connected between the bell-ringing battery B and the grounded gas pipe. The other terminal of the battery is connected directly to the insulated contact on the gas burner. When the two contacts on the burner meet, the circuit is closed, and a magnetic field is established in the laminated core of the spark coil. As the two contacts of the

burner separate, they snap apart and the circuit is broken suddenly. Consequently, a high electromotive force of self induction is produced in the spark coil. This causes a hot arc at the contacts, which ignites the gas, the gas being turned on simultaneously with the closing of the contact points and by the same mechanism.

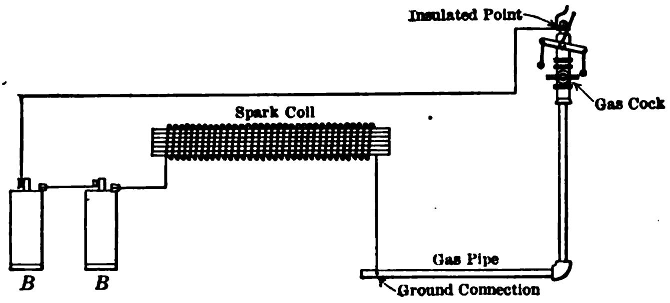


FIG. 167.—Electric gas ignition.

The spark coil may be considered as having a magnetic field which is built up as the two contacts at the gas jet wipe by each other. Energy is thus stored in the magnetic field. When this energy is released suddenly by the contacts snapping open, considerable power is developed resulting in a hot spark at the contact points.

148. Mutual Inductance.—In Fig. 168 are shown two coils, A and B. Coil A is connected to a battery through a switch S. Coil B is not connected to any source of voltage, but to a galvanometer. Coil B is placed so that its axis is nearly coincident with that of A and the two coils are close together. When the switch S is closed, current flows in coil A, building up a field which links the coil. The position of B with regard to A results in a considerable part of the magnetic flux produced by A linking B. Therefore, if the current in A be interrupted by opening the switch S, or if it be altered in magnitude, a *change of flux simultaneously* occurs in B inducing an emf. in B. This emf. is detected by the galvanometer connected across the terminals of B. Upon closing the switch S the galvanometer will deflect momentarily, and upon opening the switch S its deflection will reverse, showing that the induced voltage on opening the circuit is opposite in direction to the induced voltage on closing the

circuit. Because coil B is in such a relation to A that an emf. is induced in B due to the change of flux in A, these two coils are said to possess *mutual inductance*. The induced emf. is an

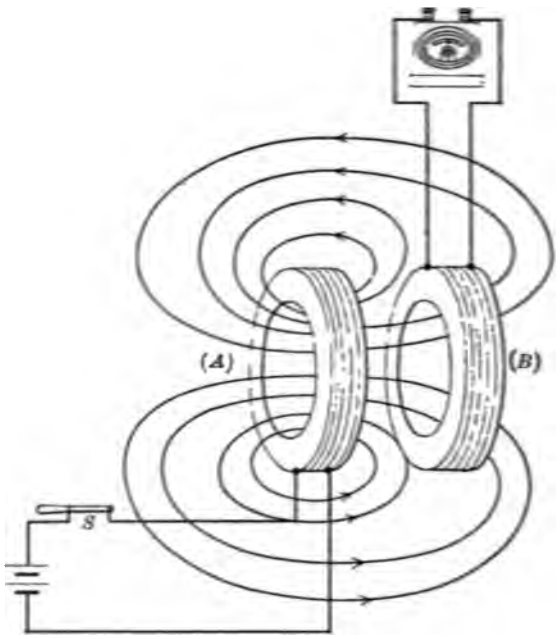


FIG. 168.—Mutual inductance between two coils.

electromotive force of mutual induction and its magnitude, equation (73), page 184, is

$$e_2 = N_2 \frac{\phi_2}{t} 10^{-8} \text{ volts}$$

where N_2 is the number of turns in coil B, ϕ_2 the change in magnetic flux from coil A which links coil B, and t the time in seconds required to change the flux by ϕ_2 lines.

Even though coils A and B be brought close together, all the flux, ϕ_1, produced by coil A does not link coil B. Only a certain proportion, K, of ϕ_1 links B, K being less than unity. That is:

$$e_2 = N_2 \frac{K\phi_1}{t} 10^{-8} \text{ volts} \qquad (78)$$

K is often called the *coefficient of coupling* of the circuits A and B. As N_2 and K are constants for any particular circuit and position and ϕ_1 may be assumed proportional to I_1, the current in coil A, equation (78), may be written

$$e_2 = M \frac{I_1}{t} 10^{-8} \text{ volts} \tag{79}$$

where M is the *mutual inductance* or coefficient of mutual induction in henrys between coil A and coil B.

$$M = \frac{KN_2\phi_1}{I_1} \tag{80}$$

The mutual inductance of two circuits may be defined as that factor which when multiplied into the time rate of change of current in one circuit, gives the induced voltage in the other circuit.

Example.—Coil A (Fig. 168) has 400 turns and coil B has 600 turns. When 5 amp. flow in coil A, a flux of 500,000 lines links with A, and 200,000 of these lines link coil B. What is the self inductance of coil A with B open-circuited, and what is the mutual inductance of the two coils?

$$L_1 = \frac{N\phi_1}{I} = \frac{400 \times 500{,}000}{5} 10^{-8} = 0.4 \text{ henry. } \textit{Ans.}$$

The induced voltage in B due to the current in A rising to 5 amp. in 1 second will be

$$e_2 = N_2 \frac{\phi_2}{t} = 600 \times 200{,}000 \times 10^{-8} = 1.2 \text{ volts.}$$

as a change of 5 amp. in coil A changes the flux in coil B by 200,000 lines. Therefore:

$$e_2 = M \frac{I_1}{t}$$

$$1.2 = M \times \frac{5}{1}$$

$$M = 0.24 \text{ henry. } \textit{Ans.}$$

or using equation (80)

$$M = \frac{0.4 \times 600 \times 500{,}000}{5} 10^{-8} = 0.24 \text{ henry.}$$

The mutual inductance of two circuits may be materially increased by linking the circuits with an iron core. Thus, if the two coils, similar to those shown in Fig. 168, be placed upon an

iron core (Fig. 169) the coefficient of coupling, K, may be made very nearly unity. That is, practically all the flux linking coil A also links coil B.

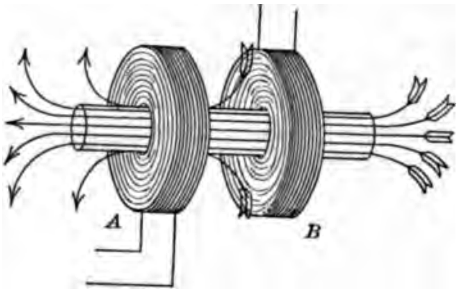


FIG. 169.—Effect of iron core upon mutual inductance.

A very common example of mutual inductance occurs in the induction coil (Fig. 170). A primary winding, P, of comparatively coarse wire and few turns, is wound on a laminated iron core C. This winding is connected to a battery B. The primary current is interrupted by passing through the contact D, against which the iron armature A is held by a spring. When the core C

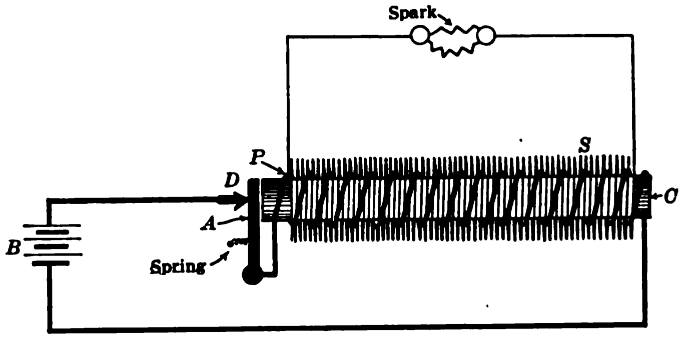


FIG. 170.—Induction coil.

is magnetized by the primary current, the armature A is drawn toward it and away from D, opening the circuit and causing the flux in the core to drop practically to zero. The spring then pulls the armature A against the contact D again, and the cycle is repeated. By this process the flux in the core C is continually being established and then destroyed.

On the same core is placed a secondary winding, S, consisting of many turns of fine wire. This winding is thoroughly insulated from the primary winding, but as it is wound on the same core as P, the two coils have a high value of mutual inductance. Because of the change of flux in the core, due to the interruptions of the primary current, a high alternating emf. is induced in the secondary. This induced electromotive force may be considered as due to the mutual inductance existing between the primary and the secondary coils. The induction coil has many practical applications. Its wide use in automobile and gas engine ignition systems is important.

149. Magnetic Pull.—It has been shown that a force exists between magnetized surfaces. This force can be accurately calculated if the surfaces are parallel and quite close together, being given by

$$f = \frac{B^2A}{8\pi}$$

where f is the force in *dynes*, A the area of each of the two surfaces in square centimeters, and B the flux density in gausses.

This becomes:

$$F = \frac{B^2A}{24.64} \text{ kilograms}$$

if B is expressed in *kilolines* per sq. cm.

$$F = \frac{B^2A}{72{,}130{,}000} \text{ lb.}$$

if B is in lines per sq. in. and A in square inches.

Example.—The core of a solenoid is 2 in. in diameter and a total flux of 200,000 lines passes from the end of the core into an iron armature of equal area. What is the pull on the armature in pounds?

$$A = \frac{\pi}{4}(2)^2 = 3.14 \text{ sq. in.}$$

$$B = 200{,}000/3.14 = 63{,}800 \text{ lines per sq. in.}$$

$$F = \frac{63{,}800^2 \times 3.14}{72{,}130{,}000} = 177 \text{ lb.} \quad \textit{Ans.}$$

CHAPTER IX

ELECTROSTATICS: CAPACITANCE

So far, electric currents, or electricity in motion, has only been considered. Electricity when in motion is called *dynamic* electricity. Electricity may, however, be stationary or at rest. Under these conditions the electricity is called *static* electricity. There is no difference in the nature of static and dynamic electricity. The static electricity usually appears different because of its extremely high potential and small quantity.

150. Electrostatic Charges.—If the terminals of an electrostatic induction machine be connected to two equal ellipsoids,

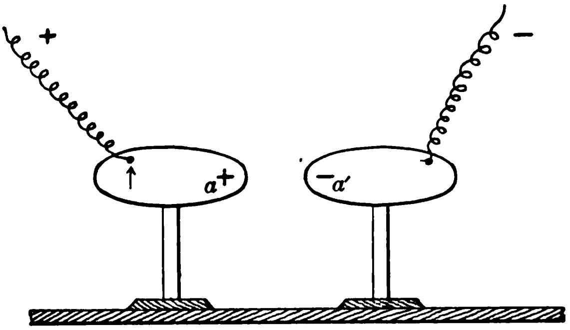


Fig. 171.—Electrostatic charges on insulated ellipsoids.

which are conducting and are insulated, Fig. 171, the ellipsoid connected to the positive terminal will be charged with positive electricity and that connected to the negative terminal will be charged with an equal amount of negative electricity. The charges will distribute themselves over the entire surface of the ellipsoids, but the density of the charges will be greatest on the ends of the ellipsoids which are adjacent. This is due to the fact that the positive and negative charges attract each other.

If the two wires from the electrostatic machine be disconnected the two charges will not be sensibly affected. In time they will leak away through the insulating supports.

If the two ellipsoids were free to move they would come together. If they were connected together with a wire a spark would be observed at the instant that contact was made, showing that current flows for an instant from one ellipsoid to the other. Both of the above effects are due to the fact that the positive and negative charges attract each other.

151. Electrostatic Induction.—If a positively charged ellipsoid A (Fig. 172(a)) be brought near another insulated ellipsoid B, which initially had no charge, a minus charge will be found on the end of B nearest A. As B did not hold any charge initially, and it is assumed to be perfectly insulated, no electricity can have gone out from B and none can have reached it from

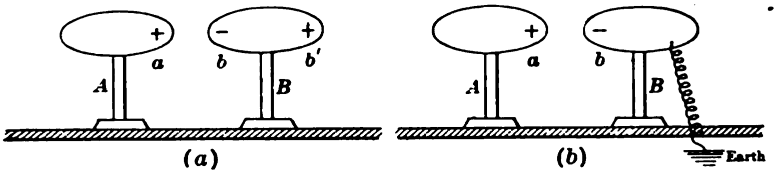


FIG. 172.—Electrostatic induction.

external sources, so that the net charge on B must still be zero. Therefore, a positive charge b' must also appear on B at the outer end farthest from A. This charge must be equal to b, and as the two are of opposite sign the net charge on B is still zero. It will be noted that the minus charge b is as near as possible to the positive inducing charge a, whereas the positive charge b' is as far away as possible from the positive charge a. This is due to the fact that the unlike charges attract each other and that like charges repel each other.

Also charges a and b are called bound charges, and charge b' is a free charge. This may be proved by connecting B to ground (Fig. 172(b)). The charge b' will be found to have escaped to ground, whereas the two charges a and b remain. Charge b' will seek a position as far away from a as possible.

If a were a negative charge, b would be a positive charge.

The above experiments are all illustrative of the following laws of electrostatics.

Charges of unlike sign attract each other and charges of like sign repel each other.

A positive charge will induce a negative charge on a body near it, or

A negative charge will induce a positive charge on a body near it.

This is similar to magnetic induction, where a north pole induces a south pole, etc. (See Par. 16.)

152. Electrostatic Lines.—Unit electrostatic charge is defined as that charge which, if placed 1 cm. distant from an equal charge in air, will be repelled with a force of 1 dyne.

If a unit positive charge, P, which can move freely, be placed at various points in the field near two oppositely charged bodies, it will be found to move along certain well defined paths, the path in each case being determined by the point at which the

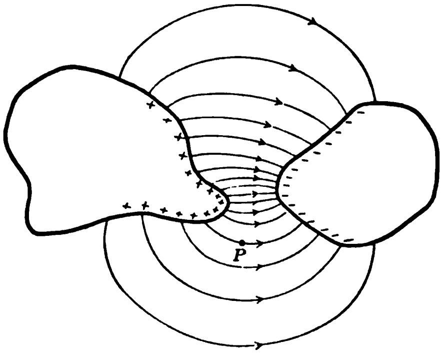


Fig. 173.—Electrostatic field between charged conductors.

unit charge starts. The unit charge starting from the positively-charged body will always move along a definite path until it reaches the negatively-charged body. The several paths which such a charge may follow are shown in Fig. 173. This is similar to the behavior of a unit north pole when placed in a magnetic field. When a difference of potential is produced between two conductors an *electrostatic* field also results. The intensity of this field at any point is equal to the force which is exerted on a unit positive charge at that point. Such a field may be represented by lines just as with the magnetic field. The density of the lines represents the ***field intensity.*** The field between two irregular bodies is sketched in **Fig. 173**, the lines of force being represented by the paths which the unit charge would follow if allowed to move freely.

An *electrostatic line of force* begins at a positively-charged conductor and ends at a negatively-charged conductor. In this respect it resembles a magnetic *line of force* which begins at a north pole and ends at a south pole. The electrostatic line of force is not like a magnetic *line of induction* which is always a closed curve. (See Par. 11.)

Electrostatic lines of force distribute themselves exactly as do the flow lines or stream lines of an electric current, or the magnetic lines in a magnetic field.

There is one difference, however, between electrostatic lines, on the one hand, and magnetic lines and electric current lines on the other. No matter how much curreot flows in a conductor, the conductor is nnt injured mechanically, provided it can be kept cool. Neither is a magnetic conductor injured, no matter how many magnetic lines exist in it. But there is a limit to the number of electrostatic lines which may exist in a medium. If the lines beoome too concentrated the medium cannot withstand the stresses which result and it is ruptured or "breaks down." This break-down may be followed by a dynamic arc, which increases the injury to the medium by burning.

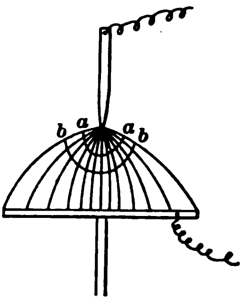


FIG. 174.—Electrostatic stress lines between a needle-point and a plate.

In a gaseous medium it is possible for a partial break-down to occur. Let a needle point in air, Fig. 174, be raised to a high potential above a plate. The electrostatic lines will be concentrated at the needle point but will be spread out over the plate. As the stress is most highly concentrated at the needle point, the air will obviously break down at this point first. This break-down can be detected by the blue glow or *corona*[1] which appears around the needle point, and at the same time an odor of ozone is evident. Complete rupture cannot occur between the point and the plate, at least at first, because the air beyond a certain region *aa* is still not stressed to the break-down point.

As the potential is raised, however, the boundary of the disrupted region will advance to *bb*, and will continue to

[1] See Chap. XII, Vol. II.

advance with increasing potential until the remaining air can no longer support the stress, when complete break-down takes place.

Dielectrics.—If electrostatic phenomenn are being considered, the medium between two conductors is called a *dielectric.* This is in distinction to the properties of the same medium, as an insulator which relates to electrical conduction. For instance, air is not a particularly good dielectric, its dielectric strength being only about 75,000 volts to the inch, but it is one of the best insulators known.

The ability of a substance to resist electrostatic break-down is called its *dielectric strength.* This is expressed in volts per unit thickness when the substance is placed between flat electrodes having rounded corners. For example, the dielectric strength of air is approximately 3,000 volts per mm. Rubber and varnished cambric have a much greater dielectric strength than air, that of rubber being in the neighborhood of 16,000 volts per mm., or 400,000 volts per in., and that of cambric being about twice as great as the value for rubber.

The volts per unit thickness impressed across a dielectric is called the *voltage gradient.* For instance, if 24,000 volts are impressed across 30 mils of insulation, the gradient is 24,000/30 or 800 volts per mil.

153. Capacitance.—Two conductors separated by a dielectric is called a *condenser.*

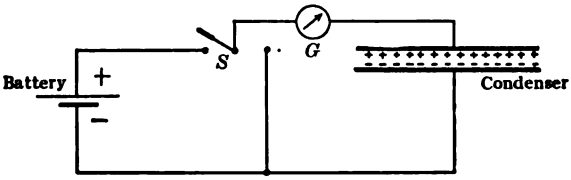


Fig. 175.—Charging and discharging a condenser.

Fig. 175 shows two conducting plates connected to a battery, the plates being separated by a dielectric. There is also a single-pole, double-throw (S.-P.D.-T.) switch S and a galvanometer G in the circuit. If the switch S be closed to the left, the galvanometer will deflect momentarily, and then come back to zero. This indicates that when the switch is closed, a quantity of electricity

passes through the galvanometer but that the current ceases to flow almost immediately. This current flows for a time sufficient to charge the condenser. After the condenser has become fully charged, the current ceases because the emf. of the condenser is equal and opposite to that of the battery. As this condenser emf. opposes the current entering the condenser it may be considered as a back emf. Any current which may flow after the condenser has become fully charged is a leakage current flowing through the insulation. If the switch S be opened for a short time, and then closed again, no deflection of the galvanometer will be noted unless there is leakage through the insulation.

This phenomenon of charging a condenser from a battery is not unlike the filling of a tank T from a reservoir R, Fig. 176.

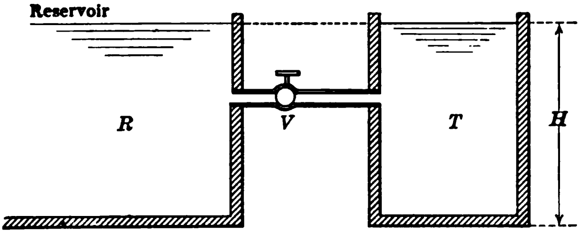


FIG. 176.—Reservoir and connected tank.

When the valve V is first opened, water will rush through the connecting pipe and will continue to flow at a diminishing rate until the level H, of the water in the tank T, is equal to the level of the water in the reservoir. If the tank does not leak, no water flows through the pipe after the water levels have become equal. In the same way the condenser, Fig. 175, takes current until its potential is equal to that of the battery, after which current ceases to flow.

To prove that electricity has actually been stored in the condenser, Fig. 175, the switch S may be closed to the right. This short-circuits the condenser through the galvanometer. The galvanometer now deflects momentarily in a direction opposite to that on charge, showing that the current now flows *out* of the positive plate. The condenser now becomes completely discharged, as is shown by there being no deflection of the galvanometer.

If the voltage of the battery, Fig. 175, be increased, the galvanometer deflection on charge and on discharge will increase also. This is due to the fact that the charge given to the condenser is proportional to the voltage across its terminals, just as the amount of water in the tank will be proportional to its height H (Fig. 176). The relation between the voltage, and the charge in a condenser may be expressed by the equation:

$$Q = CE \tag{81}$$

That is, the quantity of electricity in a condenser is equal to the voltage multiplied by a constant C. This constant C is called the *capacitance* of the condenser. The practical unit of capacitance is the *farad*. If C is in farads and E in volts, Q is in coulombs or ampere-seconds.

The farad is too large a unit for practical purposes, as a condenser having a capacitance of 1 farad would be prohibitively large. The capacitance of the earth as an isolated sphere is less than one thousandth of a farad. The microfarad, equal to one millionth of a farad, is the unit of capacitance ordinarily used.

By transposition, equation (81) may be written as follows:

$$C = Q/E \tag{82}$$

$$E = Q/C \tag{83}$$

As an example of the use of the above relations, consider the following problem:

A condenser has a capacitance of 200 microfarads and is connected across 500-volt mains. If the current is limited to 0.1 amp., how long must it flow before the condenser is fully charged?

The quantity in the condenser, when fully charged, is $Q = 0.000200 \times 600 = 0.12$ coulomb or ampere-second.

$$0.12 = 0.1t$$
$$t = 1.2 \text{ seconds.} \quad \textit{Ans.}$$

154. Specific Inductive Capacity or Dielectric Constant.—A parallel plate condenser (Fig. 177(a)), with air as a dielectric, has a measured capacitance C_1. If a slab of glass or of hard rubber be inserted between the plates so as to fill the intervening space completely (Fig. 177(b)), and the capacitance of the condenser again be measured, it will be found to be greater than its previous value. Let this new value be C_2. The increase in

capacitance obviously must be due to the presence of the glass or rubber.

The ratio $C_2/C_1 = \kappa$ is called the *specific inductive capacity*, or *dielectric constant*, of the material between the condenser plates. The specific inductive capacity of air is assumed to be unity, just as the magnetic permeability of air is likewise assumed to be unity.

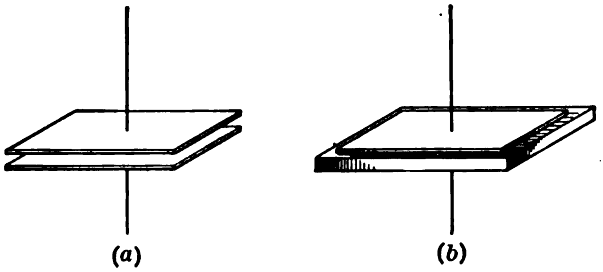


FIG. 177.—Plate condenser having air and then glass as a dielectric.

In the table are given the specific inductive capacities of some of the more common dielectrics:

*Bakelite	4.1 to 8.8	Paraffin	1.9 to 2.3
Glass	5.5 to 10	Rubber compounds	3 to 6
Ice	86.4	Hard rubber	1.5 to 3.5
Mica	2.5 to 5.5	Transformer oils	2.3 to 2.6
Paper	1.7 to 2.6		

155. Equivalent Capacitance of Condensers in Parallel.—Let it be required to determine the capacitance, C, of a number of con-

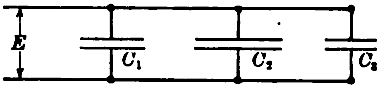


FIG. 178.—Capacitances in parallel.

densers in parallel, the condensers having respective capacitances of C_1, C_2, C_3. This arrangement of condensers is shown in Fig. 178. Let the common voltage across the condensers be E and the total resulting charge Q. Obviously,

$$Q = CE$$

and

$$Q_1 = C_1E, \quad Q_2 = C_2E, \quad Q_3 = C_3E$$

* For more complete data see "Standard Handbook," Section 4, Par. 238, *et seq.*

The total charge

$$
\begin{aligned}
Q &= Q_1 + Q_2 + Q_3 = CE \\
CE &= C_1E + C_2E + C_3E \\
CE &= E(C_1 + C_2 + C_3) \\
\therefore \quad C &= C_1 + C_2 + C_3 \qquad (84)
\end{aligned}
$$

That is, if condensers are connected in parallel, the resulting capacitance is the sum of the individual capacitances.

This is analagous to the grouping of conductances in parallel in the electric circuit.

Example.—Three condensers, having capacitances of 5, 10, and 12 microfarads, respectively, are connected across 600-volt mains. (*a*) What single condenser would replace the combination? (*b*) What is the charge on each condenser?

(*a*) $C = 5 + 10 + 12 = 27$ microfarads *Ans.*

(*b*) $Q_1 = 5 \times 600 =$ 3,000 microcoulombs

$Q_2 = 10 \times 600 =$ 6,000 microcoulombs

$Q_3 = 12 \times 600 =$ 7,200 microcoulombs. *Ans.*

Total charge = 16,200 m.c. = 27×600 m.c. (check).

156. EQUIVALENT CAPACITANCE OF CONDENSERS IN SERIES

In Fig. 179, three condensers, having capacitances of C_1, C_2, and C_3 respectively, are connected in series across the voltage E. It is desired to determine the capacitance of an equivalent single condenser. Let E_1, E_2, and E_3 be the potential differences across the condensers C_1, C_2, and C_3, respectively. After the voltage E is applied to the system, there will be $+ Q$ units of charge on the positive plate of C_1, and by the law of electrostatic induction $- Q$ units must be induced on its negative plate.

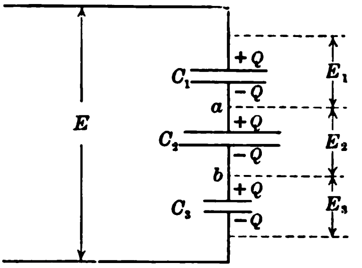


FIG. 179.—Capacitances in series.

Now consider the region a which consists of the negative plate of C_1, the positive plate of C_2, and the connecting lead. This system is insulated from all external potentials, since it is assumed that the condensers have perfect insulation. Before the voltage was applied to the system of condensers, no charge ex-

isted in the region a. After the application of the voltage, the net charge in this region must still be zero, as no charge can flow through the insulation. Therefore, $+Q$ units must come into existence in order that the net charge in the region a may remain zero. $(+Q + (-Q)) = 0$. This charge of $+Q$ units will go the plate of C_2 since it is repelled by the $+$ charge on C_1 just as the charge b', Fig. 172 (a), took a position on the end of the ellipsoid as far as possible from the positive inducing charge a. The same reasoning holds for the region b, between C_2 and C_3. Therefore, each of the three condensers in series has the same charge Q. (This is analagous to resistances in series, each of which must carry the same current if no leakage exists.)

Consider the voltages E_1, E_2, E_3.

$$E_1 = \frac{Q}{C_1},\quad E_2 = \frac{Q}{C_2},\quad E_3 = \frac{Q}{C_3} \text{ from equation (83), page 204.}$$

The sum of the three condenser voltages must equal the line voltage:

$$E_1 + E_2 + E_3 = E$$

$$E = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$$

Also $E = \frac{Q}{C}$, as by definition the equivalent condenser C must have a charge Q.

Substituting this value for E,

$$\frac{Q}{C} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$$

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \tag{85}$$

That is, the reciprocal of the equivalent capacitance of a number of condensers in series is equal to the sum of the reciprocals of the capacitances of the individual condensers.

In assuming for condensers connected in series that with direct current the potential across each condenser is inversely proportional to its capacitance, the factor of leakage is absolutely neglected. If the condensers are even slightly leaky, however,

a current flows through the series and eventually the potential distributes itself according to Ohm's Law.

$$E_1 = IR_1,\ E_2 = IR_2, \text{ and } E_3 = IR_3$$

where I is the leakage current, and R_1, R_2, and R_3 are the respective ohmic resistances of the three condensers.

Example of condensers connected in series:

Consider that the three condensers of Par. 155, having capacitances of 5, 10, and 12 microfarads respectively, are connected in series across 600-volt mains. Determine (*a*) the equivalent capacitance of the combination; (*b*) the charge on each condenser; (*c*) the potential across each condenser, assuming no leakage.

(*a*)
$$\frac{1}{C} = \frac{1}{5} + \frac{1}{10} + \frac{1}{12} = 0.383$$

$$C = 1/0.383 = 2.61 \text{ microfarads. } \textit{Ans.}$$

(*b*)
$$Q = 2.61 \times 600 = 1566 \text{ microcoulombs, on each condenser. } \textit{Ans.}$$

(*c*)
$$E_1 = \frac{1{,}566 \times 10^{-6}}{5 \times 10^{-6}} = 313 \text{ volts}$$

$$E_2 = \frac{1{,}566 \times 10^{-6}}{10 \times 10^{-6}} = 157 \text{ volts}$$

$$E_3 = \frac{1{,}566 \times 10^{-6}}{12 \times 10^{-6}} = 130 \text{ volts. } \textit{Ans.}$$

157. Energy Stored in Condensers.—As a certain quantity of electricity is stored in a condenser and a difference of potential exists between the positive and negative plates, energy must be stored in the condenser. The existence of this energy is shown by the spark resulting from short-circuiting the condenser plates. The energy in joules or watt-seconds is

$$W = 1/2\, QE \tag{86}$$

This may also be written:

$$W = 1/2\, CE^2 \tag{87}$$

$$W = 1/2\, Q^2/C \tag{88}$$

The similarity in form of (87) to the equation for the energy stored in the magnetic field should be noted. (See equation (77) Par. 147.) The energy stored in the electrostatic field is proportional to the square of the *voltage*, whereas the energy stored in the electro-magnetic field is proportional to the square of the *current*.

Example.—Determine the stored energy in each of the condensers in series of Par. 156 and the total stored energy.

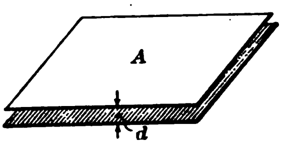
$$W_1 = \frac{1}{2}\,\frac{(1{,}566 \times 10^{-6})^2}{5 \times 10^{-6}} = 0.2453 \text{ joule}$$

$$W_2 = \frac{1}{2}\,\frac{(1{,}566 \times 10^{-6})^2}{10 \times 10^{-6}} = 0.1225 \text{ joule}$$

$$W_3 = \frac{1}{2}\,\frac{(1{,}566 \times 10^{-6})^2}{12 \times 10^{-6}} = 0.1020 \text{ joule}$$

The total energy $W = \frac{1}{2}(1{,}566 \times 10^{-6} \times 600) = 0.4698$ joule.

158. Calculation of Capacitance.—As a rule it is impossible to calculate the capacitance of a condenser, or the mutual capacitance of conducting bodies, because of the complex geometry and also because the dielectric constants of the intervening media are not always accurately known. There are some simple cases, however, where accurate calculations are possible.



$$C = \frac{\kappa A}{4\pi d \times 9 \times 10^5}$$

FIG. 180.—Capacitance of a plate condenser.

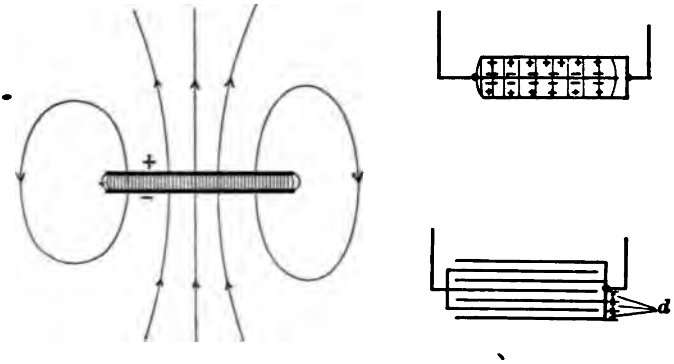
The plate condenser, Fig. 180, is the simplest form of condenser. Let A be the area of one side of each plate in square centimeters, d the distance between plates in centimeters and κ, the dielectric constant of the medium between the plates. The capacitance is

$$C = \frac{\kappa A}{4\pi d \times 9 \times 10^5} \text{ microfarads.} \tag{89}$$

In this equation it is assumed that the electrostatic lines between the two plates are parallel.

The total capacitance of a simple plate condenser of this type cannot be accurately calculated for the following reason. All the electrostatic lines do not lie between the plates as certain lines pass from the back of the positive plate to the back of the negative as shown in Fig. 181 (*a*). This results in the actual capacitance being greater than the value as just calculated. This error may be avoided by using one more plate in one group than in the other, Fig. 181 (*b*). In this case the area A, equation (89), includes both sides of all the plates with the exception of the two outside ones. As the charge on both outer plates is of the same

sign and the plates have the same potential, no electrostatic lines can pass between them. An error due to the bulging or "fringing" of the lines near the edges of the plates may occur unless the plate area is large compared with the distance between plates.



(a) Electrostatic leakage lines of a plate condenser.

(b) Multi-plate condensers.

FIG. 181.

Example of Condenser Design.—It is desired to construct a plate condenser having a total capacitance of 8 microfarads. The plates are of tin-foil 6 in. × 8 in. and 1 mil thick. The dielectric is of paper 7 in. × 9 in. and 2 mils thick and having a dielectric constant of 3. How many sheets of paper and of tin-foil are necessary? What will be the dimensions of the condenser?

The area of each plate is:

$$6 \times 8 \times (2.54)^2 = 309.6 \text{ sq. cm.}$$

The distance between plates:

$$d = 0.002 \times 2.54 = 0.00508 \text{ cm.}$$

The capacitance between two plates (from equation 89):

$$c = \frac{3 \times 309.6}{4\pi \times 0.00508 \times 9 \times 10^5} = 0.01616 \text{ mf.}$$

Therefore:

$$\frac{8}{0.01616} = 495 \text{ sections are needed.}$$

These sections are indicated at d, Fig. 181 (b). This means that 496 plates and 495 sheets of paper are necessary.

Thickness:

$$\begin{aligned} \text{Tin-foil} &= 496 \times 0.001 = 0.496 \text{ in.} \\ \text{Paper} &= 495 \times 0.002 = \underline{0.990} \text{ in.} \\ & \qquad\qquad\qquad\quad 1.486 \text{ in.} \end{aligned}$$

Volume of condenser proper = 7 in. × 9 in. × 1.49 in. *Ans.*

Of course additional outside insulation and a protective covering are necessary.

The capacitance in microfarads of two co-axial cylinders, the outer of which has a radius of R_1 cm. and the inner a radius of R_2 cm., is

$$C = \frac{0.0388\kappa}{\log_{10}\frac{R_1}{R_2}} \text{ mf. per mile.}$$

This equation is applicable to single conductor underground cables.

159. Measurement of Capacitance.—There are two common methods of measuring capacitance, the direct current or ballistic method and the alternating current or bridge method.

The direct current method employs a galvanometer which is used ballistically. It can be shown that if the moving coil of

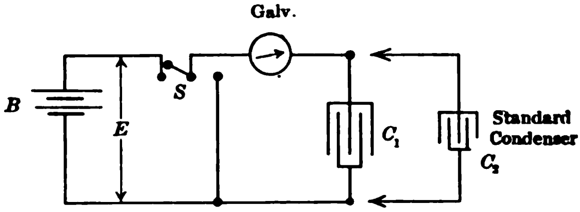


FIG. 182.—Ballistic method of measuring capacitance.

the ordinary galvanometer have considerable inertia and be properly damped, its maximum throw, due to the impulse produced by the sudden passage of a current through the coil, is proportional to the total quantity of electricity passing through the galvanometer. This assumes that the entire charge passes through the coil before the coil begins to move. Let D be the maximum galvanometer throw in centimeters. Then:

$$Q = KD \tag{90}$$

Where Q is the quantity and K the galvanometer constant.

To make the measurement, the apparatus is connected as shown in Fig. 182. A battery B supplies the current for the apparatus. The measurement may be made on either the charge or the discharge of the condenser or check measurements may be made using both the charge and the discharge. If the condenser is at all leaky, the discharge method is preferable.

When the switch S is closed to the left the condenser C_1 is

charged through the galvanometer and the maximum throw of the galvanometer is read. Several check readings should be taken. The galvanometer should return immediately to zero. If it shows a steady deflection it indicates a leaky condenser. In a corresponding manner the ballistic throw of the galvanometer may be read on discharge by closing switch S to the right. Let D_1 be the deflection of the galvanometer when C_1 is connected, Q_1 the quantity going into the condenser, and E the voltage across the condenser. Then by equation (90)

$$Q_1 = KD_1$$

Also

$$Q_1 = C_1E$$

where C_1 is the unknown capacitance.

$$\therefore C_1E = KD_1 \qquad (a)$$

If now the standard capacitance C_2 be substituted for the unknown condenser and another set of readings taken,

$$Q_2 = KD_2$$

or

$$C_2E = KD_2 \qquad (b)$$

Dividing (a) by (b),

$$\frac{C_1E}{C_2E} = \frac{KD_1}{KD_2}$$

$$C_1 = C_2\frac{D_1}{D_2}$$

$\dfrac{C_2}{D_2}$ is the galvanometer constant.

It is often desirable to use an Ayrton shunt in such measurements as it gives the apparatus greater range. When such a shunt is used, proper correction must be made for its multiplying power.

In the bridge method two capacitances form adjacent arms of a Wheatstone Bridge and two resistances form the other two arms, Fig. 183 (a). An alternating current supply is preferable. The secondary of an induction coil may be used as the source of power or a battery with a key may be made to charge and discharge the system as shown in Fig. 183 (b). A telephone is used as a detector except in (b). Let C_x be the unknown capacitance and C_2 a standard which may or may not be adjustable. R_1 and R_2 are two known resistances, one of which should be adjustable unless C_2 is so.

Either C_2 or one of the resistances is adjusted, until there is no sound in the telephone, showing that the bridge is in balance. Under these conditions:

$$\frac{C_x}{C_2} = \frac{R_2}{R_1}$$

$$C_x = C_2\frac{R_2}{R_1} \tag{91}$$

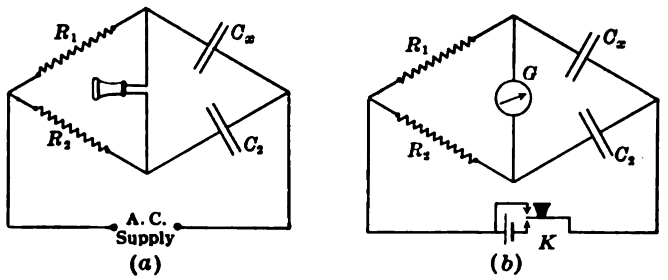


FIG. 183.—Bridge methods of measuring capacitance.

When a battery is used, a double contact key K is necessary. K is pressed and released, and until the bridge is balanced, the galvanometer will deflect both upon the charge of the system, when the key is pressed, and upon the discharge, when the key is released. The bridge is balanced when the galvanometer does not deflect on either charge or discharge. Equation (91) is then applicable.

In the above measurements, it is assumed that there is little if any leakage through the condensers.

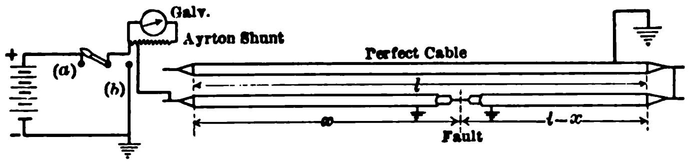


FIG. 184.—Locating an open in a cable.

160. Cable Testing—Location of a Total Disconnection.—In Chap. VII, it was shown that a grounded fault in a cable could be located by suitable resistance measurements, such as the Murray and Varley loop tests. If a cable be totally disconnected and its broken ends remain insulated these loop tests are impossible. The distance to the fault may now be determined

by capacitance measurements. The connections are shown in Fig. 184. The capacitance C_1 of the length, x, to the fault is first measured by the ballistic method. If a *similar* perfect cable parallels the faulty cable, the two are looped at the far end and the capacitance C_2 of a length l of the perfect cable plus the length $l - x = 2l - x$ of the faulty cable is measured.

Let c be the capacitance per ft. of each cable, assumed to be the same for each:

$$C_1 = xc = KD_1$$

where K is the galvanometer constant and D_1 the deflection corresponding to C_1.

Likewise,

$$C_2 = (2l - x)c = KD_2$$

Dividing one equation by the other,

$$\frac{x}{2l - x} = \frac{D_1}{D_2}$$

$$x = l\frac{2D_1}{D_1 + D_2} \tag{92}$$

The capacitance per unit length and the total capacitance do not enter into the equation, so that it is not necessary to use a standard condenser for the calibration of the galvanometer. The capacitances of the various lengths are proportional to the galvanometer deflections when corrected for the setting of the Ayrton shunt.

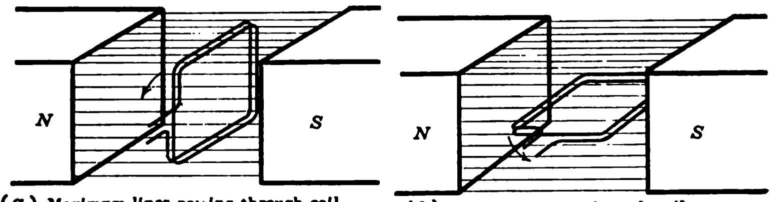
CHAPTER X

THE GENERATOR

161. Definition.—A generator is a machine which converts mechanical energy into electrical energy. This is accomplished by means of an armature carrying conductors upon its surface, acting in conjunction with a magnetic field. Electrical power is generated by the relative motion of the armature conductors and the magnetic field.

In the direct current generator the field is usually stationary and the armature rotates. In most types of alternating current generators the armature is stationary and the field rotates. Either the armature or the field is driven by mechanical power applied to its shaft.

162. Generated Electromotive Force.—It was shown in Chap. VIII that if the flux linking a coil is varied in any way, an



(*a*) Maximum lines passing through coil (*b*) No lines passing through coil

FIG. 185.—Simple coil rotating in a magnetic field.

electromotive force is *induced* in the turns of the coil. The action of the generator is based on this principle. The flux linking the armature coils is varied by the relative motion of the armature and field.

In Fig. 185 a coil revolves in a uniform magnetic field produced by a north and a south pole. In (*a*) the coil is perpendicular to the magnetic field and in this position the maximum possible flux links the coil. Let this flux be ϕ.

If the coil be rotated counter-clockwise a quarter of a revolution, it will lie in the position shown in (*b*). As the plane of

the coil is parallel to the flux no lines link the coil in this position. Therefore, in a quarter revolution the flux which links the coil has been decreased by ϕ lines. The average voltage induced in the coil during this period is, therefore,

$$e = N\frac{\phi}{t}10^{-8} \text{ (Chap. VIII, equation 74)}$$

where N is the number of turns in the coil and t the time required for a quarter revolution. But $t = \frac{1}{4R}$ where R = the *revolutions per second*. Therefore, the average voltage during a quarter revolution is

$$e = 4NR\phi 10^{-8} \qquad \text{volts}$$

The generation of electromotive force in a moving coil of this type, which is similar to those used in dynamos, may also be analyzed by considering the total electromotive force as being due to the sum of the electromotive forces generated in each side of the coil. The electromotive force of one turn is the sum of the electromotive forces in each conductor forming the sides of the turn, since these conductors are connected in series by the end connections of the turn. The individual electromotive forces are then considered as being generated in the *conductor* rather than induced in the coil. This in no way conflicts with the fact that the induced electromotive force is also due to the change of flux linked with the coil. The same total emf. is obtained under either assumption.

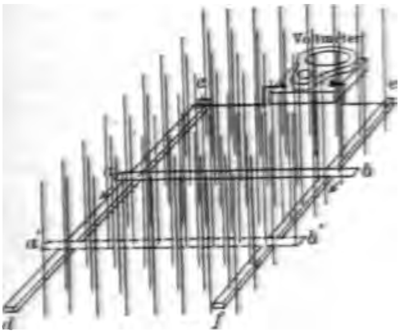


Fig. 186.—Conductor cutting a uniform magnetic field.

Consider the conductor *ab*, Fig. 186, free to slide along the two metal rails *cd* and *ef*. The rails are connected at one end *ce* by a voltmeter. A magnetic field having a density of B lines per sq. cm. passes perpendicularly through the plane of the rails and conductor.

Let the conductor *ab* move uniformly to the position *a'b'*. While this movement is taking place, the voltmeter will indicate a certain voltage. This voltage may be attributed to either of two causes.

1. As conductor *ab* moves to position *a'b'* the flux linking the conducting loop formed by *ce*, the rails and *ab*, is increased, because of the increasing area of this loop.

2. An electromotive force is generated in the conductor *ab* since it cuts the magnetic field.

Similarly, the electromotive force developed by the coil in in Fig. 185 may be attributed to the emf.'s generated in the conductors on opposite sides of the coil through their cutting of magnetic lines. These conductors are connected in series by the end conductors, or connectors, which in themselves generate no electromotive force. The direction of the electromotive forces developed in the coil sides are such that these emf.'s are additive.

The electromotive force in volts generated by a single conductor which cuts a magnetic field is

$$e = Blv10^{-8} \tag{93}$$

where B, l and v are mutually perpendicular.

B is the flux density of the field in gausses, l the length of conductor in *centimeters*, and v the velocity of the conductor in centimeters per second.

That the electromotive force induced by a change of the flux linked with a coil is the same as that obtained by considering the emf.'s generated by the cutting of magnetic lines by the conductor which make up the coil may be illustrated by a concrete example. Let the flux have a density of 100 lines per sq. cm., Fig. 186. The distance *ab* is 30 cm. and *aa'* is 20 cm. The conductor *ab* moves uniformly to position *a'b'* in 0.1 second. What is the electromotive force across *ce*?

The change of flux linking the coil is:

$$\phi = 30 \times 20 \times 100 = 60{,}000 \text{ lines.}$$

This change occurs in 0.1 second.

Then by equation (74), page 185

$$e = 1 \frac{60{,}000}{0.1} 10^{-8} = 0.006 \text{ volt.}$$

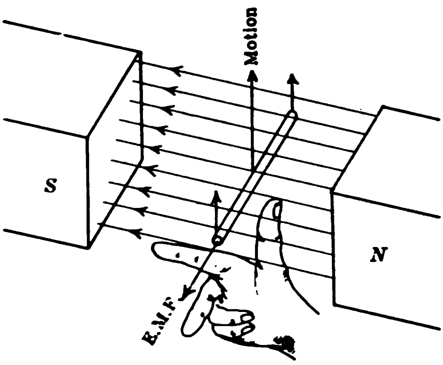
Applying equation (93),

$$v = \frac{20}{0.1} = 200 \text{ cm./sec.}$$

$$e = 100 \times 30 \times 200 \times 10^{-8} = 0.006 \text{ volt.}$$

It will be seen that the same result is obtained whether the electromotive force is considered as being generated by the con-conductor itself cutting the field or whether it is considered as being induced by the change in flux linking the coil.

163. Direction of Induced Electromotive Force. Fleming's Right-hand Rule.—A definite relation exists between the direction of the flux, the direction of motion of the conductor and the



Fore finger along lines of force. Thumb in direction of motion. Middle finger gives direction of induced emf.

FIG. 187.—Fleming's right-hand rule.

direction of the electromotive force in the conductor just as a definite relation exists between the direction of current and of the flux which it produces.

A very convenient method for determining this relation is the *Fleming right-hand rule*. In this rule the fingers of the *right* hand are utilized as follows:

Set the fore-finger, the thumb, and the middle finger of the right hand at right angles to one another (Fig. 187). If the *fore-finger points along the lines of flux and the thumb in the direction of motion of the conductor, the middle finger will point in the direction of the induced electromotive force.*

This rule is illustrated by Fig. 187.

164. Voltage Generated by the Revolution of a Coil.—A coil of a single turn is shown in Fig. 188 (*a*). The coil rotates in a counter-clockwise direction at a uniform speed in a uniform magnetic field. As the coil assumes successive positions, the electromotive force induced in it changes. When it is in position (1) the electromotive force generated is zero, for in this position neither conductor is cutting magnetic lines, but rather is moving parallel to these lines. When the coil reaches position (2), (shown dotted) its conductors are cutting across the lines obliquely and the electromotive force has a value indicated

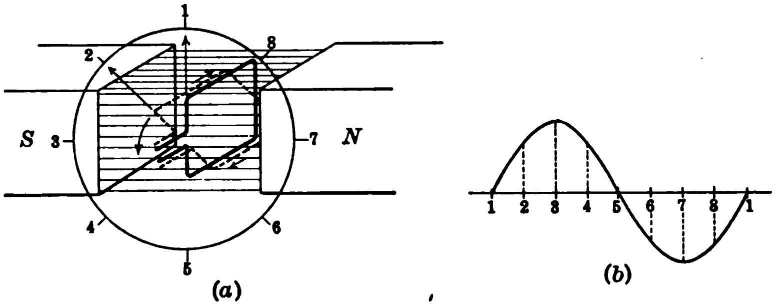


FIG. 188.—Emf. induced in a coil rotating at constant speed in a uniform magnetic field.

at (2) in Fig. 188 (*b*). When the coil reaches position (3) the conductors are cutting the lines *perpendicularly* and are therefore cutting at the maximum possible rate. Hence the electromotive force is a maximum when the coil is in this position. At position (4) the electromotive force is less, due to a lesser rate of cutting. At position (5) no lines are being cut and as in (1) there is no electromotive force. In position (6) the direction of the electromotive force in the conductors will have reversed as each conductor is under a pole of opposite sign to that for positions (1) to (5). The electromotive force increases to a negative maximum at (7) and then decreases until the coil again reaches position (1). After this the coil merely repeats the cycle.

This induced electromotive force is alternating and an emf. varying in the manner shown is called a *sine wave* of electromotive force. This alternating electromotive force may be impressed on an external circuit by means of two *slip rings*,

Fig. 189. Each ring is continuous and insulated from the other ring and from the shaft. A metal or a carbon brush rests on each ring and conducts the current from the coil to the external circuit.

If a *direct current* is desired, that is, one whose direction is always the same, such rings cannot be used. A direct current must always flow into the external circuit in the *same direction*.

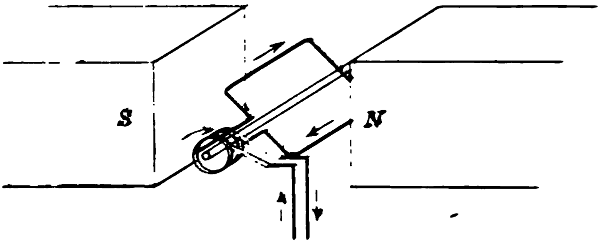


FIG. 189.—Current taken from rotating coil by means of slip-rings.

As the *coil current* must necessarily be alternating, since the *emf.* which produces it is alternating as has just been shown, this *current* must be rectified before it is allowed to enter the *external circuit*. This rectification can be accomplished by using a *split ring* such as is shown in Fig. 190. Instead of using two rings, as in Fig. 189, one ring only is used. This is split

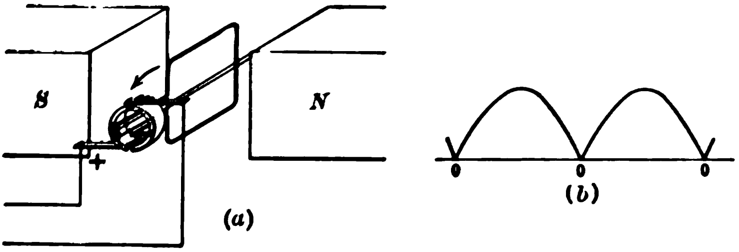


FIG. 190. Rectifying effect of a split ring or commutater.

by saw cuts at two points diametrically opposite each other. The two ends of the coil are connected one to each of the sections or segments so produced.

A careful consideration of Fig. 190 will show that, as the direction of the current in the coil reverses, its connections to the external circuit are simultaneously reversed. Therefore, the

direction of flow of the current in the external circuit is not changed. The brushes pass over the cuts in the ring when the coil is perpendicular to the magnetic field or when it is in the so-called neutral plane and is generating no voltage, as shown in Fig. 188. These neutral points are marked 0-0-0 in Fig. 190 (*b*).

By comparing Fig. 188 (*b*) with Fig. 190 (*b*) it will be seen that the negative half of the wave has been reversed and so made positive.

A voltage with a zero value twice in each cycle, as shown in Fig. 190, could not be used commercially for direct current service. Also a single coil machine would have a small output

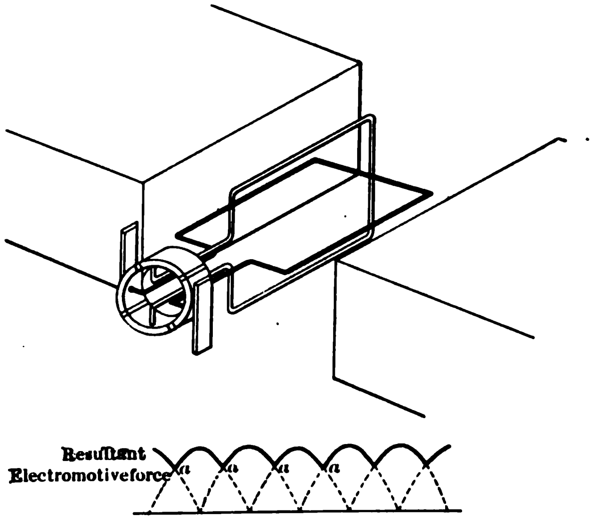


FIG. 191.—Effect of two coils and four commutator segments upon the electromotive force wave.

for its size and weight. The electromotive force wave of Fig. 190 may be improved upon by the use of two coils and four commutator segments, Fig. 191. This gives an *open circuit* type of winding, since it is impossible to start at any one commutator segment and return to this segment again by following through the entire winding. In this particular arrangement the full electromotive force generated in each coil is not utilized, as one coil passes out of contact with the brushes at points *a*, *a*, *a*, Fig. 191 (*b*), and the voltage shown by the dotted lines is not utilized.

165. Gramme Ring Winding.—This type of winding in its elementary form, Fig. 192, consists of insulated wire wound spirally around a ring (or hollow cylinder of iron) with taps taken from the wire at regular intervals and connected to commutator segments. This winding is simple, and has the advantage that a single winding is adapted to any number of poles, if the voltage limitations do not prevent. The portions of the conductors which lie inside the ring cut no flux and act merely as connectors for the active portions of the conductors. Because of the small proportion of active conductors a relatively large amount of copper is required in such a winding. In small

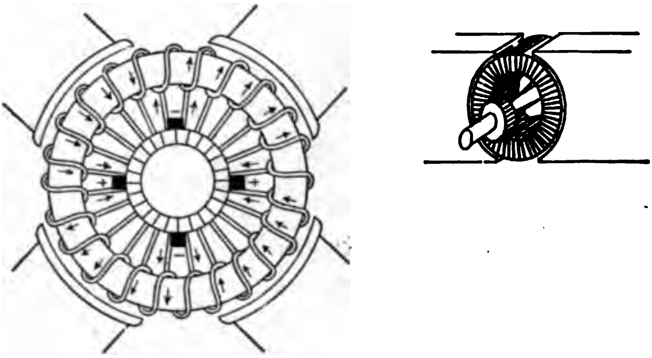


FIG. 192.—Gramme ring winding.

machines, there is not sufficient room to carry these inactive conductors back through the armature core. In a gramme ring winding formed coils cannot be used and this makes the winding expensive. This type of winding has a high inductance, which renders good commutation difficult.

It will be noted that the electromotive force between brushes in a gramme ring winding is the sum of the electromotive forces of all the coils that lie between brushes. When one coil passes a brush another moves forward to take its place. Fig. 193 shows the electromotive force between brushes due to four coils, it being assumed that the voltage curve for each is a sine wave. The electromotive force of each coil is plotted separately. These electromotive forces do not all have their zero value at the same time nor do they reach their maximum value at the same time

owing to the positions of the individual coils. The resultant electromotive force at any point is the sum at this point of these individual electromotive forces. This voltage should be compared with the electromotive force obtained with the open coil winding shown in Fig. 191, in which the resultant electromotive force does not equal the sum of the individual electromotive forces but is made up of the successive tops of the individual waves. It will be noted that a fairly smooth resultant electromotive force is obtained with four coils, the "ripples" being noticeable but comparatively small in magnitude.

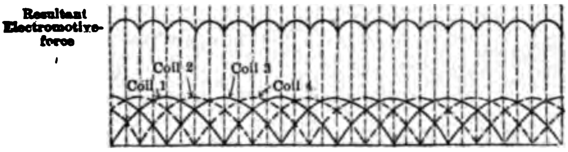


FIG. 193.—Resultant electromotive force due to four series-connected coils between brushes.

A gramme ring winding is called a *closed* winding, since it is possible to start at any one point in the winding and return to the same point again by passing continuously through the winding.

166. Drum Winding.—The objections to the ring winding are overcome by the use of the drum winding. The conductors of this winding all lie upon the surface of the armature and are connected to one another by front and back connections or coil ends (*ad* and *bc*, Fig. 194, are coil ends). With the exception of these end connections, all the armature copper is "active," that is, it cuts flux and so is active in generating electromotive force.

The sides of each coil should be about one pole pitch (the distance between centers of adjacent poles) apart. If one conductor is under a north pole the other is then under a south pole, and as both move in the same direction, but under different poles, the electromotive forces of these two conductors will be in opposite directions, Fig. 194. Due to the manner in which these conductors are connected at their ends, the electromotive forces in the individual coils are additive.

In most gramme ring windings, and in the earlier drum-wound machines, the surface of the armature core was smooth. The conductors were held in position partly by projecting pins, and were prevented by binding wires from flying out under the action of centrifugal force. The smooth core construction has been superseded by the "iron-clad" type where the conductors are embedded in slots as indicated in Fig. 197. The slots are lined with insulation and the conductors are held in firmly by wooden or non-conducting wedges in the larger machines (see Fig. 224), and by binding wires in the smaller types (see Fig. 214). These constructions are much better mechanically than the smooth

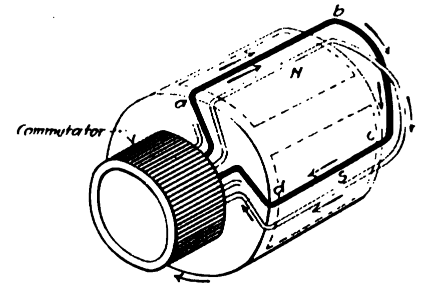


FIG. 194.—Two coils in place on a 4-pole, drum-wound armature.

core armature type and they also permit a much shorter air gap. On the other hand, as the coils are embedded in iron, they have a high inductance. This makes commutation more difficult and the flux pulsations due to the armature teeth give pole face and tooth losses.

167. Lap Winding.—Direct current armatures are usually wound with former-made coils, Fig. 195. These coils are usually wound on machines with the necessary number of turns, and are then wound with cotton or mica tape. They are then bent into proper shape by another machine. The two ends are left bare so that later they may be soldered to the commutator bars. The span of the coil, called the coil pitch, should be equal or nearly equal to the pole pitch, so that when one side of a coil is under a north pole the other is under a south pole. This span may be as low as nine-tenths the pole pitch, in which case a *fractional pitch winding* results.

Usually two coil sides occupy one slot, one coil side lying at the top and the other at the bottom of the slot. That is, if the side of one coil is in the bottom of a slot, its opposite side lies in the top of some other slot. This allows the end connections to be easily made as the coil ends can be bent around one another in a systematic manner, passing from the bottom to the top layer by means of the peculiar twist in the ends of the coils.

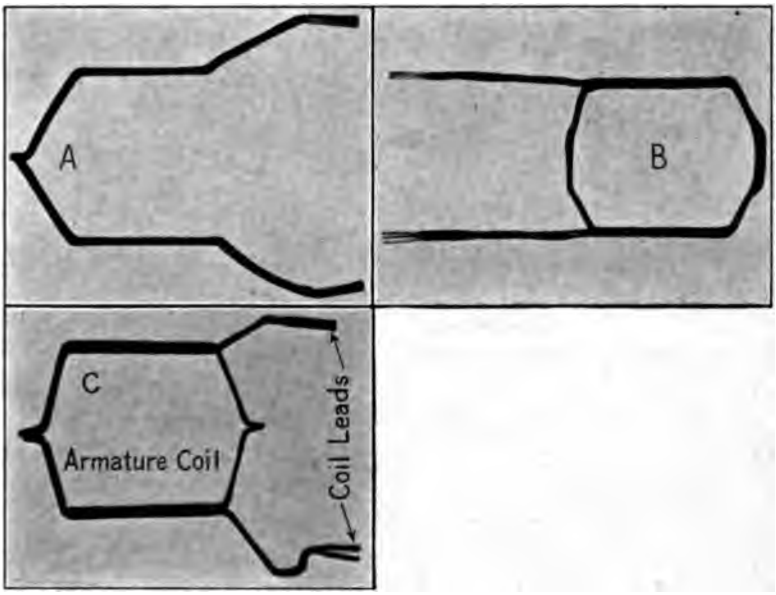


Fig. 195.—Formed armature coils.

The bundle of wires constituting one side, *ab* (Figs. 194 and 196), of a coil will be termed a winding element. This may consist of one or of several turns taped together. Even when there are several turns, they will be shown as one turn in the wiring diagram, as indicated in Fig. 196. Obviously there will be twice as many of these elements as there are coils. The number of elements that the coil advances on the back of the armature is the *back pitch* of the winding and will be denoted by y_b. This back pitch is obtained by the connection *bc*, Fig. 194. The number of elements spanned on the commutator end of the

armature is called the *front pitch* and will be designated by y_f. This may be greater or less than the back pitch *but not equal* to it. If it be greater, the winding is retrogressive, and if it be less, the winding is progressive. This is illustrated in Figs. 197 and 198. Conductor 1 is connected on the back of the armature to conductor 10. Therefore, the back pitch $y_b = 9$. Conductor 10 is then connected back to 3 on the front of the armature, the connection being made at the commutator segment.

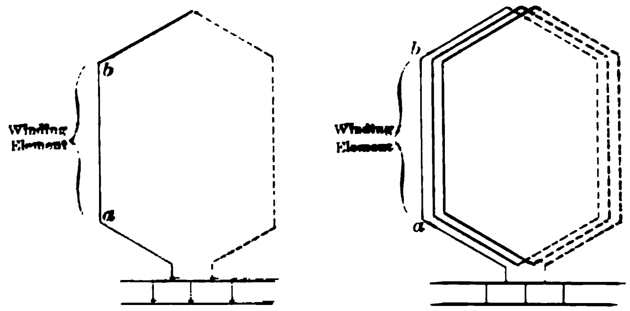


FIG. 198.—Single coil representing a 3-turn coil of an armature winding.

Therefore, the front pitch $y_f = 7$. This winding is therefore *progressive*.

As most windings are now made in two layers, only two-layer windings will be considered. The conductors or elements lying in the top of the slots will be given odd numbers and those in the bottom of the slots even numbers, Fig. 197. As one side of a coil lies in the bottom of a slot and the other side in the top of a slot, obviously y_b and y_f must both be odd. Further, if they were both even, all the conductors could lie only in *either* the odd or the even slots but could not lie in both. In a simplex lap winding having two elements per slot, the return connection cannot be made back to the original slot but it must always lead back to a slot which is next to the original slot. Thus, in Fig. 197, the connection is from the top conductor 1 to conductor 10, thence back to 3, *i.e.*, to the top of the next slot. Therefore, the front and back pitches can only differ from each other by 2.

That is,

$$y_b = y_f \pm 2 \qquad (94)$$

The average pitch

$$y = \frac{y_b + y_f}{2} \tag{95}$$

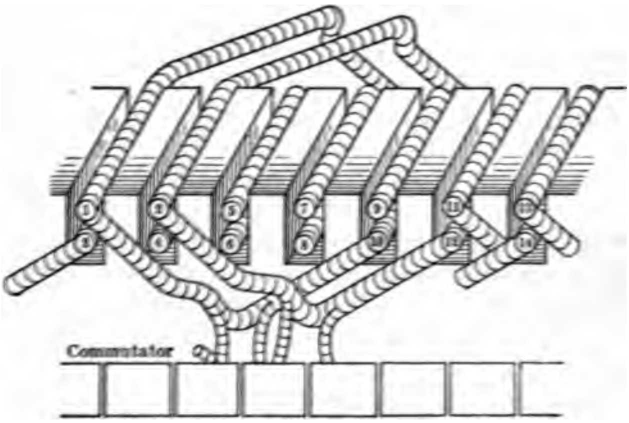


FIG. 197.—Simplex lap winding having back-pitch of 9 and front pitch of 7.

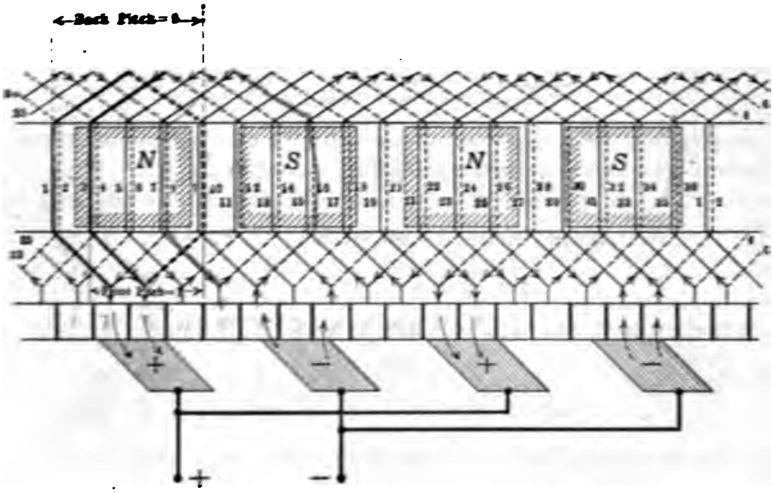


FIG. 198.—Development of a 4-pole lap winding.

The + sign in (94) indicates that the winding is progressive that is, progresses in a clockwise direction when viewed from the commutator end. The − sign indicates a retrogressive winding

whose advance is in a counter-clockwise direction when viewed from the commutator end.

It will be seen that for every coil one commutator segment is necessary. Therefore, the number of commutator segments

$$N_c = N = \frac{Z}{2}$$

where Z is the total number of winding elements on the surface of the armature and N is the number of coils.

From Figs. 197 and 198 it will be seen that the winding advances one commutator segment for each complete turn.

In designing a winding it is necessary that the opposite sides of each coil lie under different poles so that the two electromotive forces generated in the coil sides may be additive. Hence the average pitch should be nearly equal to the number of elements per pole.

The three fundamental conditions to be fulfilled by a *lap winding* are:

(1) The pitch must be such that the opposite sides of the coil lie under unlike poles.

(2) The winding must include each element once and only once.

(3) The winding must be re-entrant or must close on itself.

Example.—Assume that the armature of a 4-pole machine has 18 slots. Design a two-layer lap winding having two elements per slot.

There are 36 elements. The average pitch should be nearly equal to $\frac{36}{4} = 9$. The back pitch can be made equal to 9.

$$y_b = 9 \qquad y_f = 7$$

Starting at 1, the winding will progress as follows:

1–10–3–12–5–14–7–16–9–18–11–20–13–22–15–24–17–26–19
28–21–30–23–32–25–34–27–36–29–2–31–4–33–6–35–8–1.

The above is called a winding table. It is very useful in checking the winding. By proper checking it may be seen that each conductor is included once and only once and that the winding closes at the same conductor, 1 in this case, at which it began. The winding is shown in Fig. 198 as if it were split axially and rolled out flat. It will be noted that the brushes rest on segments to

which are connected elements which lie midway between the poles, as 11 and 19, for example.

168. Lap Winding—Several Coil Sides per Slot.—In the larger sizes of machines it is often necessary to place several coil sides or elements in one slot, usually 4, 6, or 8. More than eight coil sides per slot are rarely used. The reason for placing several coil sides in a slot is as follows: If two elements per slot were used, one in the top layer and one in the bottom layer, a large number of slots would be necessary. This would reduce the size of the slots and make the space factor (ratio of the copper cross-section to the slot cross-section) low. Also the tooth roots would be so narrow that the teeth would be mechanically weak. By placing several elements in each slot the number of slots is reduced and larger slots result. This also reduces the cost of winding.

Coils made up from several individual coils are shown in Fig. 195. The two or three coils are taped as one and are placed in the slots as a unit. A careful examination of the armature of Fig. 214 shows four wires running from each coil side to the commutator, indicating a quadruple coil.

The numbering and connections of the conductors are in no way different from those already described in the case of but two coil sides per slot.

The selection of the pitch, where several coil sides per slot are used, is more restricted than it is with two elements per slot.

Assume that a 6-pole machine has 72 slots and six elements per slot. The total number of elements on the armature surface:

$$Z = 72 \times 6 = 432$$

The pitch should be approximately

$$y = \frac{432}{6} = 72$$

Let

$$y_b = 71$$

$$y_f = 69$$

If this back pitch is used a coil must reach from conductor 1 to conductor 72 (Fig. 199). Then the next coil will obviously reach from conductor 3 to conductor 74. These two coils, therefore, span different distances on the armature and accordnigly

must have different spans, as a study of Fig. 199 will show. In practice it is desirable that the coils be all the same when possible, and further it should be possible to tape all three coils together and place them in the slots as a unit.

If in the above case $y_b = 73$ and $y_f = 71$, the coil containing conductor 1 will reach from the upper *left-hand* side of slot A to the lower *left-hand* side of slot B, that is, from conductor 1 to conductor 74. Conductor 3 will reach from the center and top of slot A to the center and bottom of slot B, and conductor 5 will reach from the upper right-hand side of slot A to the lower right-

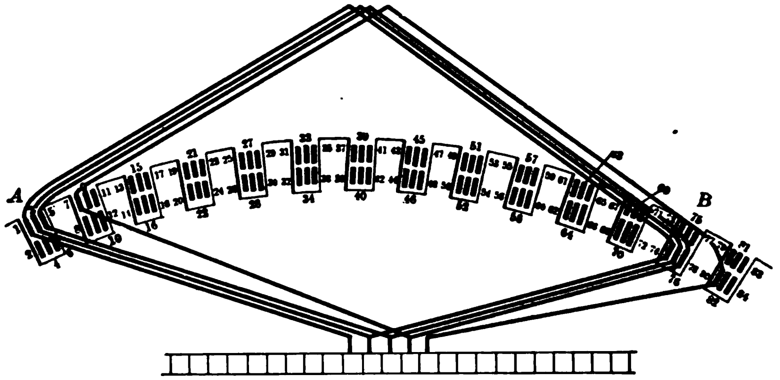


FIG. 199.—Method of connecting the conductors of a triple coil.

hand side of slot B. As all three coils now span the same distance on the armature, they will be equal in size, form, etc. Moreover, the three coils can be taped together and placed in the two slots as a unit. Therefore, if three coils have their adjacent sides in the top of one slot, their other sides should lie together in the bottom of some other slot. This condition is obtained by making the back pitch one greater than a multiple of the number of coil sides or elements per slot. For example, in the illustration just given, y_b is equal to 73, one greater than 72, 72 being a multiple of 6.

169. Paths through an Armature.—If four batteries, each having an electromotive force of 2 volts and a current capacity of 10 amp. be connected in parallel, Fig. 200 (a), there will be *four* paths for the current to follow in going through the batteries. The voltage of the combination will be 2 and the ampere capac-

ity 40; making a total power capacity of 80 watts. If now these same batteries be arranged in two groups of two in series, Fig. 200 (*b*), there result but two paths for the current to follow, but the voltage is now 4 volts. The current capacity is now 20 amp., and the power capacity is $4 \times 20 = 80$ watts, its previous value.

Similarly the conductors in an armature may be so connected that certain groups of conductors are in series. These groups may then be so connected that there are two or more paths in parallel. To determine the number of such parallel paths, start

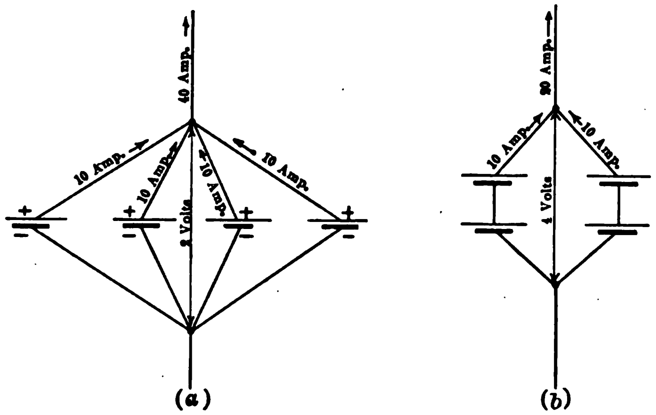


FIG. 200.—Parallel and series-parallel arrangement of batteries.

at one of the machine terminals, as for instance, the negative, and see how many different paths through the armature it is possible to follow in order to reach a positive terminal.

The simplest arrangement of conductors occurs in the gramme ring winding. Fig. 201 (*a*) shows a winding for a 4-pole machine.

Starting at the (−) terminal, one path may be followed by going to brush (*a*), through the winding at (1) to brush (*d*) and then to the (+) terminal.

A second path is obtained by going to brush (*a*), then through path (2) to brush (*b*) and then to the (+) terminal.

A third path is obtained by going to brush (*c*), through path (3), then through brush (*b*) to the (+) terminal.

A fourth path is obtained by going to brush (*c*), through path (4) to brush (*d*) and then to the (+) terminal.

This makes four separate paths between the (−) and (+) terminals, these paths being in parallel.

Assume that there are 10 amp. per path and 20 volts between brushes. The armature may be considered as being equivalent to four batteries connected as shown in Fig. 201 (*b*), each battery delivering 10 amp. at 20 volts. Battery 1 corresponds to path 1, battery 2 to path 2, etc.

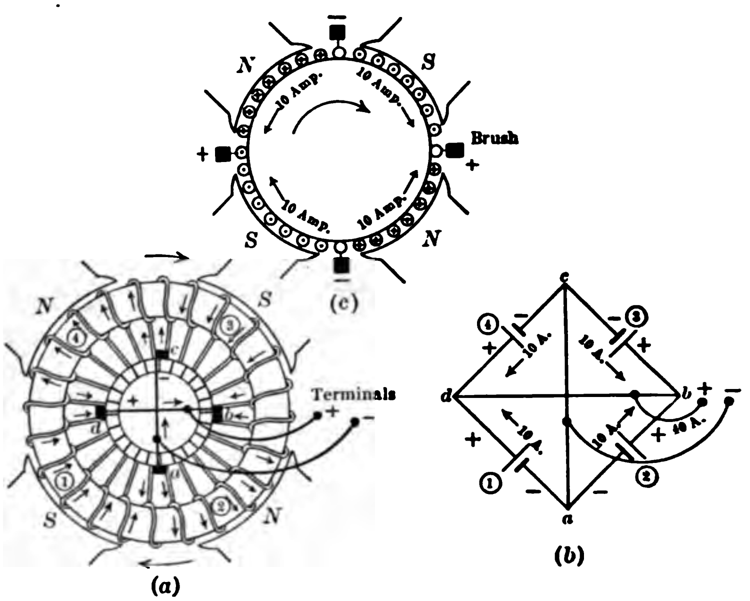


FIG. 201. Four paths in parallel through an armature.

It will be seen that the four batteries are connected in parallel because their four positive terminals and their four negative terminals are respectively connected together. The total current delivered will be 40 amp. at 20 volts. In a similar manner each path in the ring winding will deliver 10 amp., making 20 amp. per brush or 40 amp. per terminal. The potential difference between brushes will be 20 volts.

The paths through a drum winding are not as easy to follow as those through a ring winding. Fig. 202 shows the 18-slot drum winding of Fig. 198 developed in circular form. For the sake of simplicity two paths are shown with heavy lines, one

from brush *a* to brush *b*, and the other from brush *c* to brush *d*. These constitute two paths. By tracing through the lighter lines, two more paths may be found, one between brushes *c* and *b* and the other between brushes *a* and *d*, making four paths in all.

In all simplex lap windings there are as many paths through the armature as there are poles.

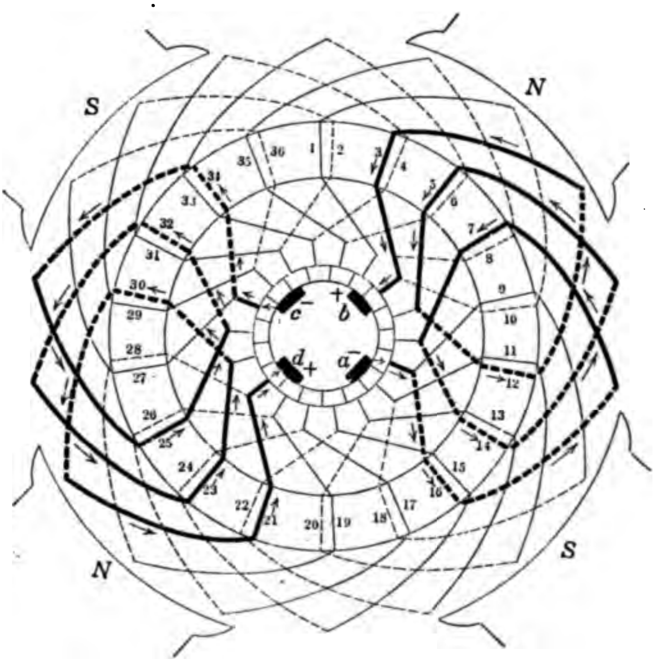


FIG. 202.—Heavy lines show two of the four parallel paths of a lap winding.

170. Multiplex Windings.—Fig. 203 shows a 36-slot, 4-pole winding, in which every alternate slot is filled. There are two conductors per slot. The back pitch, y_b, is 17, and conductor 1 connects to conductor 18 on the back of the armature. Conductor 18 then connects to 5 on the front of the armature, making the front pitch $y_f = 13$. Instead of returning to the conductor differing by *2* from the initial conductor, the return is made to a conductor differing by *4* from the initial conductor. Conductors *3* and *4* are not connected to this winding. Furthermore, only alternate commutator segments are utilized. It will be

seen that this winding closes on itself after going once around the armature; that is, this winding is re-entrant and is in itself complete in the same manner as any simplex 18-slot winding. (See Fig. 202.)

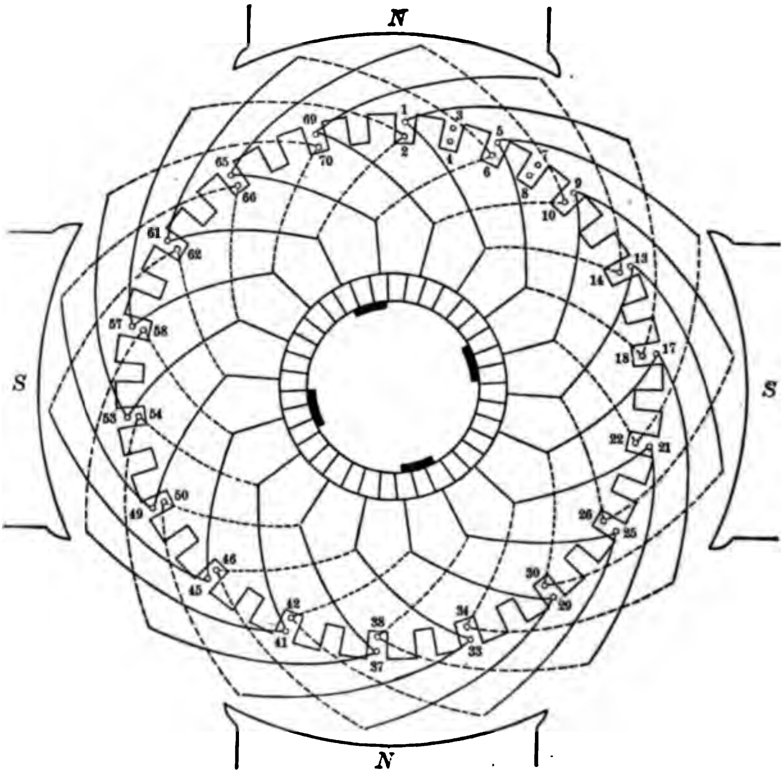
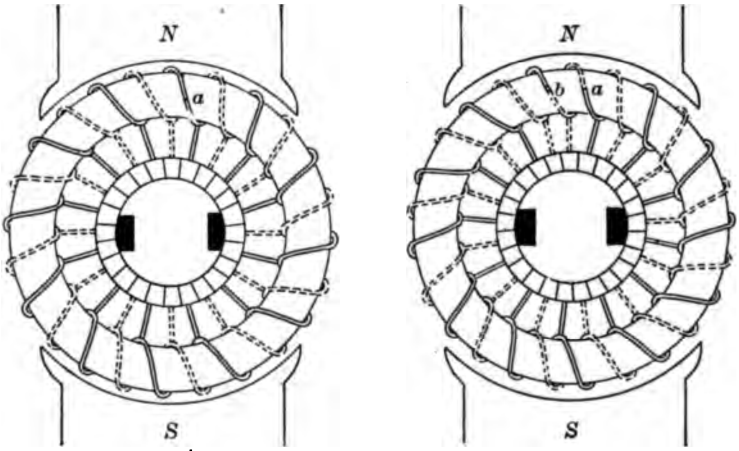


Fig. 203.—Duplex doubly-re-entrant lap winding—one winding only being shown.

As this winding only uses every alternate slot, another winding, the duplicate of this one, can be placed in the vacant slots, this new winding having the same front and back pitch as the other, and being connected to the commutator segments not utilized by the other. This winding will also close on itself, and is, therefore, re-entrant.

These two windings are separate and are insulated from each other on the armature, but are tied together electrically by the

span of the carbon brushes on the commutator. This condition is perhaps more clearly shown in the simple gramme ring winding of Fig. 204 (*a*), where one winding is in solid lines and the other in dotted lines. These two windings are in parallel, so that the number of paths is now twice what it would be in a simplex lap winding. As each of the two windings closes on itself, the winding is said to be *doubly re-entrant.* It is necessary with this type of winding that the brush span at least two commutator segments.



(*a*) Duplex doubly re-entrant gramme ring winding. (*b*) Duplex singly re-entrant gramme ring winding.

FIG. 204.

When there are two such windings in parallel, the winding is said to be *duplex.* Therefore, this is a *doubly re-entrant duplex winding.* Obviously, three or more such windings can be placed on an armature, making the winding triplex, quadruplex, etc., the number of such windings being called the multiplicity of the winding.

Let m = the multiplicity of the winding.

The number of paths p' in a lap winding is

$$p' = mp \tag{96}$$

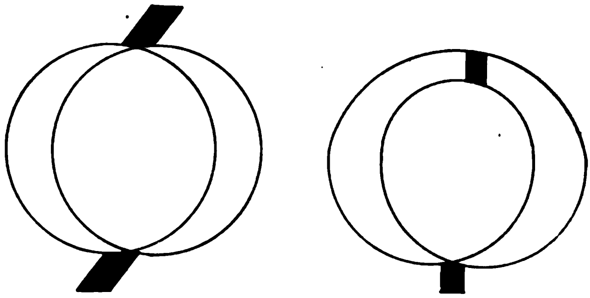
where p is the number of poles.

The relation of the back and the front pitch becomes

$$y_b = y_f \pm 2m \tag{97}$$

This should be compared with equation (94) where $m = 1$.

If the number of coils, Fig. 203, be odd, that is, if there are 35 or 37 coils or commutator segments, the winding will not close after having gone once around the armature, but will return one slot, or two conductors, to the right or to the left of the one at which it started. (If there are more than four elements per slot the winding may return to the same slot at which it started, but removed by two conductors from the conductor at which it started.) Therefore, this winding does not close or become re-entrant, after having passed once around the armature, but must pass around once again before closing. This is illustrated in Fig. 204 (*b*). The initial winding starts at *a*. After



(*a*) Duplex doubly re-entrant winding. (*b*) Duplex singly re-entrant winding.
FIG. 205.—Duplex windings in diagrammatic form.

passing once around the ring armature it does not close at *a* as does the winding in Fig. 204 (*a*), but terminates at *b*, one conductor removed from *a*. The second winding, shown dotted, starts at *b* and after passing once around the armature, closes at *a*. Although this winding passes around the armature twice, it only closes once, so is said to be *singly* re-entrant. Therefore, this constitutes a *singly re-entrant duplex* winding. The two windings are the same electrically. Their difference is best illustrated by the two simple diagrams of Fig. 205.

171. Equalizing Connections in Lap Windings.—Lap windings may consist of several paths in parallel, the parallel connections being made through the brushes. If several batteries are connected in parallel and their emf.'s are not equal, currents circulate among the batteries, even when no external load is being supplied. This means a constant loss of energy which heats the batteries.

This same condition exists in generator armatures. Because of very slight inequalities in the air gap, due to the wearing of the bearings, lack of mechanical alignment, etc., there may be slight differences of electromotive force in the different paths through the armature. These differences of emf. will cause currents to flow between different points in the armature, and these currents must flow through the brushes even when no current is being delivered by the generator. To relieve the brushes of this extra current, several points in the armature

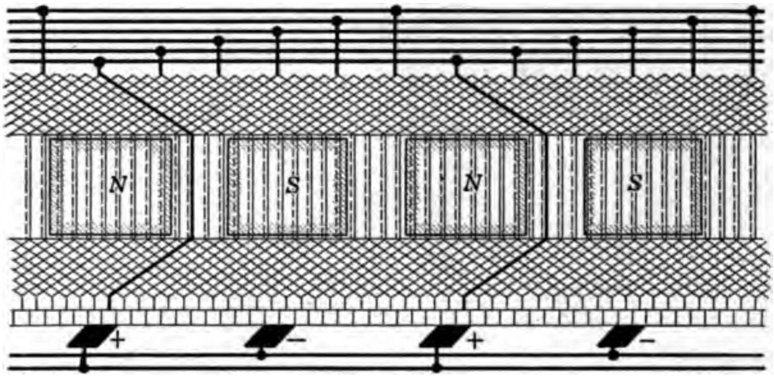


FIG. 206.—Simplex lap winding with equalizing connections.

which are simultaneously at equal potentials are connected together by heavy copper bars. This allows these circulating currents to flow from one point in the armature to another without passing through the brushes. To make these equalizer connections, the number of coils should be a multiple of the number of poles, and the coils per pole should be divisible by some small number as 2 or 3.

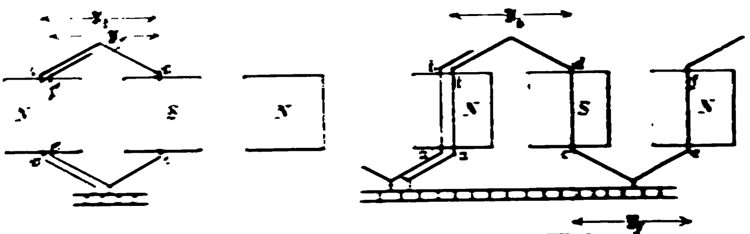
As an example, assume an 8-pole generator having 12 slots per pole and two coil sides per slot. There will be 96 slots and 192 coil sides. The number of coil sides per pole will be 24. Let $y_b = 25$ and $y_f = 23$. A portion of this winding is shown in Fig. 206. It will be noted that every fourth coil is connected to an equalizing connection. The coils that are connected to the same equalizing connection occupy the same positions relative to the poles. (See the two coils drawn with heavy lines.) This is necessary as such coils should be generating the *same*

voltage at every instant. It will be noted in Fig. 206 that the two segments under the two positive brushes are connected together by an equalizing connection.

Theoretically, every coil should be connected to an equalizing connection, but as this would require an undue number of such connections, it is sufficient, practically, to connect every third or fourth coil. This is the reason that the number of coils per pole should be divisible by a small number as 2, 3 or 4. Fig. 207 shows a large direct-current armature with the equalizer connections at the back of the armature.

FIG. 207.—General Electric Co. direct-current armature with equalizer rings.

172. Wave Winding.—It has been shown that in the case of the *lap* winding a conductor under one pole is connected directly to a conductor which occupies a nearly corresponding position under the next pole. This second conductor is then connected *back* again to a conductor under the *original* pole, but removed two or more conductors from the initial conductor. This is



(a) Lap Winding (b) Wave Winding

FIG. 208.—Lap and wave windings.

shown in Fig. 208 (a), where conductor *ab* under a north pole is connected to conductor *cd* having a corresponding position under the next south pole. Conductor *cd* is then connected to *ef* which is adjacent to *ab* under the original north pole.

Obviously it would make no difference as far as direction of induced emf. in the winding is concerned if the connection, instead of returning back to the same north pole, advanced *forward* to the next north pole, as shown in Fig. 208 (*b*). When the connection is so made, the winding passes successively every north and south pole before it returns again to the original pole, as shown at $a'b'$ in Fig. 208 (*b*). The winding after passing once around the armature reaches conductor $a'b'$ lying under the same pole as the initial conductor ab. When a winding advances from pole to pole in this manner, it is called a *wave winding*. The number of units spanned by the end connections on the back of the armature is called the *back* pitch and is denoted by y_b in Fig. 208 (*b*). This is similar to the corresponding term in the lap winding shown in Fig. 208(*a*). The number of elements which the end connections span on the commutator end of the armature is the *front* pitch and is denoted by y_f. This should also be compared with Fig. 208 (*a*). As in the lap winding, y_f and y_b must both be odd in order that one side of a coil may lie in the top of a slot and the other side in the bottom of a slot. Unlike the lap winding, y_f may equal y_b in the wave winding.

The above is illustrated as follows:

A certain wave winding may have a back pitch of 23 and a front pitch of 19. The average pitch

$$y = \frac{23 + 19}{2} = 21.$$

Likewise both the front and the back pitch may each be 21 making the average pitch 21.

In any event, the average pitch

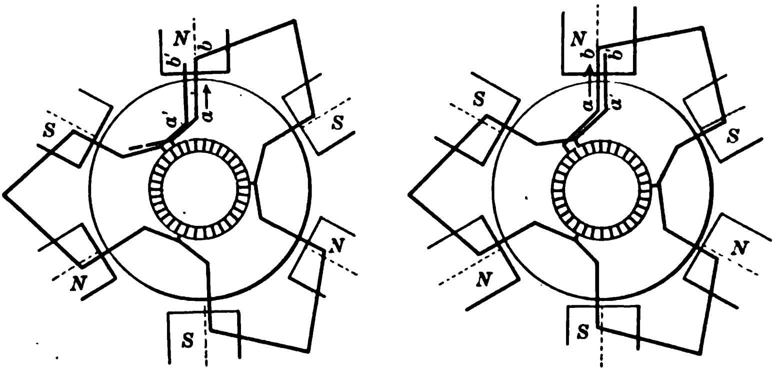
$$y = \frac{y_b + y_f}{2} \tag{98}$$

y may be either even or odd.

When the winding viewed from the commutator end falls in a slot to the left of its starting point as $a'b'$, Figs. 208(*b*) and 209 (*a*), after passing once around the armature, the winding is *retrogressive*. If, on the other hand, it falls to the right of its starting point, as shown in Fig. 209 (*b*), it is *progressive*.

The wave winding is much more restricted in its relation to the

number of slots and coils than is the lap winding, for the following reason. In a simplex wave winding, after having passed once around the armature, the winding must fall *two* conductors either to the right or to the left of the conductor at which it started. Thus in Fig. 209 (*a*), if there are two conductors per slot and conductor *ab* lies in the bottom of one slot, conductor *a'b'* must lie in the bottom of the slot next to *ab*. As there are two coil sides in each slot, this means that conductors *ab* and *a'b'* will differ from each other by 2.



(*a*) Retrogressive wave winding—34 commutator segments.

(*b*) Progressive wave winding—32 commutator segments.

FIG. 209.

Let y be the average pitch. Assume that the winding closes after passing once around the armature, which, of course, it should not do as this would constitute a short-circuit. Then:

$$py = Z$$

where p is the number of poles and Z the number of coil sides or elements. But the winding must *not* close after passing once around. In fact, it must not close until every slot is filled. Therefore, after passing once around the armature, the product py cannot equal Z but must be $Z \pm 2$. That is:

$$py = Z \pm 2$$

or

$$y = \frac{Z \pm 2}{p} \qquad (99)$$

The + sign indicates a *progressive* winding and the − sign a *retrogressive* winding.

As an illustration, assume that a 4-pole armature has 63 slots and four conductors per slot, making 252 winding elements. Let the average pitch be 63, the front and back pitch both being 63. As in the lap winding diagrams, a single-turn coil will be used to represent a coil having several turns, as indicated in Fig. 210. Starting at conductor 1, the winding will advance as follows:

1–64–127–190–(253 or 1)

That is, the winding will close on itself after going once around the armature, which condition constitutes a short circuit and

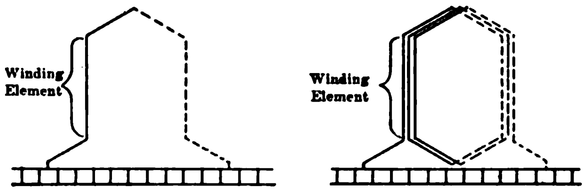


FIG. 210.—Single-turn coil representing a 3-turn coil for winding diagram.

makes the winding impossible. (The method by which a winding may be placed in these slots will be shown later.) Therefore, a wave winding is impossible in a 4-pole machine if 252 winding elements are to be included.

Let N_c be the number of commutator segments, which is also the number of coils.

$$N_c = Z/2, \quad Z = 2N_c$$

Let p_1 = pairs of poles $= p/2 \qquad p = 2p_1$
Substituting in equation (99)

$$y = \frac{2N_c \pm 2}{2p_1}.$$

$$N_c = p_1 y \pm 1 \tag{100}$$

If p_1 is odd and y is odd, the product p_1y is odd, as the product of two odd numbers is always odd. Adding or substracting unity makes N_c even.

Therefore, with a wave winding whose average pitch is *odd* and having 6, 10, 14 poles, or 3, 5, 7 pairs of poles, the number of commutator segments and coils must each be *even*. If the average pitch is *even* the number of commutator segments and coils must each be *odd*.

On the other hand, if p_1 is *even*, corresponding to 4, 8, or 12 poles, the product p_1y is even, so that N_c must be *odd*. The application of equation (100) is illustrated in Fig. 209. There are 6 poles and the average pitch, y, is 11. Applying equation (100), $N_c = 3 \times 11 + 1 = 34$; $N_c = 3 \times 11 - 1 = 32$. The 34 segments are shown in (*a*), Fig. 208, which gives a retrogressive winding, and the 32 are shown in (*b*), which gives a progressive winding. N_c is even in either case.

This is another limitation of the wave winding and shows why the 252-conductor (126-coil) winding just considered is impossible. The number of coils must be *odd* in a 4-pole winding. However, if one coil were omitted, making 250 elements, the winding would progress as follows:

1–64–127–190–(253 or 3)
–66–129–192–5, etc.

That is, the winding would advance by two conductors after each passage around the armature, which condition makes the winding possible. This, of course, reduces the number of commutator seg-

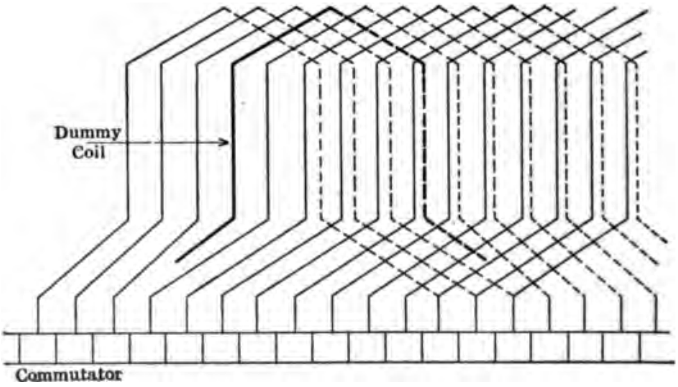


FIG. 211.—Dummy coil and "creeping" in a forced wave winding.

ments and coils from 126 to 125, an odd number. If, in this case, the armature stampings were standard, having 126 slots, the winding would be possible by omitting one coil. This coil would be inserted in the slots just the same as the other coils, except that its ends would not be connected to the commutator segments but would be taped and thus insulated from the main

winding. The coil would serve only as a filler, and is called a "dummy coil." In this case there would be a slight "creeping" of the winding with respect to the commutator, as shown in Fig. 211. This is called a *forced* winding.

If the coils used in a wave winding consist of more than one turn, they will have the ends brought out and connected in the manner shown in Fig. 195 (*b*) and in Fig. 210.

173. Number of Brushes.—Fig. 212 shows the beginning of a wave winding, which begins at positive brush *a* and advances

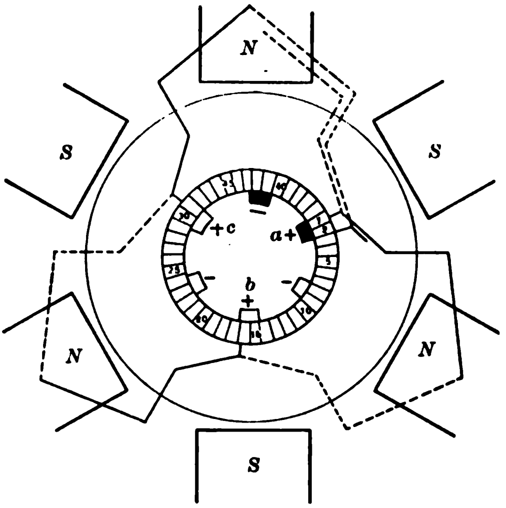


FIG. 212.—Wave winding—3 positive brushes connected by the winding itself.

once around the armature. This is a 6-pole winding in a machine having 44 commutator segments. The pitch is found from equation (100).

$$44 = 3y \pm 1$$

$$y = \frac{44 \pm 1}{3} = 15, \text{ using the + sign.}$$

This is also equal to the number of commutator segments by which the winding advances per *pair* of poles, or each time that it is connected to the commutator. Therefore, the segment connections, starting with 1, are 1–16–31–2–17, etc., as shown in Fig. 212. The winding ends one segment beyond the

starting point for each complete passage around the armature, showing that the correct pitch has been chosen.

There are three positive brush sets, *a*, *b*, and *c*, and also three negative brush sets, the same number that would be used in a lap winding. It should be noted that the three + brushes, *a*, *b*, and *c* are all connected together *directly* by the winding. Moreover, the conductors which connect these three brushes all lie between the poles in the neutral plane, where they are not cutting any magnetic lines and are for the instant, therefore, dead conductors. Hence, if brushes *b* and *c* were removed, the current could easily pass through the armature to brush *a* and thence to the external circuit. In like manner, two of the negative brushes could be removed, without serious disturbance. It is desirable to utilize all six brush sets, as two brush sets would mean a commutator three times as long in order to obtain the necessary brush area.

In a wave winding only two brushes are necessary, regardless of the number of poles, although it is usually desirable to use the same number of brushes as poles.

There are cases, however, where it is desirable to use only two brushes. The best example is in railway motors where it would be difficult to obtain access to four or six brushes. By means of a small hand hole in the motor casing, it is a comparatively simple matter to reach two brushes located on the top of the commutator.

174. Paths through a Wave Winding.—In a simplex wave winding there are always *two* parallel paths, regardless of the number of poles. Fig. 213 shows a 4-pole, 13-slot, simplex wave winding, having two coil sides per slot. One of the paths is shown by the heavy lines. Approximately half the winding is shown heavy, the other half constituting the other path. (The coils short-circuited by the brushes are not included.) A wave winding may be duplex, triplex, or have any degree of multiplicity just as the lap winding may.

The paths through the armature depend only on the degree of multiplicity and not on the number of poles. A simplex wave winding always has two paths, a duplex winding four paths, etc.

It is interesting to compare the current and voltage of an armature for the various ways of connection. Consider a 6-pole

machine. When connected as a simplex lap winding let its emf. be 300 volts and the armature current per terminal be 120 amp.

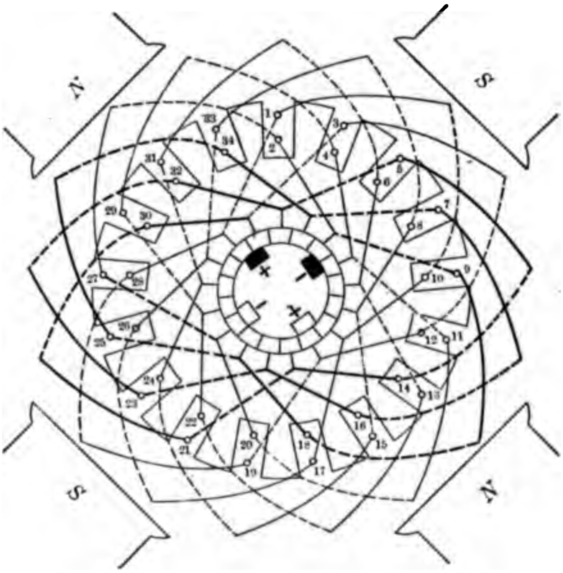


FIG. 213.—17-slot, 4-pole, simplex wave winding; back pitch = 9, front pitch = 7; one of two parallel paths shown heavy.

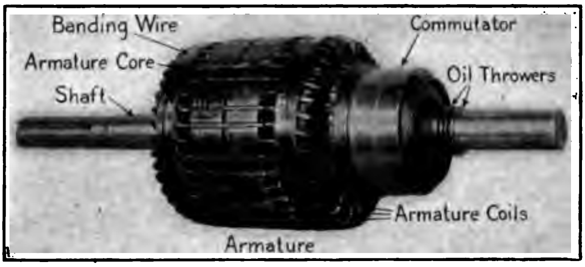
The following table gives the values of current and emf. obtainable when the winding is changed, the total number of armature conductors remaining fixed.

	Paths	Volts	Amperes	Kw.
Simplex lap	6	300	120	36
Duplex lap	12	150	240	36
Triplex lap	18	100	360	36
Simplex wave	2	900	40	36
Duplex wave	4	450	80	36
Triplex wave	6	300	120	36

It will be noted that in this particular machine the triplex wave winding gives the same result as the simplex lap winding. The kilowatt capacity is not affected by the connection used. The above relations should be kept in mind when it is desired

to change a machine from one current and voltage rating to another. This may often be done by merely changing the commutator connections.

175. Uses of the Two Types of Winding.—A wave winding has an advantage in that it gives a higher voltage with a given number of poles and armature conductors. It is used, therefore, in small machines, especially those designed for 600-volt circuits.



(*a*) 25 H.P. wave-wound Westinghouse generator armature.

(*b*) End view of an armature showing open construction—Westinghouse commutating-pole D. C. motor.

FIG. 214.

In this case a lap winding would result in a very large number of small conductors. This in turn means a higher winding cost and less efficient utilization of the space in the slots.

The wave winding has the additional advantage that the electromotive force in each path is produced by series-connected conductors, which lie under successive north and south poles.

Any magnetic unbalancing, therefore, due to such causes as air-gap variation and difference in pole strength, does not produce cross currents, because the corresponding conductors of each and

FIG. 215.—Low-voltage, high-speed G. E. armature for electrolytic work. (Note double commutator and shrink rings.)

every path are moving by the same poles and the effect of such unbalancing will be the same in each path. Hence no equalizer connections are necessary.

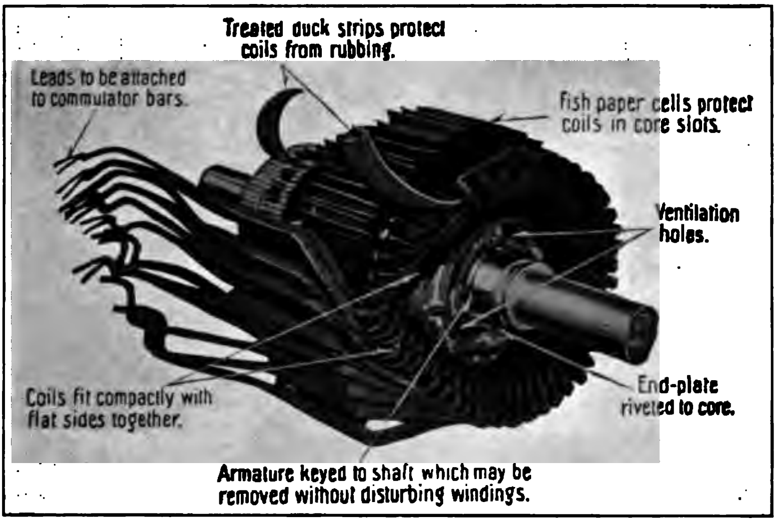


FIG. 216.—Partly wound armature showing method of assembling coils (Westinghouse).

The possibility of using only two brushes with a wave winding, and the corresponding advantage in railway motors, have already been mentioned.

When large currents are required, the lap winding is more satisfactory, since it gives a large number of paths. As 200

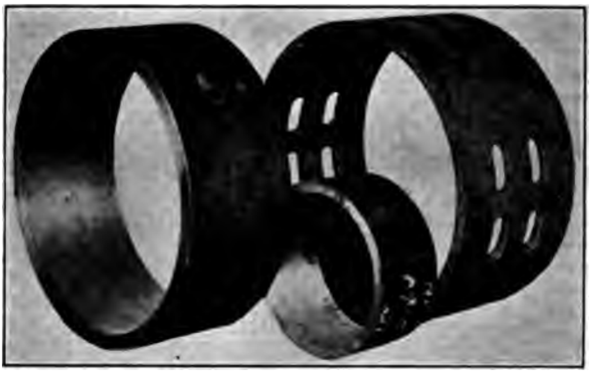


FIG. 217.—Frame rings—Westinghouse type S. K. motor.

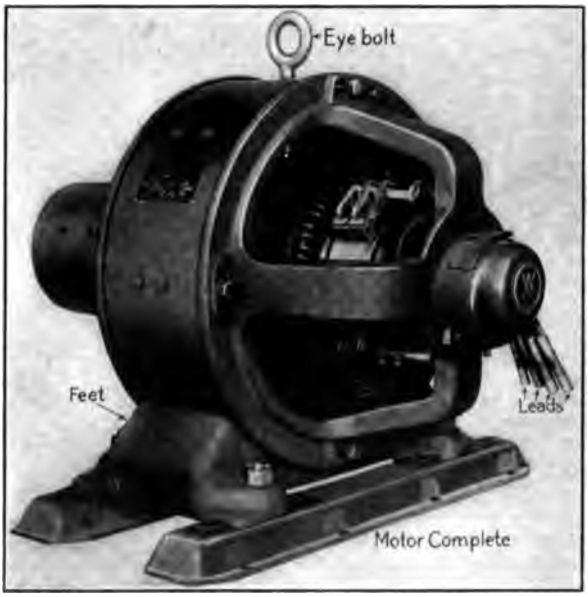


FIG. 218.—Westinghouse 230-volt, 35 H.P., 850 R.P.M., shunt motor.

amp. per path is practically the limit, a large number of paths must be used where heavy current output is desired. This is particularly true of large engine-driven multipolar generators.

Figs. 214 and 215 show two different types of armature and Fig. 216 shows an armature in the process of being wound.

DYNAMO CONSTRUCTION

176. Frame and Cores.—The frame or yoke of a dynamo has two functions. It is a portion of the magnetic circuit (see Figs.

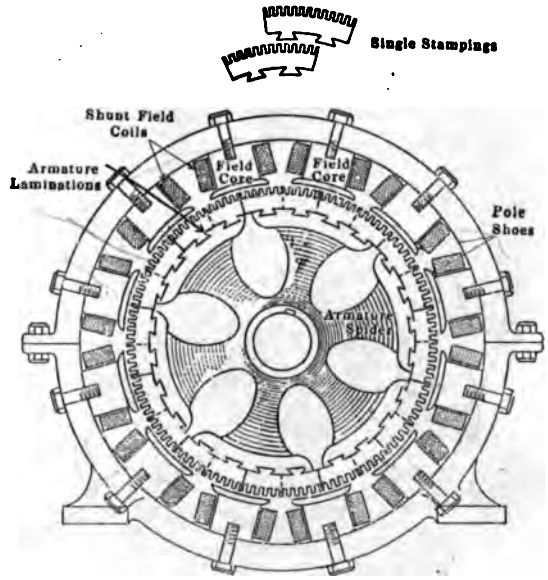


FIG. 219.—Construction of a 12-pole, direct-connected, engine-driven generator.

38, 39 and 40) and it acts as a mechanical support for the machine as a whole. In small machines, where weight is of little importance, the yoke is often made of cast iron. The feet almost always form a part of the casting. In another type of construction a steel plate is rolled around a cylindrical mandrel and then welded, Fig. 217. The feet in this case are made of steel stampings and are riveted on, Fig. 218. In larger machines the yoke is made of cast steel and is usually more or less oval in cross-section, Figs. 219 and 220. The feet are a part of the yoke casting. The yoke for the larger machine is usually cast in two pieces

which are bolted together. This facilitates the shipment of large machines and allows the armature to be removed easily.

177. Field Cores and Shoes.—The field cores are made of forged steel, cast steel and steel laminations. When made of cast or forged steel they are usually circular in cross-section, as such a section allows the minimum length of turn for a given

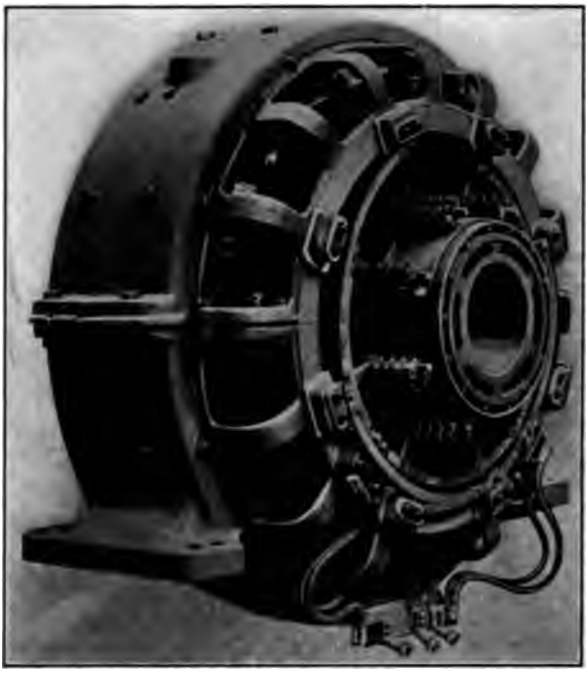


Fig. 220.—Westinghouse engine-driven, 300 K.W., 500 volt, 150 R.P.M. generator.

core section. These cores are held to the yoke by bolts, Figs. 219 and 220. The laminated cores are built of sheet steel stampings, Fig. 221. They are stacked so that the pole tip comes alternately on one side and the other. This results in there being but half the iron in a pole tip cross-section and so producing a saturated pole tip, which assists commutation. When stacked to the proper thickness, they are riveted together and dove-tailed to the yoke. In this case a separate pole shoe is not necessary. A laminated or a solid steel pole shoe may be bolted to the solid

cores, the laminated type being used on the larger machines to reduce pole face losses. (See Par. 228, page 355.)

178. The Armature.—The armature is made of sheet steel discs (14 to 25 mils thick) punched out by a die. The slots may be cut by the die or they may be cut out afterward with a

FIG. 221.—Field core lamination and pole piece assembled—Westinghouse D. C. motor.

slotting machine. In small motors these stampings are keyed directly to the shaft, Fig. 222. After every 2 or 3 inches of laminations a suitable spacer is inserted to form a ventilating duct, Fig. 223. The laminations are clamped together by end plates, Fig. 222, which are in turn held by nuts on the shaft or by bolts

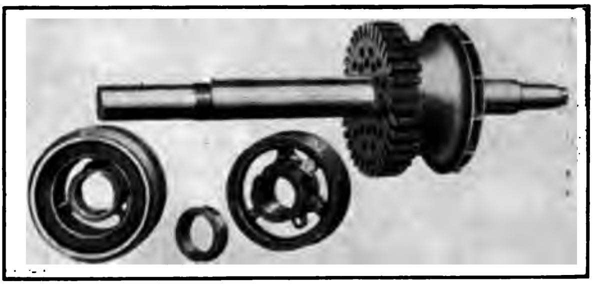


FIG. 222.—Armature construction of a small motor.

passing through the laminations. The laminations are perforated to allow air to pass through the armature axially and out radially through the ducts. Frequently a blower is attached to the end of the end plate, Fig. 222, to facilitate ventilation.

In machines of medium size, the stampings are assembled and keyed to an armature spider, which is in turn keyed to the

shaft, Fig. 223. This reduces the amount of sheet steel necessary and at the same time permits a free passage of air through the center of the armature. This air is then thrown out through the ventilating ducts by centrifugal action, as indicated by the arrows. The stampings in Fig. 223 are clamped together by

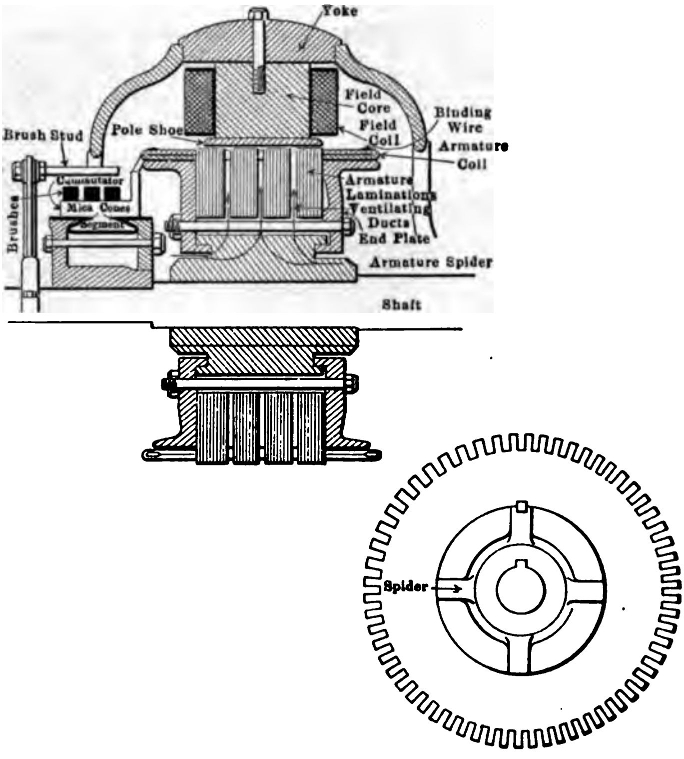


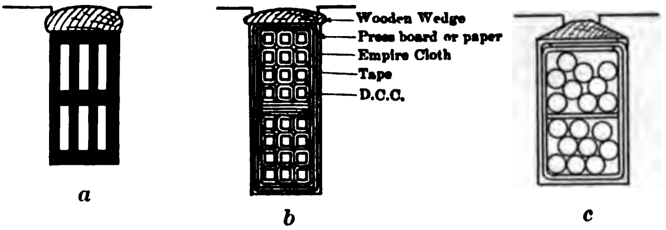
FIG. 223.—Cross-section of a moderate size generator; armature stamping.

end plates held by bolts. These end plates may also serve as supports for the overhang of the armature coils.

When the armature becomes greater than 30 in. in diameter, it is not economical to stamp out a complete ring. Such armatures are made up of segments similar to those shown in Fig. 219. These are dove-tailed to the armature spider, each segment lapping the joint in the next layer.

The slots may be straight sided, Fig. 223, in which case the

conductors are held in the slots by binding wires. In the larger machines the conductors are held in the slots by wooden wedges, Fig. 224. The slots must be well insulated, as grounds are troublesome and are expensive to repair. A layer of a hard substance such as fish paper, fiber or press board should be placed next to the laminations. This in turn should be lined with varnished cambric or empire cloth. The conductors themselves are usually covered with cotton insulation, except in the heavy bar windings. The groups of conductors are bound together in one coil by cotton tape. (See Fig. 216.)



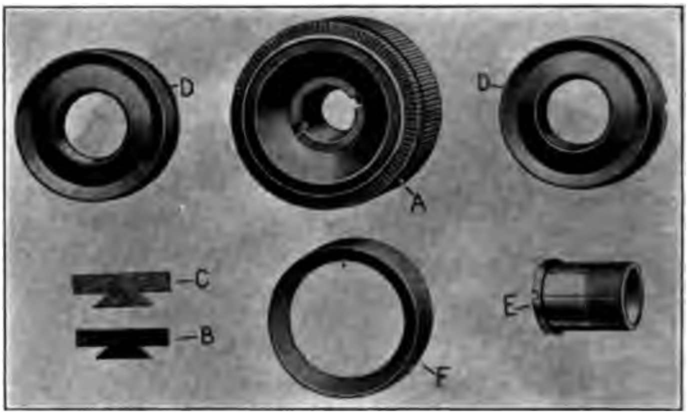
(*a*) Open slot containing triple coil.
(*b*) Open slot containing two coil sides, 12 turns per coil.
(*c*) Semi-closed slot and "mush" winding.

FIG. 224.—Types of slot.

To reduce the flux irregularities in the air gap, due to the teeth a semi-closed slot, Fig, 224*c*, is used occasionally. In this case the individual conductors must be placed in the slot one by one, so the coil ends must be taped after the coils are placed in the slots. Such a winding in called a "mush" winding. The expense of winding prevents the general use of this type of slot in direct current machines.

179. The Commutator.—The commutator is made of wedge-shaped segments of hard-drawn or drop-forged copper, insulated from one another by thin layers of mica. The segments are held together by clamping flanges (*DD*, Fig. 225), which pull the segments inward when the flanges are drawn together by through-bolts. These flanges are prevented from short-circuiting the segments by two cones of built-up mica (*F*, Fig. 225). This construction is illustrated by the commutator of the machine shown in Fig. 223.

The leads from the armature coils may be soldered into small longitudinal slits in the ends of the segments or the segments



(*A*) Assembled commutator.
(*B*) Commutator bar.
(*C*) Mica commutator insulating strip.
(*DD*) Clamping flanges.
(*E*) Drawn steel tube.
(*F*) Insulation used between clamping flanges and commutator bars.

Fig. 225.—Crocker-Wheeler commutator and details.

may have risers (Fig. 223) to which these leads are soldered. (Also see Fig. 214 (*b*).)

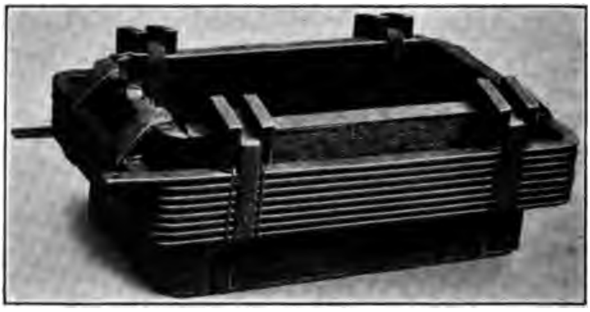


Fig. 226.—Shunt field coil and edgewise series winding.

180. Field Coils.—The field coils are usually wound with double-cotton-covered (d.c.c.) wire. The coils are dried in a vacuum and then impregnated with an insulating compound.

The outer cotton insulation is often protected by tape or cord on the outside. In the larger machines an air space is often left between layers for ventilating purposes. The coils are also wound on metal spools, Fig. 226. An edgewise series winding, set some distance from the shunt winding, also is shown here.

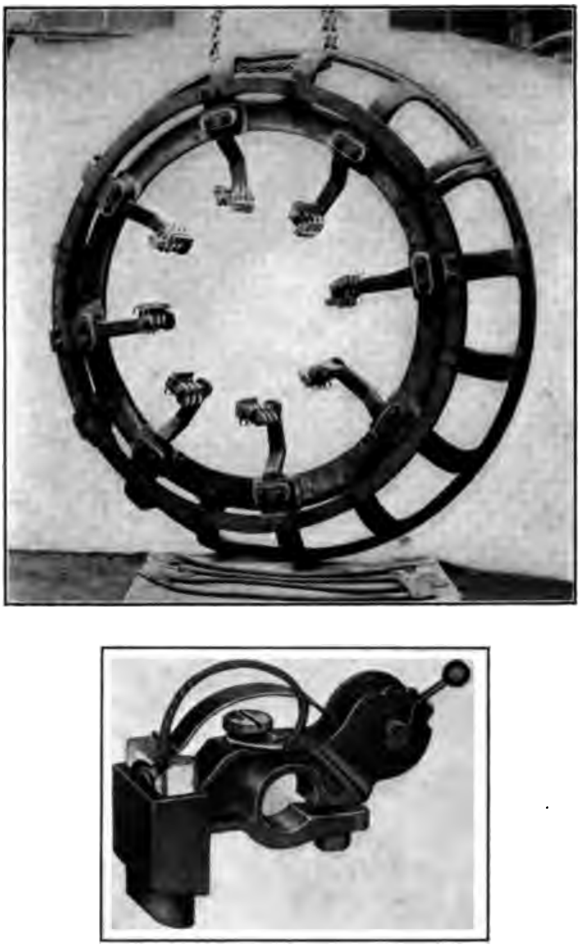


Fig. 227.—Rocker ring and brush holder.

181. The Brushes.—The function of the brushes is to carry the current from the commutator to the external circuit. They are usually made of carbon, although in very low voltage machines

they may be made of copper gauze, or patented metal compounds. The brush holder, Fig. 227, is fastened to the brush stud and holds the brush in its proper position on the commutator. The brush should be free to slide in its holder in order that it may follow any irregularities in the commutator. The brush is made to bear down on the commutator by a spring, Fig. 227. The pressure should be from 1 to 2 lb. per sq. in. To decrease the electrical resistance the upper portion of the brush is copper plated and this plating is connected to the brush holder by a pig-tail made of copper ribbon. A rocker ring with cross connections is also shown in Fig. 227.

CHAPTER XI

GENERATOR CHARACTERISTICS

182. Electromotive Force in an Armature.—The path of the magnetic flux from the poles of a generator into the armature and a curve showing the flux distribution are given in Fig. 228. The positive ordinates of the distribution curve are north pole flux entering the armature and the negative ordinates are flux leaving the armature and entering a south pole. The total flux leaving a north pole is given by the area under one of the positive parts of the distribution curve. Similarly, the total flux leaving the

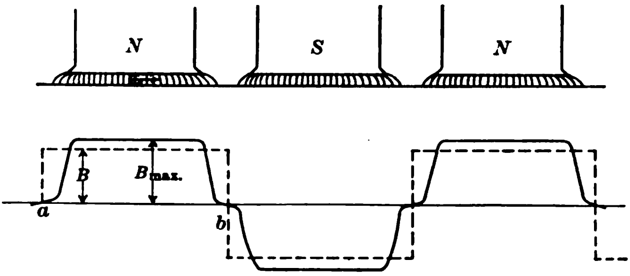


FIG. 228.—Flux distribution at no load of a D. C. generator.

armature and entering a south pole is given by the area of one of the negative parts of the distribution curve. Each positive part and each negative part of the curve may be replaced by a rectangle having the same area, as shown by the dotted line, Fig. 228. The height of this rectangle will be B maxwells, which is equal to the *average* value of the flux density under an entire pole pitch.

Let it be required to determine the average electromotive force induced in a single conductor as it passes through the flux of successive poles.

Let the flux leaving a north pole or entering a south pole be ϕ. Let A be the pole area in sq. cm., l the active length of the conductor in cm., s the speed of the armature in revolutions per *second*, and P the number of poles.

When the conductor passes through the distance ab, or one pole pitch, the average induced voltage, by equation (93), is

$$e = Blv\,10^{-8}$$

where B is the average flux density, l the active length of the conductor in cm., and v the velocity of the conductor in cm. per second.

$$v = \frac{ab}{t}$$

where t is the time required for the conductor to traverse the distance ab.

$$e = \frac{Bl(ab)}{t}\,10^{-8} = \frac{\phi}{t}\,10^{-8}$$

since $Bl\ (ab)$ gives the total flux between the points a and b as cut by the conductor and is therefore equal to ϕ.

The time $\qquad t = \dfrac{1}{sP}$

Therefore, the average voltage per conductor is

$$e = \frac{\phi}{1/sP}\,10^{-8} = \phi sP10^{-8}$$

If there are Z such conductors and p paths through the armature, there must be Z/p such conductors in series. (See Par. 169.)

Hence the total voltage generated between brushes is

$$E = \frac{\phi sPZ}{p10^8} \tag{101}$$

Example.—A 900 r.p.m., 6-pole generator has a simplex lap winding. There are 300 conductors on the armature.

The poles are 10 in. square and the average flux density is 50,000 lines per sq. in. What is the voltage induced between brushes?

$$\phi = 10 \times 10 \times 50{,}000 = 5{,}000{,}000 \text{ lines}$$
$$s = 900/60 = 15 \text{ r.p.s.}$$
$$P = 6$$
$$p = 6 \text{ (see Par. 169)}$$
$$E = \frac{5{,}000{,}000 \times 15 \times 6 \times 300}{6 \times 10^8} = 225 \text{ volts. } \textit{Ans.}$$

183. The Saturation Curve.—Equation (101) may be written as follows:

$$E = \left(\frac{PZ}{60p10^8}\right)\phi S$$

where S = r.p.m.

The quantity within the parenthesis is constant for a given machine and may be denoted by K.

Therefore:

$$E = K\phi S \tag{102}$$

The induced emf. in a machine, therefore, is directly proportional to the flux and to the speed.

If the speed be kept constant, the induced voltage is directly proportional to the flux, ϕ.

The flux is produced by the field ampere-turns, and as the turns on the field remain constant, the flux is a function of the field current. It is not directly proportional to the field current because of the varying permeability of the magnetic circuit.

Fig. 229 shows the relation existing between the field ampere-turns and the flux per pole. The flux does not start at zero ordinarily but at some value slightly greater, owing to the residual magnetism in the machine. At first the line is practically straight, as most of the reluctance of the magnetic circuit is in the air gap. At the point q the iron begins to be saturated and the curve falls away from the straight line.

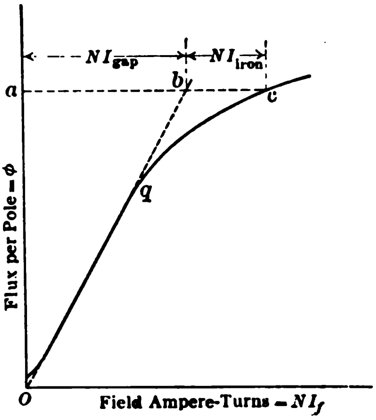


FIG. 229.—Saturation curve.

The number of field ampere-turns for the air gap and for the iron can be approximately determined for any point on the curve.

Let it be required to determine the ampere-turns for the gap and for the iron at the point c. From the origin draw ob tangent to the saturation curve and also draw the horizontal line ac. The line ob is the magnetization curve of the air gap, if the reluctance of the iron at low saturation be neglected. Therefore, the ampere-turns required by the gap are equal to ab and those required by the iron are equal to bc.

From equation (102) the induced voltage is proportional to the flux, if the speed is maintained constant. Therefore, if the induced voltage be plotted against field current as abscissæ,

a curve similar to that of Fig. 229 is obtained. This is shown in Fig. 230 and differs from the curve of Fig. 229 only by a constant quantity (KS). Two curves are shown in Fig. 230, one plotted for 1,200 r.p.m. and the other for 900 r.p.m. The curves are similar, any ordinate of the lower curve being 900/1,200 of the value of the corresponding ordinate of the upper curve. Thus, at ordinate ac,

$$\frac{ab}{ac} = \frac{900}{1,200}$$

Also at ordinate $a'c'$

$$\frac{a'b'}{a'c'} = \frac{900}{1,200}$$

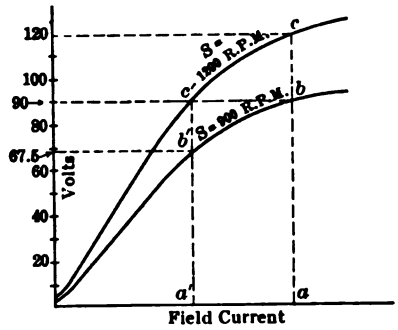


FIG. 230.—Saturation curves for two different speeds.

If the saturation curve of a generator for one speed has been determined, saturation curves for other speeds may be readily found by the method just indicated.

184. Hysteresis.—The saturation curve *oab*, Fig. 231 (*a*), is determined for *increasing* values of the field current. If when

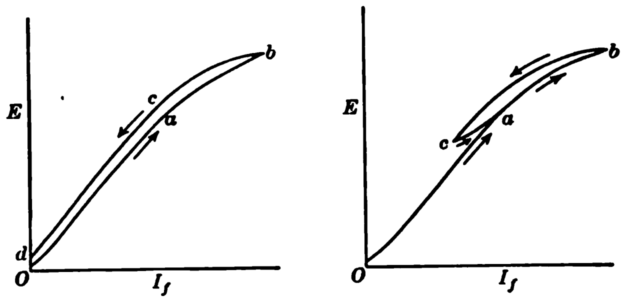


FIG. 231.—Hysteresis loops.

point *b* is reached the field current be decreased, the curve will not retrace its path along the curve *bao*. For any given field current, the corresponding induced voltage will now be greater than it was for *increasing* field currents. This is shown by the curve *bcd*. This is due to hysteresis in the iron.

Fig. 231 (*b*) shows the effects obtained when the curve is

carried up along the path *oab*, back to *c*, and at *c* the field current is again increased, the curve ultimately coming back to *oab* at the point *a*.

It is evident that for any given value of field current, there is no single value of flux. The value of flux for any given field current depends upon whether the field current was *increased* until it reached the value in question or whether it was *decreased*. This characteristic of the magnetic circuit should be carefully borne in mind, for the operating characteristics of both generators and motors are affected to a considerable degree by hysteresis in the magnetic circuit.

185. Determination of the Saturation Curve.—To determine the saturation curve experimentally, connect the field, in series

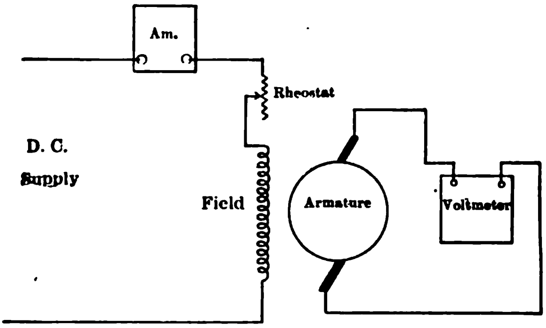


FIG. 232.—Connections for obtaining saturation curve.

with an ammeter, across a direct current source of power. A voltmeter should be connected across the armature terminals. Obviously the ammeter measures the field current, values of which are plotted as abscissæ; the voltmeter reads the values of induced armature voltage, which are plotted as ordinates. These connections are shown in Fig. 232. As the voltage drop within the armature due to the voltmeter current is negligible, the terminal volts and the induced volts under these conditions are identical. During the experiment the speed should be determined each time that the other readings are taken. If the speed cannot be maintained constant, corrections can be easily made for any variation, by the method described in Par. 183.

When the saturation curve of a shunt generator is determined,

it may be difficult to obtain a sufficiently high resistance to reduce the field current to its lower values. A drop wire connection, Fig. 233, allows field currents as low as zero to be obtained without the use of excessive resistance. Such a connection is easily made with the well-known "3-point" type of field rheostat, shown in Fig. 233.

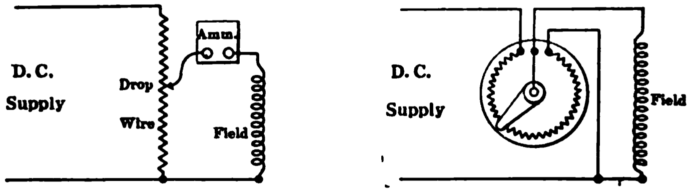


FIG. 233.—Drop-wire connections for obtaining field current.

In determining the saturation curve experimentally, the field current should be varied continuously in one direction, either up or down, as shown in Fig. 231 (*a*). Otherwise minor hysteresis loops, such as shown in Fig. 231 (*b*), will be introduced.

The field current in this experiment should be obtained from a supply other than the generator itself, for two reasons: If the generator excited its own field, the voltage and field current would be inter-dependent and it would be difficult to adjust the field current without the voltage in turn changing this adjustment. Also a voltage drop would exist in the armature due to the field current. The voltmeter would not then be reading the true induced voltage, although the error from this cause would be slight.

186. Field Resistance Line.—By Ohm's Law the current in a circuit is proportional to the voltage, for a constant resistance. If the current be plotted against volts, Fig. 234, a straight line passing through the origin results. For example, if the resistance of a field circuit be 50 ohms, the current will be 2 amp. when the voltage is 100 volts; 1.5 amp. when the voltage is 75 volts, and 1 amp. when the voltage is 50 volts. This relation is shown in Curve II, Fig. 234. Curve I shows the resistance line for 80 ohms field resistance. It will be noted that at 80 volts the current is 1.0 amp., at 40 volts it is 0.5 amp., etc. Curve III shows the same relation for a field resistance of 40 ohms.

It will be noted that the higher the resistance the greater the slope of the resistance line. In fact the slope of the line is equal

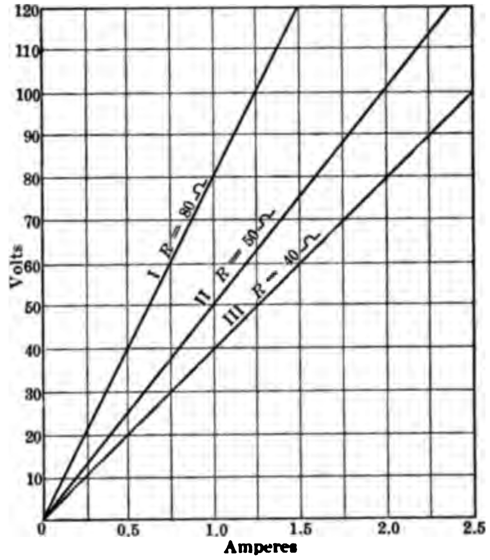


Fig. 234.—Field resistance lines.

to the field resistance in ohms since the tangent, of the angle which the line makes with the axis of abscissæ is $\frac{E}{I}$.

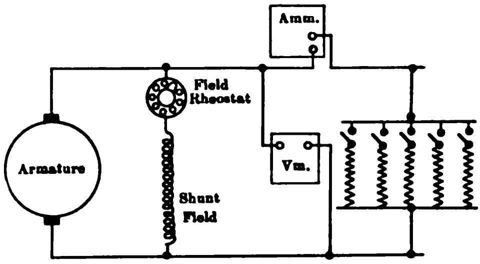


Fig. 235.—Shunt generator connections.

187. Types of Generators.—There are three general types of generator in common use, the shunt, the compound and the series. In the shunt type the field circuit is connected across the armature terminals, usually in series with a rheostat, Fig. 235. The

shunt field, therefore, must have a comparatively high resistance in order that it may not take too great a proportion of the generator current. The compound generator is similar to the shunt, but has an additional field winding connected in series wth the armature or load, Fig. 270. The series generator is excited entirely by a winding of comparatively few turns connected in series with the armature and load.

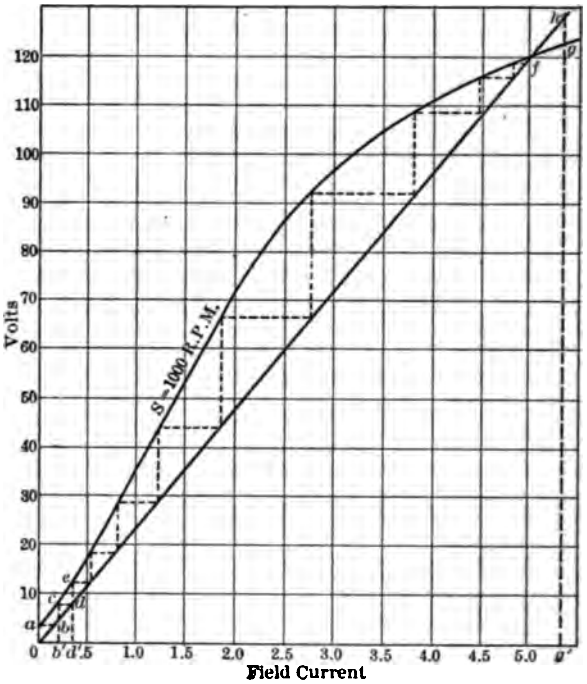


Fig. 236.—Method of shunt generator building up.

188. The Shunt Generator.—Fig. 236 shows the saturation curve of a shunt generator and its shunt field resistance line drawn on the same plot. This field, it will be observed, has a resistance of 24 ohms, so that at 120 volts it takes 5 amp.; at 60 volts 2.5 amp.; etc.

At the instant of starting a generator the induced voltage is zero. The generator may come up to voltage in the following manner: As the generator is brought up to speed there will be a

small voltage *o-a*, in this instance about 4 volts, induced in the armature due to the residual magnetism of the machine. This 4 volts also exists across the field, because the field is connected across the armature terminals. The value of field current which flows in virtue of this 4 volts can be obtained by drawing a horizontal line from *a* until it meets the field resistance line at *b*. The current in this particular case is about 0.2 amp. By consulting the saturation curve it will be seen that for this field current the induced voltage, *b'c*, is about 8 volts. The 8 volts produces about 0.33 amp. in the field, as may be seen by projecting across to the field resistance line at *d*. This field current *od'* produces a voltage *d'e*, which in turn produces a higher value of field current. Thus it will be seen that each value of field current produces a voltage in excess of its previous value and this increased voltage in turn increases the field current, that is, the action is cumulative. The machine will continue to build up until point *f* is reached, where the field resistance line crosses the saturation curve. The machine cannot build up beyond this point for the following reasons:

Consider a point *h* on the field resistance line, above *f*. This point represents a field current *og'* of about 5.3 amp. To produce this field current requires a voltage *g'h* of about 128 volts. But this field current of 5.3 amp. produces an induced voltage *g'g* of only 122 volts. If 128 volts are required to produce the field current of 5.3 amp and the machine can only produce 122 volts at this field current, it is obvious that the machine cannot build up to the point *h*.

It is evident that a machine would build up indefinitely if its iron did not become saturated.

189. Critical Field Resistance.—If the resistance of the field be increased to 60 ohms, the field resistance line will be represented by *oa*, Fig. 237. This line crosses the saturation curve at point *a'*, corresponding to about 6 volts. Therefore, with this value of field resistance, the generator will not build up beyond *a'*. If the field resistance be slowly decreased until the field resistance line reaches *ob*, the generator will be observed to start building up rapidly. It will of course stop building up at the point *b'*. The value of the field resistance corresponding to *ob* is called the *critical field resistance*. In this particular case the

resistance is 120/3.25 or 36.1 ohms. If the field resistance exceeds the critical value, the generator cannot build up.

190. Generator Fails to Build Up.—There are three common reasons for a generator failing to build up. (1) The shunt field may be connected in such a way that the current sent through it on starting is in such a direction as to "buck" or reduce the residual magnetism instead of increasing it. Under these conditions, the generator cannot of course build up. To test for this,

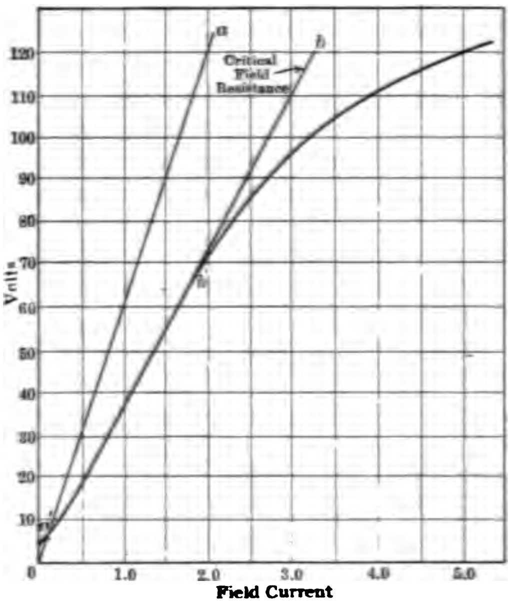


FIG. 237.—Critical field resistance.

open the field circuit. If the voltage rises when the field is opened, the current is bucking the residual magnetism and the field should be reversed. If opening and closing the field produces no effect upon the voltmeter it may be assumed that the field circuit is open.

(2) The field resistance may be greater than the critical field resistance. In this case, the procedure is to reduce the field resistance until the machine builds up.

(3) There may be no residual magnetism in the machine, due to jarring or to too long a period of idleness. If the armature cir-

cuit is not open and the voltmeter is known to be all right, the absence of residual magnetism will be indicated by the voltmeter's not reading. To remedy the difficulty it may be necessary to connect the field terminals temporarily across a separate supply circuit in order to build up the residual magnetism. This is called "flashing" the generator. If the generator has a series field, a convenient method is to connect a low voltage source, such as a storage battery or even a dry cell, across the series field. This may produce enough magnetism to cause the machine to begin to build up. One or two trials may be necessary in order to secure the proper polarity.

191. Armature Reaction.—Fig. 238 (*a*) shows the flux passing through an armature when there is no current in the armature conductors. This flux is produced entirely by the ampere-turns of the field. The neutral plane, which is a plane perpendicular to the flux, coincides with the geometrical neutral of the system. At the right is shown a vector F which represents the mmf. producing this flux, in magnitude and direction. At right angles to this vector F is the neutral plane.

In Fig. 238 (*b*) there is no current in the field coils, but the armature conductors are shown as carrying current. This current is in the same direction in the armature conductors as it would be were the generator under load. The current obviously flows in the same direction in all the conductors that lie under one pole. The current is shown as flowing into the paper on the left-hand side of the armature. (This current direction may be checked by Fleming's right-hand rule, Par. 163.) These conductors combine their mmf.'s to send a flux *downward* through the armature, as shown in the diagram, this direction being determined by the corkscrew rule. The conductors on the right-hand side of the armature are shown as carrying current coming out of the paper. They also combine their mmf.'s to send a flux *downward* through the armature. That is, the conductors on both sides of the armature combine their magnetomotive forces in such a manner as to send flux down through the armature. The direction of this flux is perpendicular to the polar axis. To the right of the figure the armature mmf. is represented in direction and magnitude by the vector F_A.

Fig. 238 (*c*) shows the result obtained when the field current

and the armature current are acting simultaneously, which occurs when the generator is under load. The armature magnetomotive

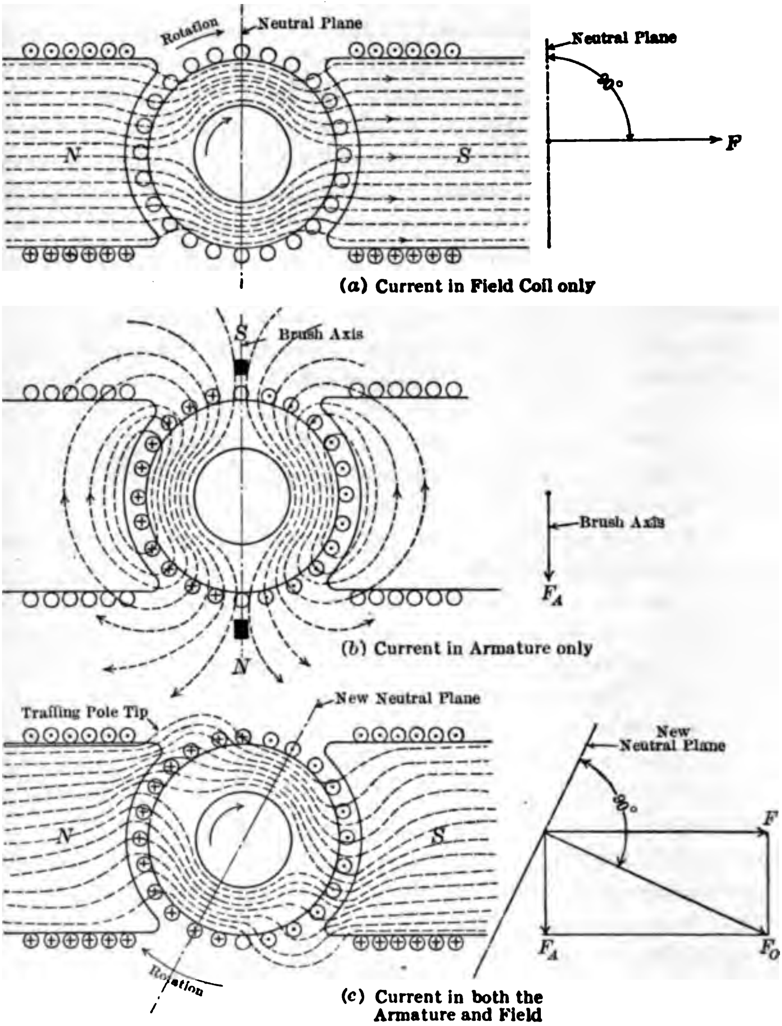


FIG. 238.—Effect of armature reaction upon the field of a generator.

force crowds the symmetrical field flux shown in (*a*) into the upper pole tip in the north pole and into the lower pole tip in the south pole. As the generator armature is shown rotating in a clockwise

direction, it will be noted that the flux is crowded into the *trailing* pole tip in each case. On the other hand, the flux is weakened in the two *leading* pole tips.

The effect of the armature current is to displace the field in the direction of rotation of the generator. It should be borne firmly in mind that the flux is not pulled around by the mechanical rotation of the armature.

To the right of Fig. 238 (*c*) the armature reaction is shown by vectors. The field vector F and the armature vector F_A combine at right angles to form the resultant field vector F_o. The direction of F_o is downward and to the right, which corresponds to the direction of the resultant flux in the drawing. The neutral plane is at right angles to F_o.

As the neutral plane is perpendicular to the resultant field, it will be observed that it too has been advanced. It was shown

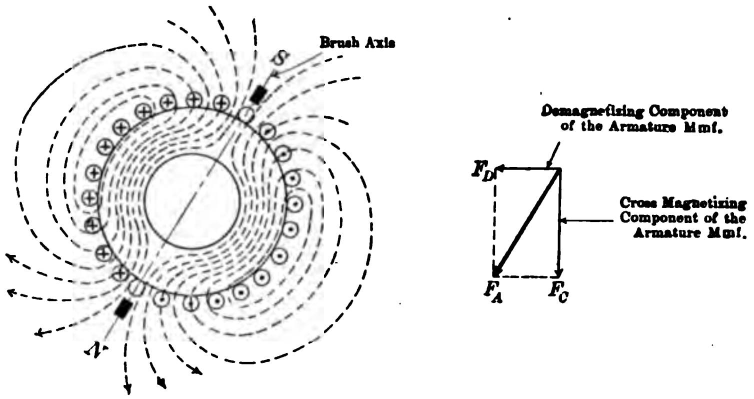


FIG. 239.—Relation of armature field to brush axis.

in Chap. X that the brushes should be set so that they short-circuited the coil just as it was passing through the neutral plane. When the generator delivers current the brushes should be set a little ahead of this neutral plane, as will be shown later. If the brushes are advanced to correspond to the advance of the neutral plane, all the conductors to the left of the two brushes must still carry current into the paper, and those to the right must carry current out of the paper. The result is shown in Fig. 239. *The direction of the armature field moves with the brushes.* Its axis

always lies along the brush axis. Therefore F_A, instead of pointing vertically downward, now points downward and to the left, as is shown by the vectors. F_A may be resolved into two components, F_D parallel to the polar axis and F_C perpendicular to this axis.

It will be noted that F_D acts in direct opposition to F, the main field, Fig. 238. Therefore, it tends to reduce the total flux

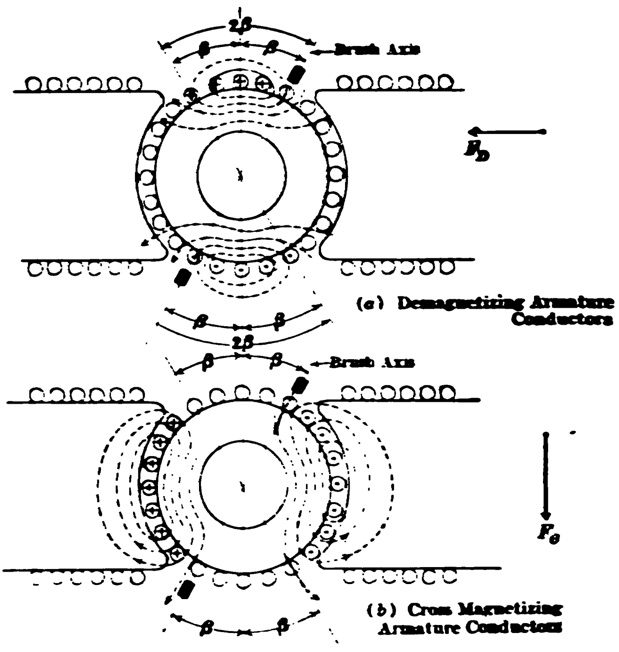


FIG. 240.—Demagnetizing and cross-magnetizing components of armature reaction.

and so is called the *demagnetizing* component of armature reaction. F_C acts at right angles to F and produces distortion. Therefore, it is called the *cross-magnetizing* component of armature reaction.

The exact conductors which produce these two effects are shown in Fig. 240. In (*a*) the brushes are shown as advanced by an angle β to correspond to the advance in the neutral plane. All the conductors within the angle 2β, both at the top and at

the bottom of the armature, carry current in such a direction as to send a flux through the armature from right to left. This may be checked by the corkscrew rule. These conductors thus act in direct opposition to the main field and are therefore called the *demagnetizing* armature conductors. Their magnetomotive force is represented by the component F_D, Fig. 239.

Fig. 240 (*b*) shows the flux produced by the conductors not included within twice the angle of brush advance. The direction of this flux is downward and perpendicular to the polar axis. These conductors cross-magnetize the field. The mmf. producing this flux is represented by the component F_C, Fig. 239. The resultant of F_D and F_C is F_A.

It should be remembered that the sum of both the demagnetizing and cross-magnetizing *ampere-turns* is equal to one-half the number of *ampere-conductors*.

Example.—A 4-pole dynamo has 288 surface conductors. The machine is lap wound and delivers 120 amp. to the external circuit. The brushes are advanced 15 mechanical degrees. How many demagnetizing and how many cross-magnetizing armature ampere-turns are there?

Twice the angle of brush lead is 30°. There are four brushes, so that the total number of degrees covered by the demagnetizing conductors is 120°. Therefore ⅓ the conductors on the armature, or 96 conductors, are demagnetizing conductors.

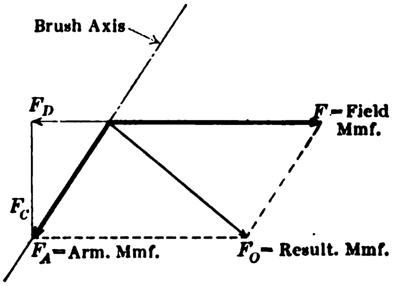


Fig. 241.—Resultant effect of armature and field mmf's.

As the machine is lap wound there are four paths through the armature. The current per path = 120/4 = 30 amp.

Demagnetizing ampere-conductors = 30 × 96 = 2,880.

Demagnetizing *ampere-turns* = 2,880/2 = 1,440. *Ans.*

The number of cross-magnetizing conductors must be ⅔ of the conductors on the armature. Therefore, the number of cross-magnetizing ampere-turns is

$$\frac{192 \times 30}{2} = 2{,}880. \quad \textit{Ans.}$$

Fig. 241 shows the method of finding the resultant magnetomotive force acting on the armature. F is the field magnetomotive force and F_A is the armature magnetomotive force, acting along the brush axis after the brushes have been advanced. F_o is

the resultant of the two, being less than F due to the demagnetizing component of F_A. F_A can be resolved into two components at right angles to each other, F_D the demagnetizing component of the armature mmf. and F_C the cross-magnetizing component of the armature mmf.

192. Armature Reaction in Multipolar Machines.—Reactions occur in multipolar machines in the same manner as in the bipolar machines that have just been described. The picture to the eye may be a little different, however. In Fig. 242 the armature and

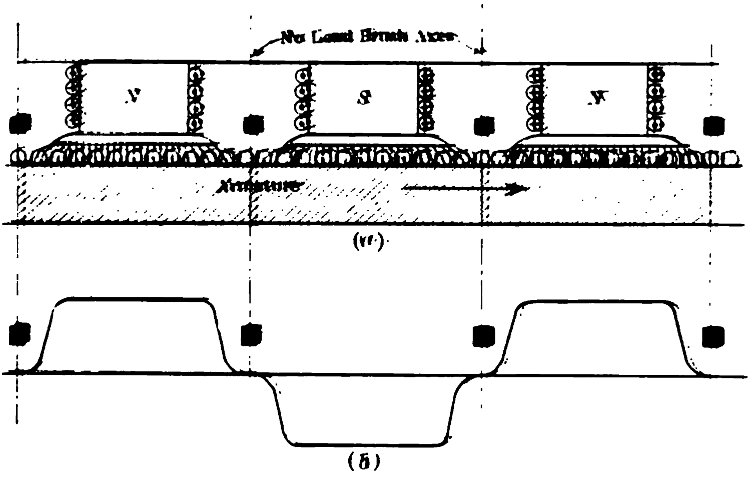


Fig. 242. Field flux of a multipolar generator.

the field poles of a multipolar machine are shown, the armature being shown as a flat surface, for convenience.

In (a) are shown the alternate north and south poles, together with the magnetic flux entering the armature. There is no current flowing in the armature conductors. In (b) the flux distribution is shown. It will be observed that it is symmetrical about the polar axis. It is substantially constant under the pole shoe and drops off gradually at the edges, due to fringing. It falls to zero and reverses in the inter-polar spaces. The neutral plane is the region where the flux is zero and under no-load conditions is midway between the poles.

Fig. 243 (a) shows the armature conductors carrying current, the field current being zero. These armature conductors

produce a flux in a manner similar to that shown in Fig. 238 (*b*). The magnetomotive force of the armature is not uniform, but varies uniformly from zero at the pole axis to a maximum in the center of the inter-polar spaces. The armature conductors be-

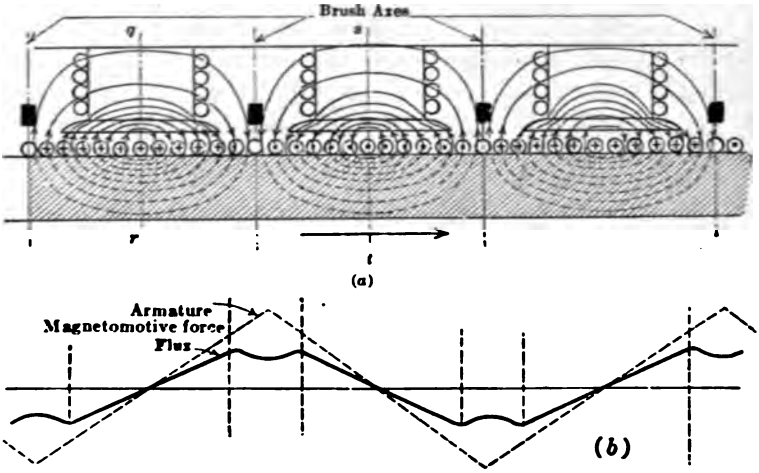


FIG. 243.—Flux due to armature reaction in a multipolar generator.

tween the lines *qr* and *st* may be considered as constituting a pancake coil, the current flowing into the paper in the conductors on the left and out of the paper in the conductors on the right. Obviously at the center of the space the magnetomotive force

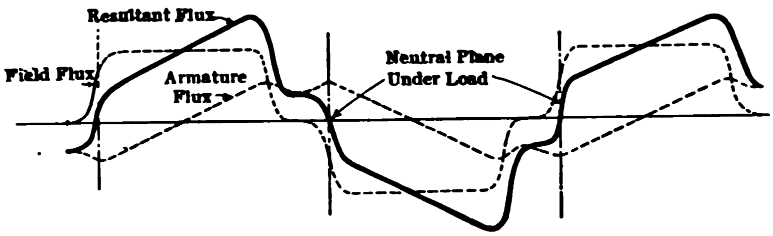


FIG. 244.—Resultant flux found by combining field flux (Fig. 242) and armature flux (Fig. 243).

will be a maximum, as the magnetomotive forces of all the conductors on both sides are acting together at this point. This is shown by the dotted line, Fig. 243 (*b*). Owing to the high air reluctance, the *flux* curve has not the same shape as the mmf.

curve but droops in the inter-polar space as shown in Fig. 243 (*b*).

The resultant flux is found by adding the two flux curves of Figs. 242 and 243, as is done in Fig. 244. (This assumes constant permeability in the iron.) It will be noted that the flux peaks on the trailing pole tip, Fig. 244, as in the case of the bi-polar generator. Also the neutral plane has advanced in the direction of the rotation. In order to keep the brushes in the neutral plane they should be advanced as this neutral plane advances. Fig. 245

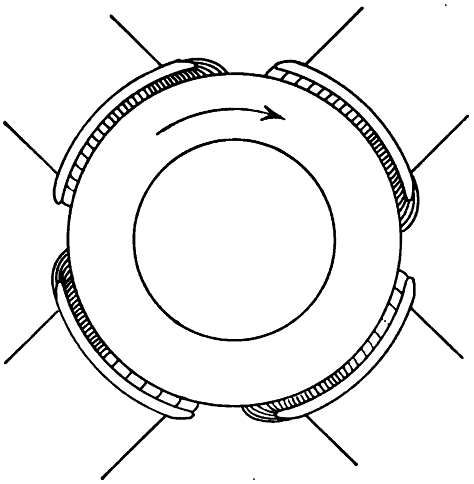


FIG. 245.—Field distortion in a 4-pole generator.

shows the crowding of flux into the trailing pole tips in a 4-pole generator.

193. Compensating Armature Reaction.—As the cross-magnetizing effect of the armature usually necessitates the shifting of the brushes with load, it is desirable to minimize armature reaction if this can be done conveniently. One practical method, when laminated pole cores are used, is to use a stamping having but one pole tip, as shown in Fig. 221. These are alternately reversed when the core is built up. This leaves spaces between the pole tip laminations, resulting in the pole tips having but one-half the cross-section of iron along their lengths. Therefore, the pole tip becomes highly saturated and its permeability

greatly reduced. This tends to prevent the flux from crowding into the trailing tip.

Another method is to introduce longitudinal slots in the pole faces, as shown in Fig. 246. These slots introduce high reluctance in the path of the armature flux but have little effect on the field flux.

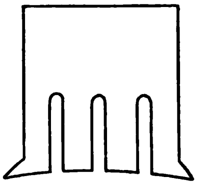
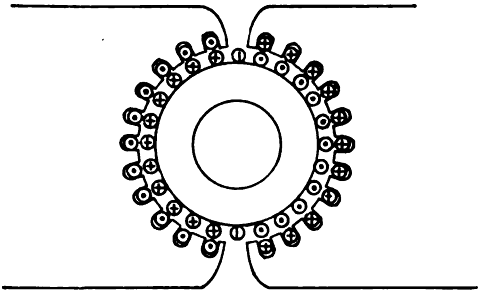


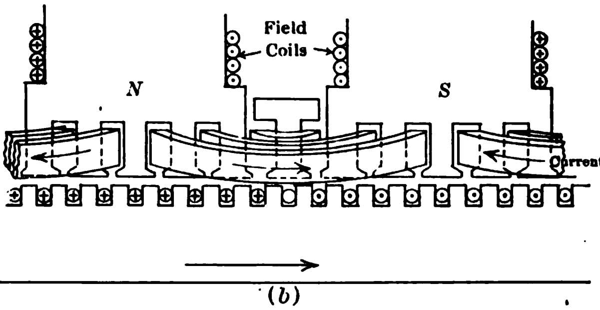
FIG. 246.—Longitudinal slots in pole-face for reducing armature reaction.

The Thompson-Ryan method is to compensate armature reaction by magnetomotive forces equal and opposite to those of the armature. In order to be effective, these compensating magnetomotive forces should be equal and opposite to those of the armature at every point. This principle is illustrated by Fig. 247 (*a*), which shows



(*a*)

FIG. 247*a*.—Compensation of armature reaction with pole face conductors.



(*b*)

FIG. 247*b*.—Turns in pole-faces to compensate armature reaction.

conductors embedded in the pole faces close to the armature. Each conductor carries a current opposite to that of its corre-

sponding armature conductor. This winding is connected in series with the armature so that the magnetomotive forces are opposite and equal at all loads. These windings allow the use of a very short air gap, with the accompanying reduction in field copper and in field loss.

The Thompson-Ryan principle has been applied to many modern machines where commutation difficulties are unusually great, as in a.c. series motors and in large rolling mill motors. The conductors are installed in the pole faces in the manner indicated in Fig. 247 (*b*). This type of construction is used in the Ridgeway dynamo.

The conductors are connected in series with the armature and are so adjusted that their ampere-turns are in almost exact opposition to the armature ampere-turns at each point. They do not as a rule have the same number of conductors as the armature because the armature current at each point is that of one armature path, whereas the current through this compensating winding is the sum of the currents in the various parallel paths through the armature. The small poles between the main poles, Fig. 247 (*b*), are saturated for any flux tending to leak between the main poles.

Armature reaction is also reduced by increasing the length of the air gap, thus offering higher reluctance to the armature flux. A longer air gap means more field copper and a greater field current, however.

194. Commutation.—It has been shown that the electromotive force induced in any single coil of a direct current generator is alternating, and in order that the current may flow always in the same direction to the external circuit, a commutator is necessary. Fig. 248 shows the changes of current in an armature coil as it approaches and recedes from the brushes. It is assumed that ideal commutation is being realized. The load is such that 20 amp. flow in each path of the armature, making 40 amp. leaving the machine by this one brush. The current distribution throughout the brush is also assumed to be uniform.

When in positions (1), (2) and (3) each coil (and, therefore, successive positions of any one particular coil) carries 20 amp. As the brush covers four segments and the current distribution is uniform, 10 amp. must flow into the brush from each segment. Therefore, when passing from position (3) to (4), the

coil must lose the 10 amp. which pass from this segment into the brush. Hence, in position (4) the coil carries only 10 amp.

Before reaching position (5) the coil gives up another 10 amp. so that the current is zero when the coil reaches position (5). When the coil reaches position (6) the current flows through the coil in the reverse direction, due to current entering the brush from another armature path. The current reaches 20 amp. in position (7) and remains 20 amp. in the further positions (8), (9) and (10).

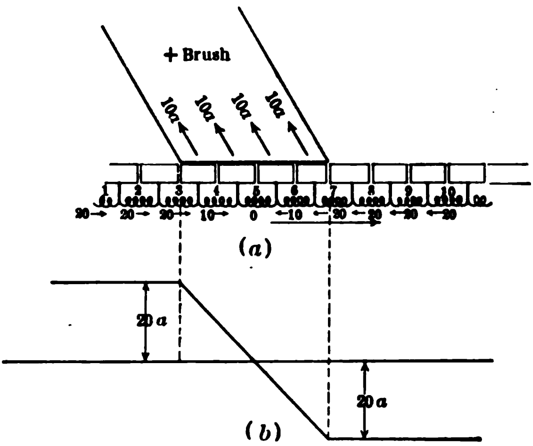


FIG. 248.—Current in coil undergoing commutation—ideal conditions.

Therefore, commutation consists of two parts:

1. Reversing the current in any coil from its full positive value to an equal negative value. This reversal must take place in the short time interval required for a segment to pass under the brush.

2. The current supplied by the two paths meeting at the brush must be conducted to the external circuit.

Part (1) is illustrated by Fig. 248 (*b*). The current in the coil is + 20 amp. until the brush is reached, when it reverses at a uniform rate to a value of −20 amp. This is perfect commutation.

The foregoing ideal commutation is only approximated in practice. There are two causes preventing its realization.

It will be noted that when the coil is in positions (4), (5) and (6) it is short-circuited by the brush. If any voltage is being induced in the coil when it is in these positions, a large current will necessarily flow, since the resistance of the circuit is very

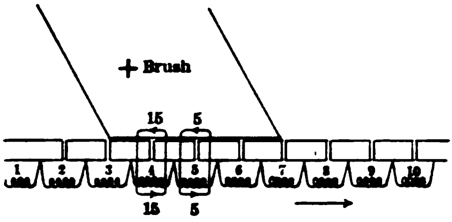


Fig. 249.—Short-circuit currents through brush.

low. This resistance consists merely of the resistance of the coil plus the contact resistance of the brush. This contact resistance constitutes the major portion of the total resistance. Fig. 249 shows assumed currents of 15 and 5 amp. flowing

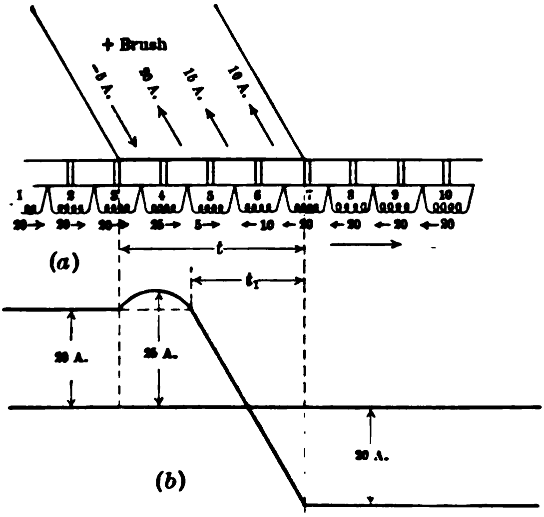


Fig. 250.—Change of current in coil when brushes are too far back of the neutral plane.

in coils (4) and (5) respectively, due to voltages induced in them while they are being short-circuited by the brush.

If now the currents of Fig. 249 be superposed upon those of Fig. 248 (a) the current distributes itself over the brush in the

manner shown in Fig. 250 (*a*). There are now 45 amp. entering the brush and 5 amp. leaving it. Therefore, the brush has to handle 50 amp. instead of 40, and in one place there are 20 amp. per segment, or twice that which occurred under the ideal conditions of Fig. 248. This will tend to produce heating and undue sparking under the heel of the brush.

Fig 250 (*b*) shows the manner in which the current in the coil varies under these new conditions. Instead of dropping uniformly from 20 amp. it first *rises* to 25 amp. before starting

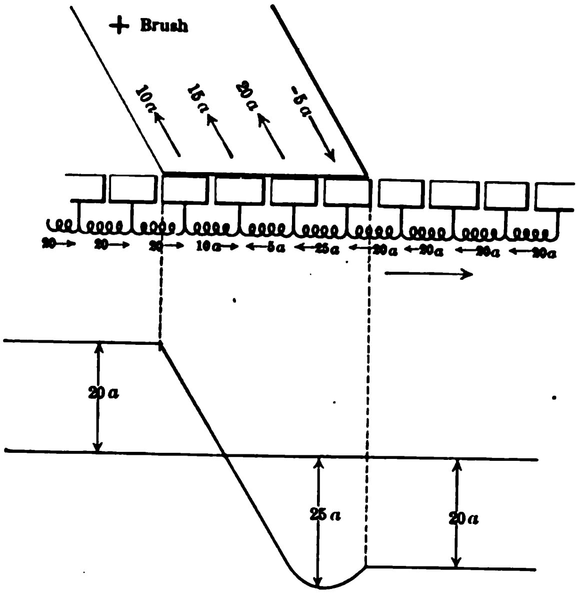


Fig. 251.—Commutation with the brushes too far ahead.

to reverse. It will be noted that the time for reversing from +20 amp. to −20 amp. has been reduced from time t to time t_1, which makes commutation more difficult. The curve of Fig. 250 occurs when the brush is too far back of the neutral plane. Voltages are then induced in the coils as they are undergoing commutation.

The curves of Fig. 248 (*b*) and 250 (*b*) are called commutation curves.

If the brushes are placed too far ahead of the neutral plane, short-circuit currents flow under the toe of the brush, resulting

in the current distribution and commutation curve of Fig. 251. This condition produces undue sparking under the toe of the brush.

If the brushes are too wide, both the heel and the toe will short-circuit coils in which voltages are induced, resulting in the commutation curve of Fig. 252. Moving the brushes either backward or forward does not assist matters in this case. The only remedy is a narrower brush.

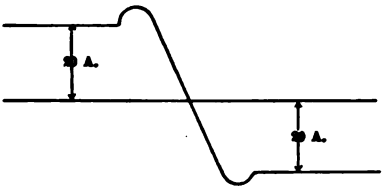
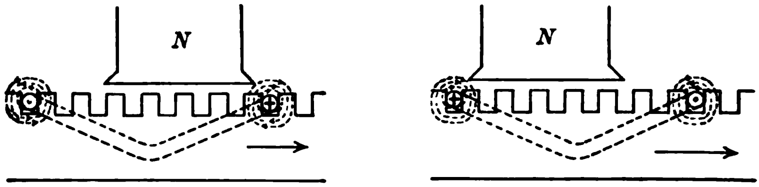


FIG. 252.—Commutation with too wide a brush.

195. The Electromotive Force of Self Induction.—Fig. 253 (*a*) shows an armature coil just as it is entering the commutation zone. The slot conductors are embedded in iron and, due to the current flowing in the coil, considerable flux passes through the coil, in this case upward. Let the value of the flux be ϕ_1. In Fig. 253 (*b*) the same coil is shown just after it has left the



(*a*) Before commutation (*b*) After commutation

FIG. 253.—Change of flux through a coil undergoing commutation.

commutation zone. The current through the coil is the same as its previous value, but it now flows in the reverse direction. The flux is still ϕ_1 but it has been reversed in direction.

Therefore, in the time *t* seconds required for a segment to pass the brush or commutating zone, the flux has changed by $2\phi_1$ lines. This is shown in Fig. 254, where the ideal commutation curve is assumed. This change of flux will induce a voltage,

$$e = -N\frac{2\phi_1}{t}10^{-8} \text{ volts (from equation 74)}$$

N being the number of turns in the coil.

This voltage, with its proper direction, is shown in Fig. 254. It is called the voltage of self induction.

Instead of looking upon this as a voltage phenomenon, it may be considered as follows: The armature coil has self and mutual inductance. This inductance tends to prevent the current reversing in the same manner that it tries to prevent any change of current in a circuit.

Therefore, even though the brushes are set exactly in the neutral plane and the coils undergoing commutation are cutting no magnetic lines, there will be a voltage induced in the coil due to its own self inductance. To eliminate this voltage it is

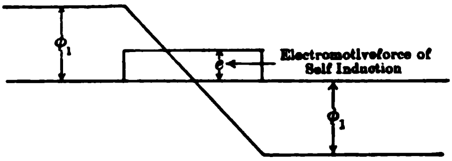


FIG. 254.—Electromotive force of self induction in a coil undergoing commutation.

necessary to set the brushes *ahead* of the neutral plane in a generator. When the coil is undergoing commutation it finds itself in a field of the same polarity as that which the conductors leaving the commutation zone are about to enter. Therefore, this field induces a voltage which assists the current to reverse.

Another way of stating it is that the electromotive force induced in the coil due to its being ahead of the neutral zone is in exact opposition to the electromotive force of self induction, shown in Fig. 254, and so neutralizes it.

It is therefore necessary that the brushes be kept *ahead* of the neutral plane in a *generator*, in order to obtain satisfactory commutation under load conditions.

196. Sparking at the Commutator.—The voltages induced in a coil due to the shifting of the neutral plade and also due to its own self inductance are comparatively low in value, being of the order of magnitude from a few tenths of a volt to perhaps 4 or 5 volts. But they are acting in a circuit having a very low resistance. The resistance of one coil is extremely low so that most of the circuit resistance is at the brush contact. If the brush contact resistance is too low, these short-circuit

currents may reach such excessive values as to produce severe sparking at the brushes. On the other hand, a low resistance brush is desirable from the standpoint of carrying the current out to the external circuit with minimum contact loss.

Copper brushes have a very low contact resistance, but the short-circuit currents are excessive when they are used. Therefore, their application is limited to very low voltage, high current machines. In this case copper gauze is often used. Another disadvantage of using copper brushes is that they "cut" the commutator mechanically.

Carbon brushes have a much higher contact resistance than copper and therefore limit the short-circuit currents, giving much more satisfactory results. In addition, they are more or less graphitic in their composition and so lubricate the commutator to a certain extent. Unusually hard carbon brushes may cut the commutator. Different grades of carbon are required for different machines.

The passage of the current from the commutator to the brush is more of an arc phenomenon than it is one of pure conduction. A careful examination will show myriads of minute arcs existing between the brush surface and the commutator. The voltage drop between the commutator and the brush, instead of being proportional to the current (as it would be with straight conduction) is substantially constant and is equal to about 1 volt per brush. Bits of copper may be found in the positive brush due to the arcing. The voltage drop across the negative brush is different from that across the positive brush, due to the copper being positive in one case and negative in the other. These facts all substantiate the arcing theory.

Another proof is the so-called "high mica." After a machine has been in operation for a considerable time, it often happens that the mica insulation between the commutator segments protrudes above the surface of the commutator, resulting in so-called "high mica," Fig. 255. It was long supposed that this was due to the mica being harder than the copper, which resulted in the wearing away of the copper more readily than the mica. The fallacy of this supposition is of course evident. Even though the mica is much harder than the copper, the two must always wear evenly for the brush cannot grind the copper until it comes in

contact with it. Hence the brush must grind down the mica before it can touch the copper if "high mica" is due to mechanical abrasion alone.

The rational explanation of high mica is dwelt upon in some detail by B. G. Lamme in a paper presented before the American Institute of Electrical Engineers.[1] The copper is not worn away as generally supposed, but is carried away by the minute arcs that exist between the brush and the commutator, as shown in Fig. 255. This may be proved by running two similar machines for the same periods of time, one of the machines delivering current and the other having no current at all in the brushes and com-

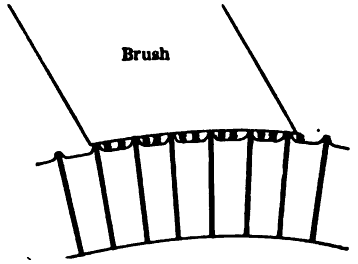


FIG. 255.—Commutator with high mica.

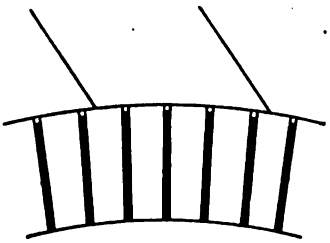


FIG. 256.—Undercut mica commutator.

mutator. High mica will ultimately appear on the commutator which carries current, if conditions warrant, whereas it will be found impossible to produce high mica on the machine which carries no current.

High mica may be reduced by the use of fairly hard brushes which grind the mica down. In modern practice the mica is under-cut by many manufacturers, that is, the top of the mica is below the commutator surface, as is shown in Fig. 256. There is some disadvantage in this construction, in that small bits of copper, carbon and dirt collect in the groove and may ultimately short-circuit the segments. These grooves can be easily cleaned out, however.

The result of any arcing under the brush is to pit the commutator. As irregularities and depressions in the commutator tend to prevent the brush making intimate contact with the

[1] "Physical Limitations in D.C. Commutating Machinery," by B. G. LAMME, A. I. E. E. *Trans.*, Vol. XXXIV, Part II (1915), page 1739.

commutator, arcs of increasing magnitude will be formed. The deeper the depressions, or the higher the mica, the larger and more vigorous these arcs become. Hence, any condition which produces sparking and so roughens the commutator only increases the sparking and roughening, or, these actions are cumulative. If a commutator is sparking badly and the cause of the

Fig. 257.—Proper method of fitting brushes.

sparking is not corrected, the commutator will deteriorate very rapidly and soon become inoperative.

The brushes should be fitted very carefully to the commutator surface by grinding with sandpaper in the manner shown in Fig. 257. Carbon on the surface of the commutator should be removed with an oily cloth. Do not use waste. A slightly roughened commutator may be partially smoothed with fine sandpaper. Do not use emery, as the particles of emery are conducting and may short-circuit the commutator bars. If the

commutator is grooved by the brushes, or is otherwise in poor condition, it should be turned down in a lathe.

Other difficulties, such as loose mica and loose segments, are more serious in character. It is often possible to rectify these difficulties by tightening up the commutator clamp bolts.

197. Commutating Poles (Interpoles).—Fig. 258 shows the geometrical neutral or no-load neutral plane and the neutral plane under load. It will be noted that this is merely Fig. 244 reproduced. If the brushes remained in the no-load neutral plane, there would be severe sparking under load conditions, because of the very appreciable flux, ϕ_2, now existing in the neutral zone. The brushes will not commutate properly even if placed

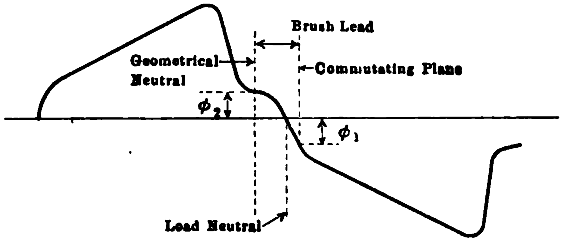


FIG. 258.—Brush advance to proper commutating plane.

in the load neutral plane. This is due to the fact that the electromotive force of self induction still exists in the coils undergoing short circuit, even if the voltage due to the pole flux is zero. The brushes must be moved ahead into the flux of the next pole as is shown by ϕ_1, in order to have a voltage generated in them which will balance the electromotive force of self induction. It will be noted that this position is in the fringe of the next pole flux. A very slight movement of the brushes in either direction makes a very marked change in the flux so it is difficult to get good commutation under these conditions. In fact, it may be impossible to obtain satisfactory commutation because of the steepness of the flux curve. When the best position of the brushes is obtained the trailing tip of each brush may be in too strong a field and the leading tip in too weak a field.

If a flux having the same value as ϕ_2, but opposite to it in direction, could be produced in the geometrical neutral, it is obvious that the flux in the neutral plane could be brought to zero in spite of armature reaction. If a flux having a value $\phi_2 + \phi_1$ were produced, satisfactory commutation would be obtained

without moving the brushes. It is the function of commutating poles to produce just this flux.

Commutating poles consist of narrow poles located between the main poles. They send a flux into the armature which is of the proper magnitude to produce satisfactory commutation. For instance, in Fig. 258, the commutating pole must first produce a flux equal to ϕ_2 so as to neutralize, in the neutral zone, the in-

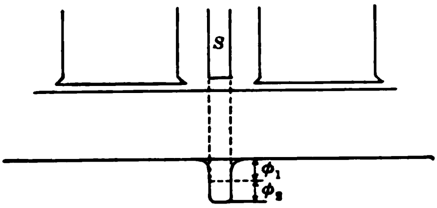


FIG. 259.—Flux produced by commutating pole.

crease of flux due to armature reaction. It must also produce an additional flux ϕ_1 to balance the electromotive force of self induction in the coil undergoing commutation. This commutating pole flux is shown in Fig. 259. The pole producing it at this point must be a south pole. Fig. 260 shows the resultant flux obtained by combining Figs. 258 and 259.

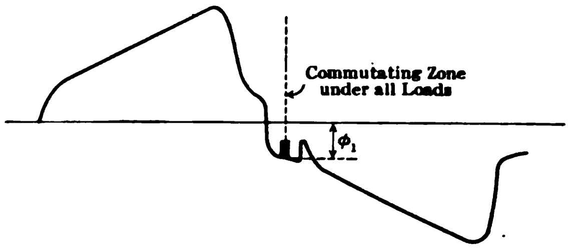


FIG. 260.—Resultant of main flux and commutating-pole flux—machine loaded.

As the armature reaction and the electromotive force of self induction in the coils undergoing commutation are both proportional to the armature current, the compensating flux produced by the commutating poles must also be proportional to the armature current. The commutating poles are wound, therefore, with a few turns of comparatively heavy wire and are connected in series with the armature, as shown in Fig. 261. The air gap

between these poles and the armature is large, so that the commutating pole flux is nearly proportional to the armature current at all loads.

It should be noted that the sequence of poles in the direction of rotation in a generator is *Ns* and *Sn*, where the capitals refer to the main poles and the small letters refer to the commutating poles. Fig. 262 shows an interpole separate from the machine. Fig. 263 shows the frame and field coils for a commutating pole motor. It will be noted that only two commutating poles are

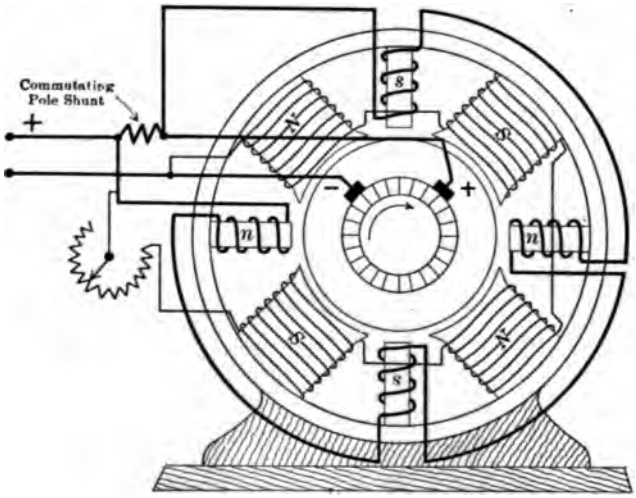


Fig. 261.—Connections of shunt field and commutating poles.

necessary in this 4-pole machine. Each pole has twice the strength that it would have if four such poles were used. Therefore, the proper commutating voltage is induced in but *one* side of the coil undergoing commutation.

In practice, commutating poles are so designed that they produce a flux of greater magnitude than is necessary. The entire commutating pole circuit is then shunted by a low resistance shunt, this shunt being adjusted until the best condition of commutation is obtained. The commutating pole shunt is shown in Fig. 261. This shunt is sometimes made inductive so that the proper proportion of current will flow to the commutating poles on sudden changes of load, such as occur in railway generators.

198. The Shunt Generator: Characteristics.—If a shunt generator, after building up to voltage, be loaded, the terminal voltage will drop. This drop in voltage will increase with in-

Fig. 262.—Commutating pole and winding.

crease of load. Such a drop in terminal voltage is undesirable, especially when it occurs in generators which supply power to incandescent lamps.

Fig. 263.—Frame and field coil for Westinghouse 30 H.P., direct current, interpole motor.

It is very important to know the voltage at the terminals of a generator for each value of current that it delivers, because the ability to maintain its voltage under load conditions determines

in a large measure the suitability of a generator for certain specified service.

To test a generator, in order to determine the relation of terminal volts to current, it is connected as shown in Fig. 235. The machine is self excited and a voltmeter is connected across its terminals to indicate the terminal volts. An ammeter is connected in the line to measure the load current. In performing this test it is often desirable to connect an ammeter in the field circuit so as to be able to follow the change in the field current as the load is applied.

In starting the test, rated load should first be applied and the field current adjusted until rated voltage is obtained. The

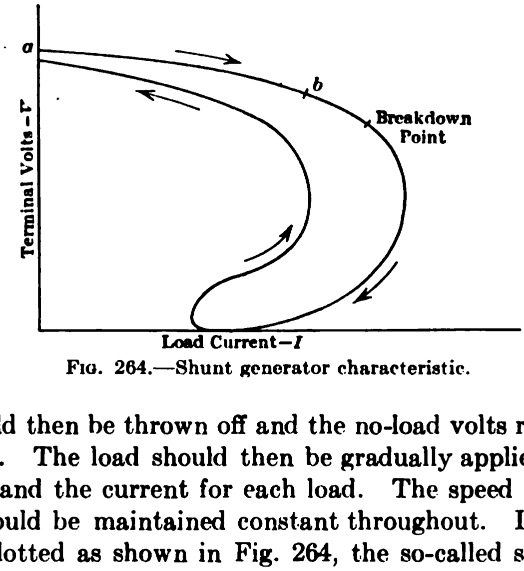


FIG. 264.—Shunt generator characteristic.

load should then be thrown off and the no-load volts read on the voltmeter. The load should then be gradually applied, reading the volts and the current for each load. The speed of the generator should be maintained constant throughout. If the readings be plotted as shown in Fig. 264, the so-called shunt characteristic results. If, in a small generator, the load be carried far enough, a rapid decrease of voltage will occur, as shown in Fig. 264. This is called the break-down point of the generator. Further application of load results in a very rapid decrease of voltage and any attempt at further increase of load results in a *decrease* of current rather than an increase. The load may even be carried to short-circuit conditions and yet the current will actually decrease as short circuit is approached. This is due to the fact that the field is short-circuited and any current flowing

at short circuit is due to the residual magnetism of the machine only.

If the external resistance be now increased, the voltage will rise slowly and will ultimately reach a value not far below that at which it started. The fact that the voltage follows a different curve when the short circuit is removed is primarily due to hysteresis. When the load is being applied, the voltage is dropping and the iron is on the part of the cycle represented by *c*, Fig. 231 (*a*). When the voltage starts to increase, it returns along the path *a*, Fig. 231 (*a*). There is less magnetism and consequently

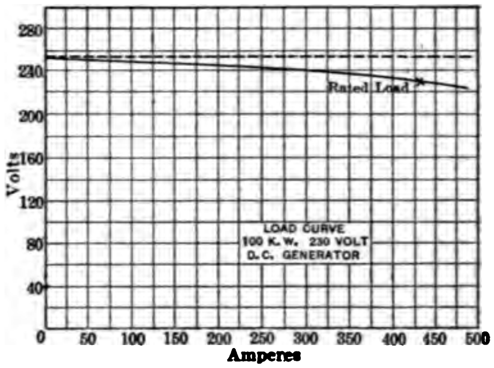


FIG. 265.—Typical shunt characteristic.

less voltage is induced in the machine upon the return curve. This, together with a lesser field current resulting from the lower voltage, accounts for the return curve lying below the other.

In practice, machines are operated only on the portion *ab* (Fig. 264) of the characteristic. Fig. 265 shows this portion of the curve for a 100-kw., 230-volt generator. The rated current is $100{,}000/230 = 435$ amp. The generator field rheostat is set so that the generator terminal voltage is 230 when it is delivering this load of 435 amp.

There are three reasons for the drop in voltage under load of a shunt generator:

(1) The terminal voltage is less than the induced voltage by the resistance drop in the armature. That is, the terminal voltage

$$V = E - I_aR_a \qquad (103)$$

where E is the induced volts, I_a the armature current and R_a the armature resistance.

Example.—The voltage induced within the armature of a shunt generator is 600 volts. The armature resistance is 0.1 ohm. What is the terminal voltage when the machine delivers 200 amp.?

Applying equation 103,

$$V = 600 - (200 \times 0.1) = 600 - 20 = 580 \text{ volts.}$$

(2) Armature reaction weakens the field and so reduces the induced voltage.

(3) The drop in terminal voltage due to (1) and (2) results in a decreased field current. This in turn results in a lesser induced voltage.

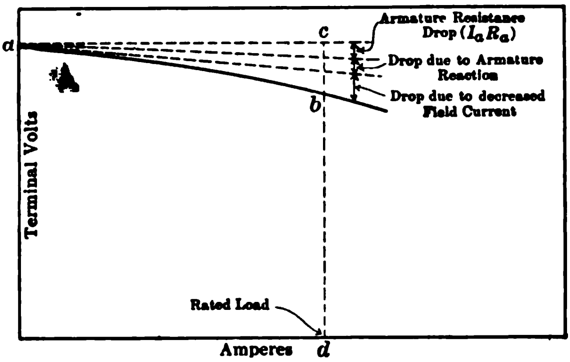


Fig. 266.—Voltage drops in shunt generator.

The effect of each of these three factors is shown in Fig. 266. It might appear that the voltage of the generator would drop to zero, or practically so, of its own accord when the load first is applied, because the foregoing cycle is cumulative. That is, a lesser terminal voltage results in a weaker field, and a weaker field results in a lesser induced voltage and therefore a lower terminal voltage which still further weakens the field, etc. The above cycle *would* result in the terminal volts reaching zero, if the iron were not in some measure saturated. If a 10 per cent. drop in terminal voltage resulted in a 10 per cent. drop in flux, the generator would be unable to supply any appreciable load. However, a 10 per cent. drop in terminal voltage, hence in field current, probably results only in a 1 or 2 per cent. drop in

flux, due to saturation and also hysteresis, as is illustrated in Fig. 231 (*a*). Therefore, a generator when operating at high saturation maintains its voltage better than when running at low saturation.

This is illustrated by Fig. 267, which shows two saturation curves for a 230-volt generator, one at 900 r.p.m. and the other at 1,200 r.p.m. If the no-load voltage of the generator in each case is 230 volts, the generator will be operating at point (*a*) on the 1,200 r.p.m. curve and at point (*b*) on the 900 r.p.m. curve. As point (*b*) corresponds to a much higher saturation

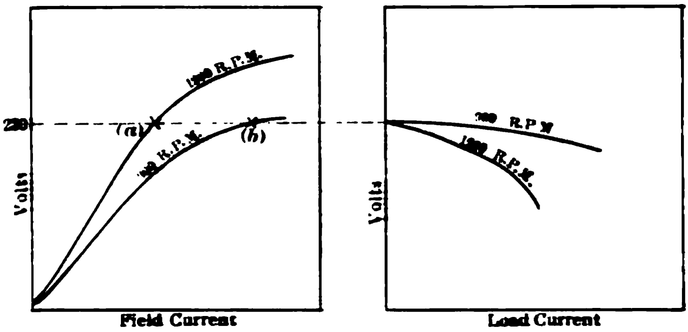


FIG. 267.—Relation of shunt characteristics to speed.

of the armature and field iron than (*a*), the generator will maintain its voltage better at 900 r.p.m. than at 1,200 r.p.m., as shown by the characteristics in Fig. 267.

199. Generator Regulation.—The ability of a generator to maintain its voltage under load is a measure of its suitability for constant potential service. The *regulation* shows quantitatively the amount the voltage varies from rated load to no load.

The definition of regulation according to the A. I. E. E. Standardization Rules is the rise in voltage between rated load and no load. This is usually expressed as a percentage. Regulation may be more specifically defined as follows:

$$\text{Regulation} = 100\,\frac{\text{no load} - \text{rated load}}{\text{rated load}}\ \text{volts (per cent.)} \quad (104)$$

As an example, in Fig. 266, the rise in voltage from b to a = bc.

$$\text{Per cent. regulation} = 100\,\frac{bc}{db}.$$

In the 100-kw. generator whose characteristic is shown in Fig. 265, the no-load voltage is 252 volts. The rated load voltage is 230.

$$\text{Per cent. regulation} = 100\frac{252 - 230}{230} = 100\frac{22}{230} = 9.6 \text{ per cent.}$$

200. Total Characteristic.—Reference is often made to the total characteristic of a shunt generator. The *shunt* characteristic, to which reference has already been made, is the relation existing between *load* current and *terminal* volts. The *total* characteristic is the relation between *armature* current and *induced* volts.

The armature current differs from the load current by the amount of current flowing in the field.

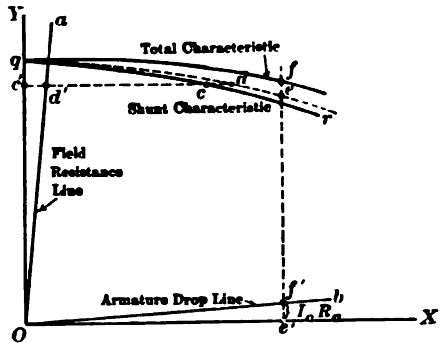


FIG. 268.—Total characteristic of shunt generator.

The armature current

$$I_a = I + I_f$$

when I is the load current and I_f the shunt field current.

The induced volts

$$E = V + I_aR_a \tag{104}$$

where V is the terminal voltage and R_a the armature resistance, including brush and brush contact resistance. The total characteristic is the curve showing the relation of I_a and E. It may be found graphically from the shunt characteristic as follows:

Let qr, Fig. 268, be the shunt characteristic. Draw the field resistance line oa, as was done in Figs. 236 and 237. The line will have the appearance of being nearly vertical, owing to the fact that the abscissæ are plotted to armature current scale.

The horizontal distances from the OY axis, Oq to Oa, give the value of field current for each value of voltage. By adding these distances horizontally to the shunt characteristic, the total current is given by the resulting characteristic qe. For example, at point c on the shunt characteristic the distance $c'd'$ is added horizontally, giving point d on the characteristic qe.

The armature resistance drop line ob is then plotted, assuming that the brush contact resistance is constant. The voltage drop in the armature is then proportional to the current. It is only necessary to determine the drop $e'f'$ at some value of current oe'. That is, the voltage drop

$$e'f' = (Oe')R_a$$

Draw the line Of'. The vertical distances from the OX axis, Oe' to Of', give the armature drop for each value of current. Adding these drops to the characteristic qe, as $ef = e'f'$ is added at the point e, the total characteristic qf is obtained.

It should be borne in mind that the total *induced* voltage multiplied by the total current gives the total power developed within the armature. All of this power is not available, however, for two reasons:

(1) Some of this power is lost in the armature itself, appearing as $I_a{}^2R_a$ loss in the armature copper.

(2) Some of the armature output is consumed in heating the shunt field.

Example.—A 20-kw., 220-volt, shunt generator has an armature resistance of 0.07 ohm and a shunt field resistance of 100 ohms. What power is developed in the armature when it delivers its rated output?

Rated current

$$I = \frac{20{,}000}{220} = 90.9 \text{ amp.}$$

Field current

$$I_f = \frac{220}{100} = 2.2 \text{ amp.}$$

Armature current

$$I_a = 90.9 + 2.2 = 93.1 \text{ amp.}$$

Induced volts

$$E = 220 + (93.1 \times 0.07) = 226.5 \text{ volts.}$$

Power developed in armature

$$P = 226.5 \times 93.1 = 21.1 \text{ kw.} \quad \textit{Ans.}$$

The same result may be obtained by adding power losses as follows:
Field loss

$$P_f = \frac{(220)^2}{100} = 484 \text{ watts.}$$

Armature loss $\quad P_a = (93.1)^2 0.07 = 607$ watts.

Power developed in armature

$$P = 20{,}000 + 484 + 607 = 21{,}091 \text{ watts} = 21.1 \text{ kw.} \quad \textit{Ans.}$$

201. The Compound Generator.—The drop in voltage with load, which is characteristic of the shunt generator, makes this type of generator undesirable where constancy of voltage is essential. This applies particularly to lighting circuits, where a very

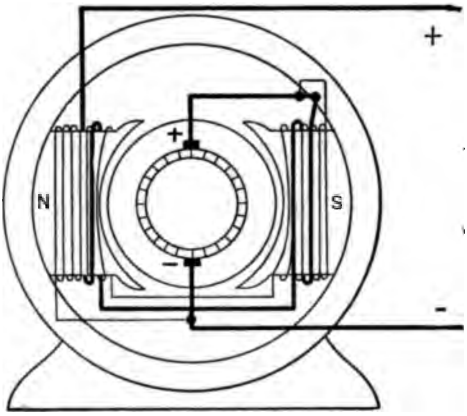


FIG. 269.—Connections of a compound generator (short shunt).

slight change of voltage makes a material change in the candle-power of incandescent lamps. A generator may be made to produce a substantially constant voltage, or even a rise in voltage as the load increases, by placing on the field core a few turns which are connected *in series* with the load. These turns are connected so as to *aid* the shunt turns when the generator delivers current, Fig. 269. As the load increases, the current through the series turns also increases and, therefore, the flux through the armature increases. The effect of this increased flux is to increase the induced voltage. By proper adjustment of the series ampere-turns, this increase in armature voltage may be made to balance the drop in voltage due to armature reaction

and that due to the resistance drop in the armature. If the terminal voltage is maintained substantially constant, the field current will not drop as the load increases. Therefore, the three causes of voltage drop, namely, armature reaction, I_aR_a drop,

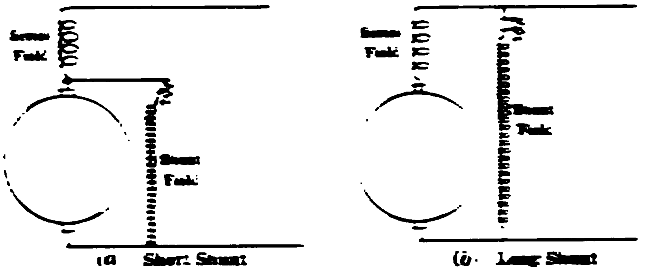


Fig. 270.—Compound generator connections.

and drop in field current (Fig. 266), are neutralized more or less completely by the effect of the series ampere-turns.

The shunt field may be connected directly across the armature terminals, Fig. 270 (a), in which case the machine is called *short* shunt. If the shunt field be connected across the machine

Fig. 271.—Compound generator characteristics.

terminals outside the series field, Fig. 270 (b), the machine is *long* shunt. The operating characteristic is about the same in either case.

If the effect of the series turns is to produce the same voltage at rated load as at no load, the machine is said to be *flat* compounded. (See Fig. 271.) It is seldom possible to maintain a

constant voltage at all points from no load to rated load. The tendency is for the voltage first to rise and then to drop again, reaching the same voltage at rated load as was obtained at no load. The particular shape of the characteristic is due to the iron becoming saturated, so that the added series ampere-turns do not increase the flux at full load as much as they do at light load. When the rated-load voltage is greater than the no-load voltage, the machine is said to be *over* compounded. When the rated-load voltage is less than the no-load voltage, the machine is said to be *under* compounded. Generators are seldom under compounded.

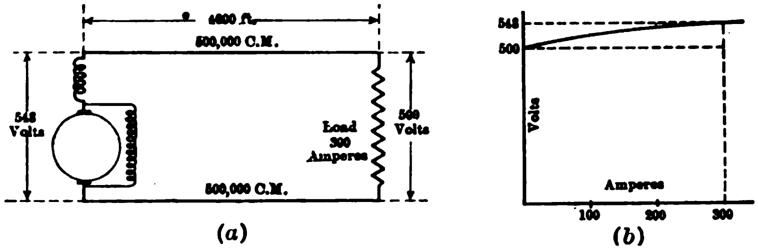


FIG. 272.—Over-compounded generator maintaining constant voltage at the end of a feeder.

Flat compounded generators are used principally in isolated plants, such as hotels and office buildings. The size of the conductors in the distribution system of such plants is determined almost entirely by underwriters' requirements as to carrying capacity. Wires conformng to these requirements are usually of such size that only a very small voltage drop takes place between the generator and the various loads.

Over compounded generators are used where the load is located at some distance from the generator. As the load increases, the voltage at the load tends to decrease, due to the voltage drop in the feeder. If, however, the generator voltage rises just enough to offset this feeder drop, the voltage at the load remains constant.

Example.—Consider the conditions shown in Fig. 272 (*a*). A certain load is 4,000 ft. distant from the generator. The load is supplied over a 500,000 cm. feeder. The no-load voltage of the generator is 500 volts. It is desired to maintain the load voltage at a substantially constant value

of 500 volts from no load to the maximum demand of 300 amp. What must be the characteristic of the generator?

If the cables were operated at the "normal" density the current would be 500 amp. or 0.001 amp. per cir. mil (Par. 68), and the drop would be 0.01 volt per foot, making a total drop of 80 volts.

The actual drop is

$$\frac{300}{500} \times 80 = 48 \text{ volts.}$$

The generator terminal voltage should rise from a no-load value of 500 volts to 548 volts when 300 amp. are being delivered to the load, Fig. 272 (*b*).

Compound generators are usually wound so as to be somewhat over compounded. The degree of compounding can then be regulated by shunting more or less current away from the series field. To do this a low resistance shunt, called a *diverter*, is used, Fig. 273.

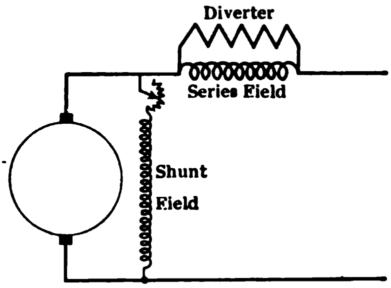


Fig. 273.—Series-field diverter.

Compound generators which supply 3-wire distribution systems usually have two series field windings, one connected to each side of the armature. There are two separate series windings on each pole, one winding being connected to the positive terminal and the other to the negative terminal of the machine. (See Fig. 349.)

In a compound generator the induced voltage in the armature is:

$$E = V + I_sR_s + I_aR_a \tag{105}$$

where V is the terminal voltage, I_s the series field current, I_a the armature current, and R_s and R_a the series field and armature resistance respectively. (R_s is the equivalent parallel resistance of the series field and diverter, if a diverter is used. I_s then equals the combined current in the diverter and series field.) In a long shunt generator $I_s = I_a$.

Example.—A compound generator, connected short shunt, has a terminal voltage of 230 volts when it is delivering a current of 150 amp. The shunt field current is 4 amp., the armature resistance 0.03 ohm and the

series field resistance 0.01 ohm. Determine the induced voltage in the armature, the total power generated in the armature and the disposition of this power.

The series field current I_s = 150 amp., and the armature current I_a = 154 amp.

$$E = 230 + (150 \times 0.01) + (154 \times 0.03) = 236.1 \text{ volts.}$$

Total power generated

$$P_a = 236.1 \times 154 = 36{,}400 \text{ watts} = 36.4 \text{ kw.}$$

Armature loss

$$P'_a = 154^2 \times 0.03 = 711 \text{ watts.}$$

Series field loss

$$P_s = 150^2 \times 0.01 = 225 \text{ watts.}$$

Shunt field loss

$$P_{sh} = (230 + 1.5)4 = 926 \text{ watts.}$$

Power delivered

$$P = 230 \times 150 = \underline{34{,}500} \text{ watts.}$$

Total 36,362 watts (check).

202. Effect of Speed.—Fig. 274 shows the saturation curve of a 230-volt, compound generator, taken at 900 r.p.m. The

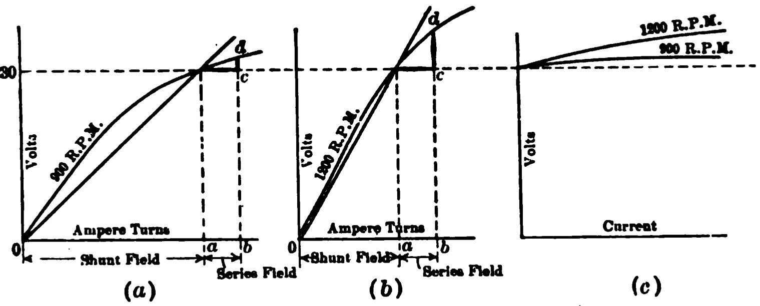


FIG. 274.—Effect of speed upon compound characteristic.

shunt field rheostat is so adjusted that the machine builds up to a no-load voltage of 230 volts. To produce this result a certain number of shunt field ampere-turns are necessary, as indicated by the distance *oa*. When load is applied to the machine a certain number of series ampere-turns are added. Let the number of series ampere-turns be represented by the distance *ab*. Neglecting armature reaction, the induced voltage will be increased by a value *cd* shown in heavy lines.

Let this same machine be speeded up to 1,200 r.p.m., Fig. 274 (*b*), and let the no-load terminal voltage still be 230 volts.

The distance *oa* will now be less than it was in Fig. 274 (*a*), owing to the increased speed. But the distance *ab* will be the same in each case, as the increase of series turns depends solely on the load. The *increase of voltage cd* is much greater in (*b*) than in (*a*), owing to the lesser saturation of the iron. Therefore, the higher speed machine will have the more rising characteristic, as is shown in Fig. 274 (*c*). It will be noted that the effect of speed upon the compound characteristic is just opposite to the effect of speed upon the shunt characteristic. (See Fig. 267.) This is due to the fact that saturation opposes change of the flux in each case.

203. Determination of Series Turns: Armature Characteristic.—It is often desired to determine the number of series turns which it is necessary to place upon the poles of a shunt generator in order to make it either flat compounded or to give it any desired degree of compounding.

To make the determination, adjust the no-load voltage to its proper value. Let this value of shunt field current be I_1. Load the generator to its rated load and by means of the field rheostat bring the terminal volts to the desired value. Let the corresponding value of field current be I_2. The necessary increase of field ampere-turns is

$$(I_2 - I_1)N_{sh}$$

where N_{sh} = shunt field turns (either turns per pole or total turns may be used).

Let I be the rated load current of the machine, and N_s the necessary series turns.

Then
$$N_s I = (I_2 - I_1)N_{sh}$$
$$N_s = \frac{(I_2 - I_1)}{I} N_{sh}$$

The number of series turns for flat compounding may also be obtained by means of the armature characteristic. The load is applied to the armature in the usual way. It is preferable to excite the field separately as shown in Fig. 275. Load is applied and the terminal voltage is maintained constant by means of the shunt field rheostat. Corresponding values of field current and armature current are noted. When the two are plotted (as

shown in Fig. 276) the resulting curve is the *armature characteristic.* The field current increases more rapidly than the armature current owing to saturation.

To determine the number of series turns necessary, multiply the increase of field current bc by the shunt turns and divide by the current Oa.

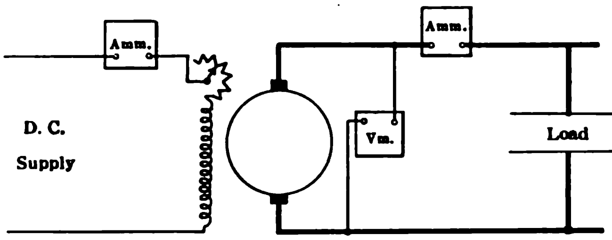


FIG. 275.—Connections for obtaining armature characteristic.

Series field turns for flat compounding

$$\therefore \quad N_s = N_{sh}\frac{bc}{Oa}$$

where N_{sh} is the number of turns of the shunt field.

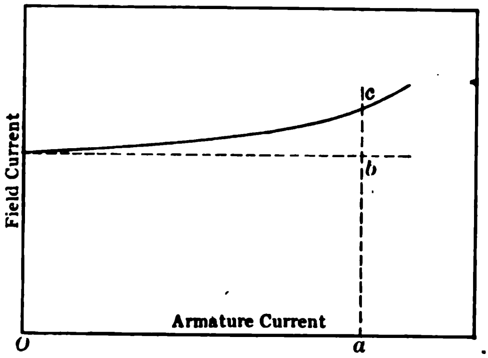


FIG. 276.—Armature characteristic.

204. The Series Generator.—In the series generator the field winding is connected in series with the armature and the external circuit. It must consist necessarily of a comparatively few turns of wire having a sufficiently large cross-section to carry the rated current of the generator.

The *series* generator in most instances is used for *constant current* work, in distinction to the *shunt* generator which maintains *constant potential*. Fig. 277 shows the saturation curve of a series generator and also its characteristic. The saturation curve differs in no way from that of the shunt generator. The external characteristic is similar in shape to the saturation curve for low saturation. The voltage at each point is less than that shown

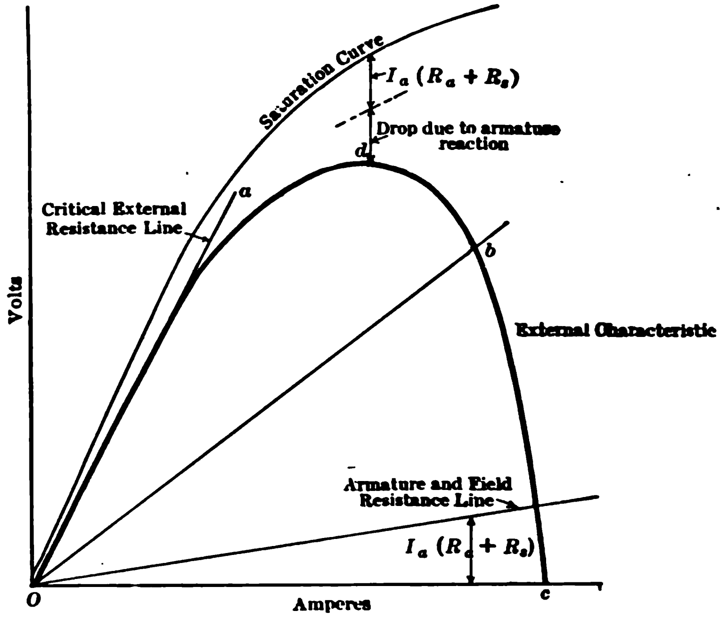


FIG. 277.—Series generator characteristic.

by the saturation curve by the amount due to the drop through the armature and field, $I_a(R_a + R_s)$, and the drop due to armature reaction. The curve reaches a maximum beyond which armature reaction becomes so great as to cause the curve to droop sharply and the voltage drops rapidly to zero. These machines are designed to have a very high value of armature reaction.

The machine builds up as follows:

If the series field is connected in such a manner that the current due to the residual magnetism aids this residual magnetism, the generator will build up, provided the external resistance equals or

is less than that indicated by the external resistance line Oa. The line Oa is therefore called the *critical* external resistance line. As the external resistance decreases, the external resistance line swings down to the right, as has already been discussed for the shunt generator, Par. 189. The line Ob is such a line. It would be practically impossible to operate with an external resistance corresponding to the line Oa, or to any line cutting the curve to the left of d, as a small increase in external resistance would swing the resistance line away from the curve resulting in the generator's dropping its load. The machine is designed to operate along the portion bc of the curve, which corresponds to substantially constant current. The current is not affected by a considerable change in external resistance, corresponding to the line Ob swinging up or down. To obtain close regulation the series field is shunted by a rheostat. The resistance of this rheostat is controlled by a solenoid connected in series with the line. In this way the current delivered by the generator may be held substantially constant.

In the past, the series generator has been much used in series arc lighting. The Brush Arc machine and the Thomson-Houston generator are common examples of such machines. Both of these have open-circuit armatures. (See Par. 164.) As the voltage on the commutator ranges from 2,000 volts to 10,000 volts, the commutators have wide gaps between segments. In the Brush Arc generator there are as many as two or three separate commutators connected in series so as to reduce the voltage per commutator and also to smooth out the ripples in the voltage wave. (See Fig. 191.) There are but four segments per commutator. (For a more complete description see "Dynamo Electric Machinery," S. P. Thompson, Vol. I.)

In Europe, power is transmitted by direct currents at potentials as high as 50,000 volts, in the Thury System.[1] This high voltage is obtained by connecting several generators in series and transmitting at constant current. The voltage increases with the load. The generators have two commutators, one at each end of the armature. The potential may run as high as 5,000 volts per commutator. Regulation is obtained by shunting the fields.

[1] See "Standard Handbook," Fourth Edition, Chap. XI, McGraw-Hill Co.

The power is utilized by series motors connected at the desired points in series with the line.

Series generators are often used as boosters on direct current feeders. When a drop on a particular feeder becomes excessive, it may be cheaper to install a booster, and utilize it at the peak load, than to invest in more copper. The booster is a series generator operating on the straight portion of the magnetization

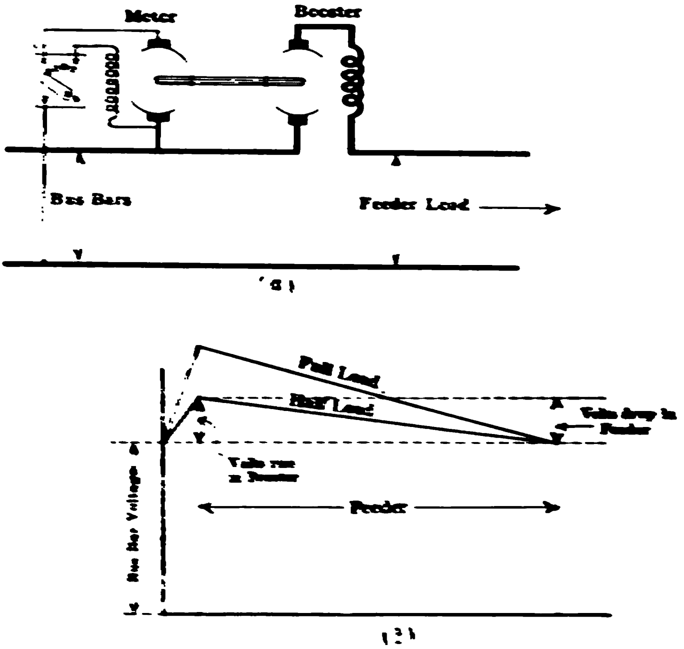


Fig. 278.—The series booster.

curve, the terminal voltage being proportional to the current flowing through the machine. Likewise the voltage drop in the feeder is proportional to the current in the feeder. If the generator be connected in series with the feeder, Fig. 278 (a), and adjusted properly, its terminal volts may be made always equal to the drop in the feeder as shown in Fig. 278 (b). Therefore, the voltage at the load may be maintained constant. The booster is direct-connected to a shunt motor taking its power from the bus-bars. If the driving power should in any way be removed,

the series generator will reverse and operate as a motor. The speed of a series motor without load is practically unlimited, so that it will run away and tear itself to pieces. Therefore, such a booster should never be belt-driven and should have some protective device to prevent its running away.

205. Effect of Variable Speed upon Characteristics.—When a generator is being tested to determine its characteristic or its regulation, it is assumed that the generator speed is maintained at a constant value, the rated speed of the generator. Any drop in voltage resulting from a drop in speed of the prime mover or driving motor is not chargeable to the generator.

In practice, a drop in speed with load in the case of the prime mover is often unavoidable. Therefore, the regulation of the generator is made to include the voltage drop due to this decreased speed. When making out specifications, the regulation of the generator when driven by its prime mover should be specified. Speed correction applied to characteristics of generators is somewhat involved, because of the many factors which enter the computation. For a more complete discussion see "A Solution of an Acceptance Test Problem," by W. B. Kouwenhoven, Flect. Wld., Vol. 71, Jan. 19, 1918.

206. The Unipolar or Homopolar Generator.[1]—In the ordinary direct current generator, the voltage as generated is alternating and the current must be rectified or commutated. In the unipolar generator, however, a direct current is generated, and no commutator is necessary.

The principle of the unipolar generator is that of Faraday's disc dynamo, Fig. 279 (*a*). If a disc be rotated between the poles of a magnet, an emf. is generated between the center and the rim of the disc. A current can be taken from the disc by placing a brush at the center and another at the rim. The disc shown in Fig. 279 (*a*) would not be practicable because the electromotive force is generated only at one portion, so that current can flow back through the disc even when the external circuit is open. If an annular pole be used, Fig. 279 (*b*), an equal electromotive force is generated along each radius, so that the current has no return path in the disc itself.

[1] For more complete discussion see the "Standard Handbook," Fourth Edition, Section 8, Par. 228.

Fig. 279 (*c*) shows a cross-section of a unipolar machine. The brushes *bb* are of one polarity and the brush *a* is of the opposite polarity. A hole in the casting allows access to brush *a*. Such generators are sometimes made with a rotating cylinder and are said to be of the axial type.

The chief disadvantage of the unipolar type of generator is the very low voltage generated, even at high speeds. It is necessary to connect several discs in series in order to obtain working

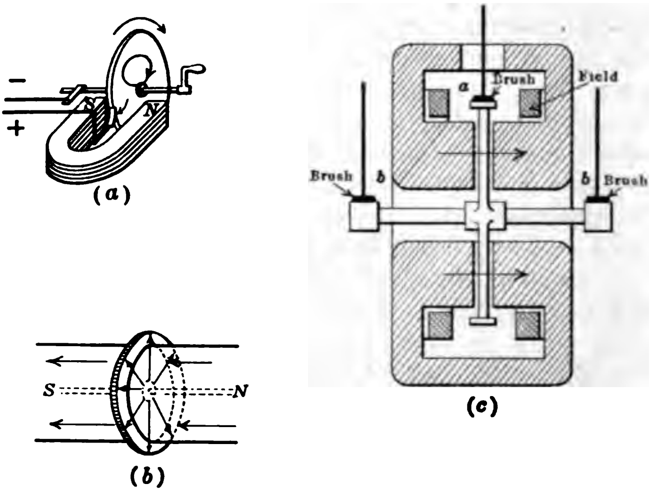


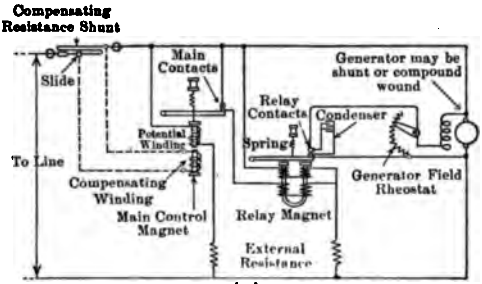
Fig. 279.—The unipolar generator.

voltages. The generator in Fig. 179 (*c*), having an armature diameter of about 20 in., and running at 3,000 r.p.m., would give only about 40 volts. Another disadvantage is the difficulty of conducting the current from the disc at the high speeds at which these machines are necessarily run.

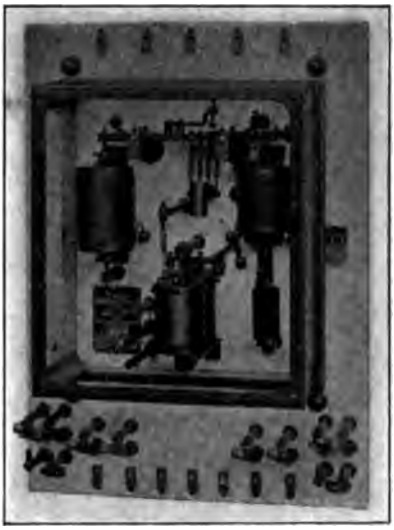
Such generators are manufactured by both the General Electric Co. and the Westinghouse Co. Their field of application is that of a high speed, turbo-driven generator, designed for high currents at low voltages.

207. The Tirrill Regulator.—It has been pointed out that the voltage of a generator varies with the load, speed, etc. By means of a Tirrill regulator, the voltage of a generator can be maintained constant even under rapid fluctuations of load.

In addition, compensation may be made for line drop. The voltage is controlled by small relay contacts, which short-circuit the shunt field rheostat, the duration of the short circuit depending upon the amount of regulation required. The field



(a)



(b)

FIG. 280a and b.—The Tirrill regulator.

rheostat is usually set so that the generator voltage is 35 per cent. below normal when the regulator is disconnected.

The diagram of the apparatus is shown in Fig. 280. The relay magnet is U-shaped and has two solenoids, differentially wound, upon its core. One winding is directly across the line. The

other is connected across the line through the main contacts. The relay contacts intermittently short-circuit the generator field rheostat.

The main control magnet can open the main contacts or allow them to close. These contacts are normally held closed by a spring. Assume that the voltage rises. The potential winding of the main control magnet strengthens this magnet and opens the main contacts. This opens one of the windings on the relay magnet and so nullifies the differential action. The relay contacts are then pulled open and the short circuit removed from the generator field rheostat. This immediately reduces the generator voltage. The reverse action takes place when the voltage drops.

As a matter of fact both relays are constantly vibrating so that the changes in the generator voltage are very small.

The relay contacts are shunted by a condenser to reduce sparking. Owing to the fact that these contacts can carry only a very small current, it is usually necessary to have the regulator act on an exciter field, and so maintain the bus-bar voltage constant through the exciter.

A compensating winding on the main control magnet may be connected across a series shunt to give the system a rising voltage characteristic and so compensate for line drop.

CHAPTER XII

THE MOTOR

208. Definition.—It was stated in Chap. XI that a generator is a machine for converting mechanical energy into electrical energy.

In a similar way the motor is a machine for converting *electrical* energy into *mechanical* energy. The same machine, however, may be used either as a motor or as a generator.

209. Principle.—Fig. 281(*a*) shows a magnetic field of constant strength or intensity in which is placed a conductor that

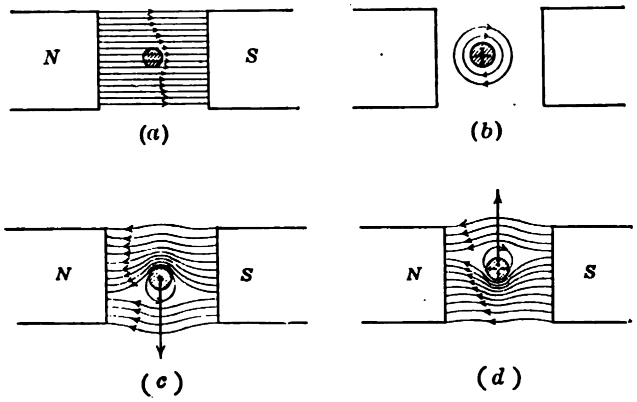


Fig. 281.—Force acting on a conductor carrying current in a magnetic field

carries no current. In (*b*) the conductor is shown as carrying a current into the paper, but the field due to the *N* and *S* poles has been removed. A cylindrical magnetic field now exists about the conductor due to the current in it. The direction of this field is determined by the corkscrew rule.

Fig. 281(*c*) shows the resultant field obtained by combining the main field and that due to the current. The field due to the current in the conductor acts in conjunction with the main field above the conductor, whereas it opposes the main field below the conductor. The result is to crowd the flux above the conductor and to reduce the flux density in the region below the conductor.

It will be found that a force acts on the conductor, trying to push the conductor *down*, as shown by the arrow.

It is convenient to think of this phenomenon as due to the crowding of the lines on one side of the conductor. Magnetic lines of force may be considered as acting like elastic bands under tension. These lines always are endeavoring to contract so as to be of minimum length. The tension in these lines on the upper side of the conductor is tending to pull it down as shown in the figure.

If the current in the conductor be reversed, the crowding of the lines will occur *below* the conductor, which will tend to move it *upward*, as shown in Fig. 281(*d*).

The operation of the electric motor depends upon the principle that a conductor carrying current in a magnetic field tends to move at right angles to the field.

210. Force Developed by Conductor Carrying Current.—The force acting on a conductor in a magnetic field is directly proportional to three quantities: the strength of the field, the magnitude of the current, and the length of the conductor lying in the field. The force in *dynes* is given by

$$F = BlI/10 \text{ dynes.} \tag{106}$$

where B is the flux density in lines per sq. cm. or gausses, l the active length of the conductor in centimeters and I the current in amperes. The direction of the field, the conductor, and the direction of the force are all mutually perpendicular to one another.

Example.—A coil consisting of 20 turns lies with its plane parallel to a magnetic field (see Fig. 286), the flux density in the field being 3,000 lines per sq. cm. The axial length of the coil is 8 in. The current per conductor is 30 amp. Determine the force in pounds which acts on each side of the coil. (See arrows in Fig. 286*a*.)

$$B = 3{,}000$$
$$l = 8 \times 2.54 = 20.32 \text{ cm.}$$
$$I = 30$$
$$F_1 = 3{,}000 \times 20.32 \times 30/10 = 182{,}900 \text{ dynes.}$$

As there are 20 turns

$$F = 20 \times 182{,}900 = 3{,}658{,}000 \text{ dynes}$$
$$3{,}658{,}000/981 = 3{,}730 \text{ grams}$$
$$= 3.73 \text{ kg.}$$
$$3.73 \times 2.204 = 8.23 \text{ lb.} \quad \textit{Ans.}$$

211. Fleming's Left-hand Rule.—The relation between the direction of a magnetic field, the direction of motion of a conductor in that field and the direction of the *induced* electromotive force is given by Fleming's Right-hand Rule.

In a similar manner, the relation between the direction of a magnetic field, the direction of a current in that field and the

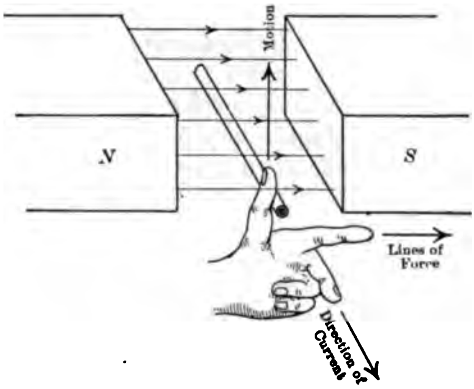
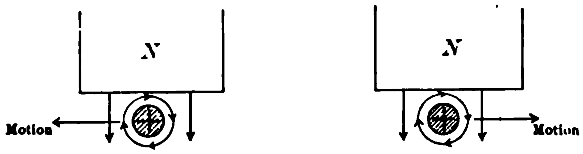


FIG. 282.—Fleming's left-hand rule.

direction of the resulting motion of the conductor can be determined by using Fleming's Left-hand Rule.

Fleming's Left-hand Rule:

Point the forefinger in the direction of the field or flux, the middle finger in the direction of the current in the conductor, and the thumb will point in the direction in which the conductor tends to move. This is illustrated by Fig. 282.



(*a*) Motor (*b*) Generator

FIG. 283.—Motor and generator action.

Another convenient method for determining the above relation is to make use of the fact that the crowding of the magnetic lines behind the conductor tends to push it along. It is necessary merely to sketch the main field and the lines about the conductor, as shown in Fig. 283(*a*). It is evident that the lines will be

crowded at the right of the conductor so that the direction of motion is to the left.

In Fig. 283(*b*) is shown a similar condition for a generator. In this case the conductor, as a generator, moves to the right. Hence in a generator the conductor must move *against* a force tending to oppose its motion, and so the conductor requires a driving force to keep it in motion. This driving force is supplied by the prime mover to which the generator is connected.

212. Torque.—When an armature, a fly wheel or any other device is revolving about its center, a tangential force is necessary to produce and maintain rotation. This force may be developed within the machine itself as in a motor or steam engine, or it may be applied to a driven device such as a pulley, a shaft, a

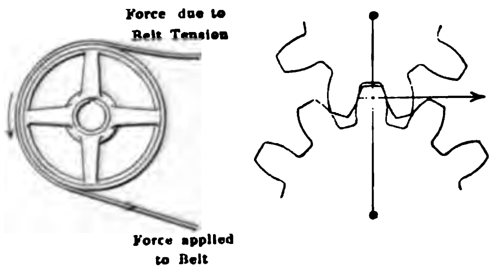


FIG. 284.—Torque developed by a belt and by gears.

generator, the driving gears on the wheels of a street car, etc. Fig. 284. The total effect of the force is determined not only by its *magnitude* but also by its *arm*, or radial distance from the center of the pulley or gear to the line of action of the force.

The product of this force and its perpendicular distance from the axis is called *torque*.

Torque may also be considered as a mechanical couple tending to produce rotation. It is expressed in units of force and distance.

In the English system, torque is usually expressed in pounds-feet. (This distinguishes it from foot-pounds which represent *work*.)

In the c.g.s. system the unit of torque is the dyne-centimeter (a very small unit), and in the metric system the unit is the kilogram-meter.

Example.—A belt is driving a 36-in. pulley as shown in Fig. 285. The tension in the tight side of the belt is 90 lb. and that in the loose side is 30 lb. Determine the torque applied to the pulley.

The two sides of the belt are acting in opposition so that the net pull on the rim of the pulley is

$$90 - 30 = 60 \text{ lb.}$$

This force is acting 18 in. or 1.5 ft. from the center of the pulley. Therefore the torque

$$T = 60 \times 1.5 = 90 \text{ lb.-ft.} \quad \textit{Ans.}$$

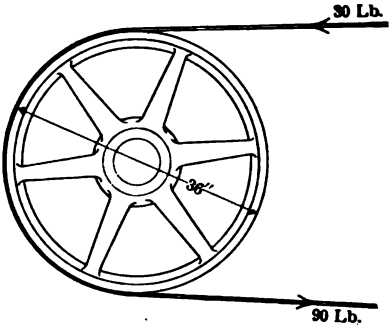


FIG. 285.—Example of torque produced upon a pulley by a belt.

213. Torque Developed by a Motor.—Fig. 286 (*a*) shows a coil of a single turn, whose plane lies parallel to a magnetic field. Current flows into the paper in the left-hand side of the coil and out of the paper in the right-hand side of the coil. Therefore, the left-hand conductor tends to move downward with a force F_1 and the right-hand conductor tends to move upward with a

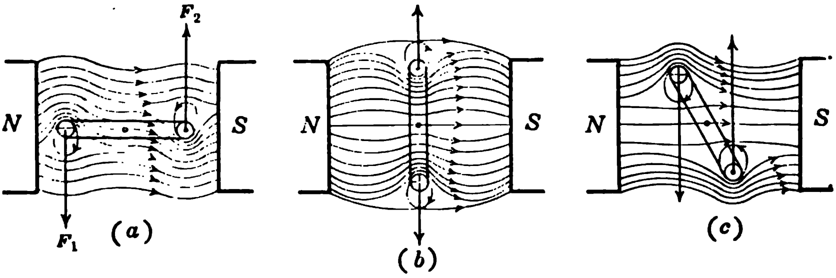


FIG. 286.—Torque developed at different positions of a coil.

force F_2. These two forces tend to rotate the coil about its axis. Both act to turn it in a counter-clockwise direction and so develop a torque. As the current in each of these conductors is the same and they lie in magnetic fields of the same strength, force $F_1 = F_2$. In (*a*) the coil is in the position of maximum torque because the perpendicular distance from the coil axis to the forces acting is a maximum.

When the coil reaches the position (*b*) neither conductor can move any farther without the coil itself spreading. This is a

position of zero torque because the perpendicular distance from the coil axis to the forces is zero.

If, however, the current in the coil be reversed when the coil reaches position (*b*) and the coil be carried slightly beyond the

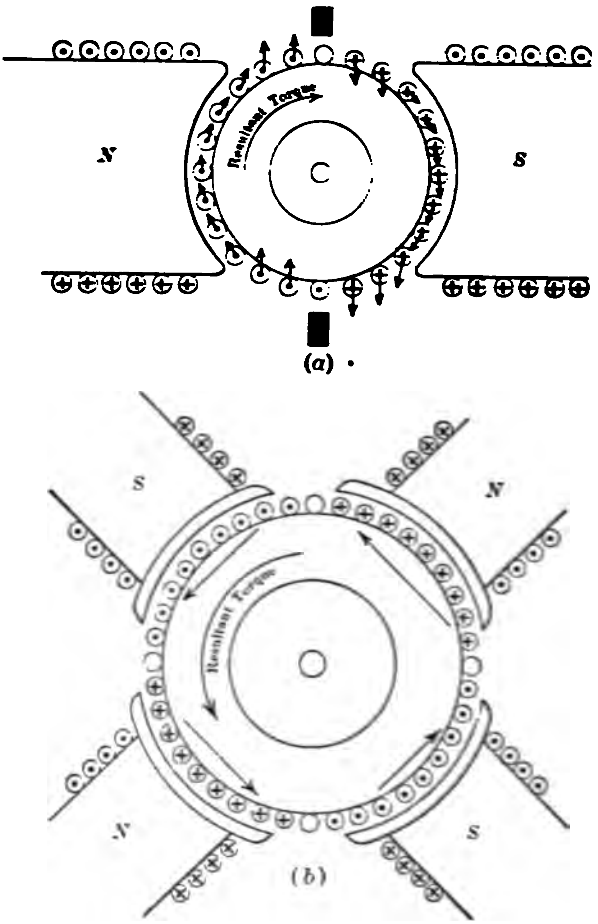


FIG. 287.—Torque developed by belt conductors in motor armatures.

dead center, as shown in (*c*), a torque is developed which still tends to turn the coil in the counter-clockwise direction.

To develop a continuous torque in a motor, the current in each coil on the armature must be reversed just as it is passing through the neutral plane or plane of zero torque and *a commutator is therefore necessary*. This is analogous to using a com-

mutator in connection with a generator in order that the current delivered to the external circuit may be uni-directional.

A single-coil motor, like that shown in Fig. 286, would be impracticable as it has dead centers and the torque which it develops is pulsating. A two-coil armature would eliminate the dead centers, but the torque developed would still be more or less pulsating in character.

The best conditions are obtained when a large number of coils is used, just as in the armature of a generator. In fact there is no difference in the construction of a motor armature and a generator armature. In Fig. 287 (*a*) an armature and a field are shown for a 2-pole machine and the torque developed by each individual conductor is indicated. Fig. 287 (*b*) shows an armature and a field for a 4-pole machine. The direction of the torque developed by each belt of conductors is indicated by the arrow at that belt.

In armatures of this type a very small proportion of the total number of coils is undergoing commutation at any one instant. Therefore, the variation in the number of active conductors is so slight that the torque developed is substantially constant, for constant values of armature current and main flux.

From equation (106), the torque developed by any armature can be shown to be

$$T = K'_t Z I \Phi \tag{107}$$

where K'_t = a constant of proportionality, involving the diameter of the armature, the parallel paths through the armature, the choice of units, etc.

Z = number of conductors on the surface of the armature.

I = current supplied to the armature, in amperes.

Φ = flux from one north pole entering the armature.

For any particular machine Z is a fixed quantity, so that the torque

$$T = K_t I \Phi \tag{108}$$

where K_t is a new constant of proportionality.

That is, in a given motor, *the torque is proportional to the armature current and to the strength of the magnetic field.*

This is a very important relation to keep in mind, for by its use the variation of torque with load in the various types of motors can be readily determined.

Example.—When a certain motor is taking 50 amp. from the line it develops 60 lb.-ft. torque. If the field strength is reduced to 75 per cent of its original value and the current increases to 80 amp., what is the new value of the torque developed?

If the current remained constant the new value of torque, due to the weakening of the field, would be

$$0.75 \times 60 = 45 \text{ lb.-ft.}$$

Due to the increase in the value of the current, however, the final value of torque will be

$$\frac{80}{50}\,45 = 72 \text{ lb.-ft.} \quad \textit{Ans.}$$

It must be remembered that the torque expressed by the above equations is the entire torque developed by the armature. The torque available at the pulley will be slightly less than this, due to the torque lost in overcoming friction and in supplying the iron losses of the armature.

214. Counter Electromotive Force.—The resistance of the armature of a 10-hp., 110-volt motor is about 0.05 ohm. If this armature were connected directly across 110-volt mains, the current, by Ohm's Law, would be

$$I = \frac{110}{0.05} = 2{,}200 \text{ amp.}$$

This value of current is not only excessive but unreasonable, especially when one considers that the rated current of such a motor is in the neighborhood of 90 amp. When a motor is in operation, *the current through the armature is evidently not determined by its ohmic resistance alone.*

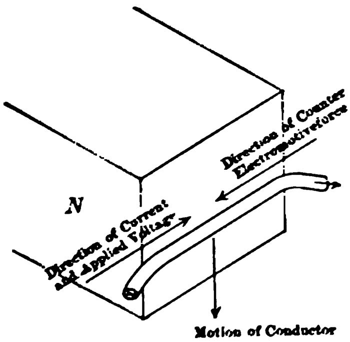


FIG. 288. Relation of the direction of currents and voltages in a motor conductor.

The armature of a motor is in every way similar to that of a generator. The conductors on its surface, in addition to carrying current and so developing torque, are cutting flux. Therefore, they *must* be generating an electromotive force.

If the right-hand rule be applied to determine the direction of this electromotive force (see Fig. 288), it will be found that it is

always in *opposition* to the current. That is, it *opposes* the current entering the armature. This induced emf. is called the counter electromotive force or back electromotive force. As the counter electromotive force opposes the current it must also oppose the line voltage. Therefore, the net electromotive force acting in the armature circuit is the difference of the line voltage and the back electromotive force. Let V equal the line voltage and E the back electromotive force. The net voltage acting in the armature circuit is

$$V - E$$

The armature current follows Ohm's Law and is

$$I_a = \frac{V - E}{R_a} \tag{109}$$

where R_a is the armature resistance.

This equation may be transposed and written

$$E = V - I_aR_a \tag{110}$$

This should be compared with equation (104), page 293, which is the similar equation for a generator.

In a generator the induced emf. is equal to the terminal voltage *plus* the armature resistance drop. In a motor the induced emf. is equal to the terminal voltage *minus* the armature resistance drop. The counter electromotive force must always be less than the terminal or impressed voltage if current is to flow *into* the armature at the positive terminal.

Example.—Determine the back electromotive force of a 10-hp. motor when the terminal voltage is 110 volts and its armature is taking 90 amp. The armature resistance is 0.05 ohm.

$$E = 110 - (90 \times 0.05) = 110 - 4.5 = 105.5 \text{ volts. } \textit{Ans.}$$

An interesting experiment for demonstrating the existence of counter electromotive force is shown in Fig. 289. A lamp bank is connected in series with the armature of a shunt motor. First close switch S_2 which closes the field circuit. Then close S_1. At the instant of closing S_1 the lamps will burn brightly, being practically up to candle-power. As the armature speeds up, these lamps will become dimmer and dimmer, showing that

the armature is generating a *counter* electromotive force which opposes the line voltage and so leaves less voltage for the lamps. When the armature is up to speed, the lamps will be very dim. If, however, the field switch S_2 now be opened, the flux and, therefore, the counter electromotive force will be immediately reduced to zero practically, which will be shown by the lamps again

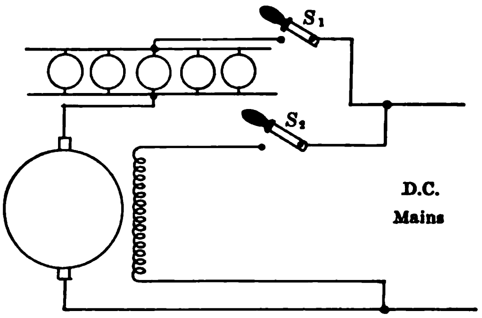


FIG. 289.—Demonstration of counter electromotive force.

coming up to full candle-power. (In practice when a motor is in operation, the field circuit should *not be opened under any conditions whatsoever.*)

Equation (101), page 258, for the induced electromotive force in a generator will obviously apply to a motor. That is, the counter electromotive force

$$E = \frac{\phi sPZ}{p\,10^8}\text{ volts}$$

where ϕ is the total flux entering the armature from one north pole, s the speed of the armature in revolutions per second, P the number of poles, Z the number of conductors on the surface of the armature, and p the parallel paths through the armature.

As Z, P, p, and 10^{-8} are all constant for any given motor, the counter electromotive force becomes

$$E = K_1\phi S$$

which is identical with equation (102), page 259, S being given in R. P. M.

Solving for speed

$$S = K\frac{E}{\phi} \tag{111}$$

where

$$K = 1/K_1$$

The speed of a motor is directly proportional to the counter electromotive force and inversely proportional to the field.

Substituting for E in (111) its value given in (110), the speed becomes

$$S = K\frac{V - I_aR_a}{\phi} \tag{112}$$

This is a very important equation for it shows the law of speed variation of a motor with changes of load.

Example.—A certain motor has an armature resistance of 0.1 ohm. When connected across 110-volt mains and taking 20 amp. its speed is 1,200 r.p.m. What is its speed when taking 50 amp. from these same mains, with the field increased 10 per cent.?

Applying (112)

$$\frac{S_2}{S_1} = \frac{K\dfrac{110 - 50 \times 0.1}{\phi_2}}{K\dfrac{110 - 20 \times 0.1}{\phi_1}} = \frac{\dfrac{105}{\phi_2}}{\dfrac{108}{\phi_1}} = \frac{105}{\phi_2} \cdot \frac{\phi_1}{108}$$

$$S_1 = 1,200$$

Therefore:

$$S_2 = 1,200\frac{105}{108} \cdot \frac{\phi_1}{\phi_2}$$

But $\phi_2 = 1.10\ \phi_1$

Therefore:

$$S_2 = 1,200\frac{105}{108} \cdot \frac{\phi_1}{1.10\phi_1} = 1,060 \text{ r.p.m.} \quad \textit{Ans.}$$

215. Armature Reaction and Brush Position in a Motor.—Fig. 290(a) shows a motor armature carrying current. Due to the armature ampere-turns, a mmf. F_A is produced in the armature, and the direction of flux produced by this mmf. is at right angles to the polar axis. Fig. 290(b) shows the vectors representing the magnitudes and directions of the armature mmf. F_A and the field mmf. F. By adding these two vectorially, the resultant mmf. F_o is obtained. The total flux produced by F_o is distorted as shown in Fig. 290(c). It will be noted that (1) the flux has been crowded into the *leading* pole tips, and (2) the neutral plane perpendicular to the resultant field has moved *backward*. Therefore in a motor it is necessary to move the brushes *backward* with increase of load, whereas in a generator they are moved *forward*. Were it not for the electro-

motive force of self induction (see Par. 195), the brush axis would coincide with the neutral plane. Due, however, to the necessity of counteracting this last electromotive force, the brushes are set ahead of this load neutral plane, as is shown in Fig. 290(*c*). That is, in both the motor and the generator it is necessary

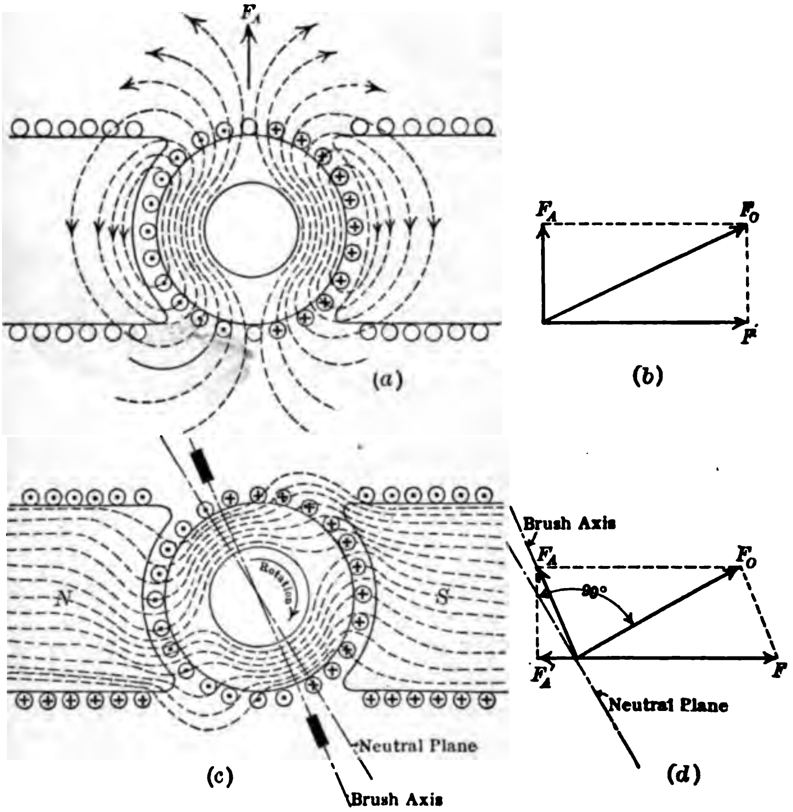


Fig. 290.—Armature reaction in a motor.

to set the brushes *ahead* of the load neutral plane in order to counteract this electromotive force of self induction. In the case of the motor they will be nearer the geometrical neutral than they are in the case of the generator.

This backward movement of the brushes is accompanied by a demagnetizing action of the armature upon the field, as indicated in Fig. 290(*d*), where F'_A is the demagnetizing component of

F_A. Therefore, as the load is increased on a motor the armature reaction tends to increase the motor speed. In fact instances have been known where motors with short air gaps (producing high armature reaction) have run away when the load was applied.

Fig. 291 shows the armature conductors carrying current and passing under successive north and south poles. It will be noted that the armature reaction F_A in the first inter-polar space is *upward*. (See Fig. 243.) Therefore, if a commutating pole is to be used it must be a *north* pole, in order to oppose this

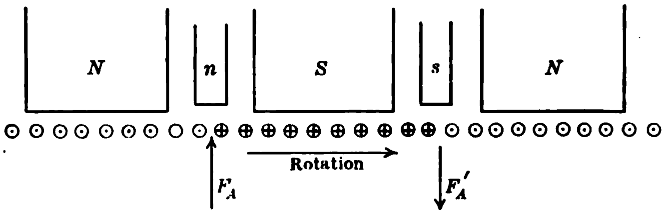


Fig. 291.—Relation of commutating poles to main poles in a motor.

magnetomotive force of the armature by tending to send a flux down into the armature. F'_A must then be opposed by a south pole. Therefore in a motor, the relation of main poles and commutating poles, in the direction of rotation, is *Nn Ss*, or opposite to the corresponding relation for a generator. (See Fig. 261, page 287.)

The polarity of the interpoles should be carefully investigated with a compass, if a motor happens to be sparking badly from some unknown cause, as the sparking may be due to their being incorrectly connected.

216. The Shunt Motor.—The shunt motor is connected in the same manner as a shunt generator, that is, its field is connected directly across the line in parallel with the armature.

A field rheostat is usually connected in series with the field.

If load is applied to any motor it immediately tends to slow down. In the case of the shunt motor this decrease of speed lowers the back electromotive force, as the flux remains substantially constant. If the back electromotive force is decreased, more current flows into the armature (see equation 109, page 317). This continues until the increased armature current produces sufficient torque to meet the demands of the increased load.

The suitability of a motor for any particular duty is determined

almost entirely by two factors, the variation of its *torque* with load and the variation of its *speed* with load.

In the shunt motor the flux is substantially constant. Therefore, from equation (108), the torque will vary almost directly with the armature current. For instance, in Fig. 292, when the armature current is 30 amp. the motor develops 40 lb.-ft. torque, and when the current is 60 amp. the motor develops 80 lb.-ft. torque. That is, when the current doubles the torque doubles.

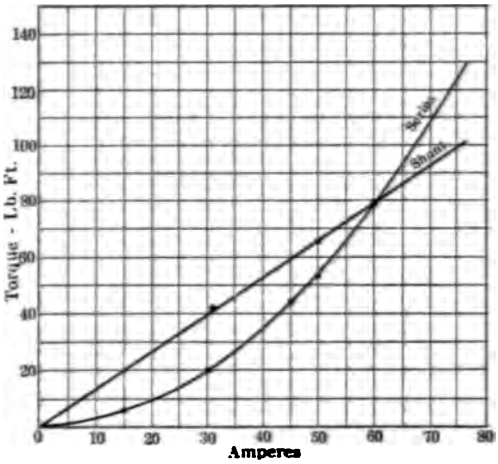


Fig. 292. — Shunt and series motors; torque-current curves.

The speed of a motor varies according to equation (112), where

$$S = K\frac{V - I_aR_a}{\phi}$$

In the case of the shunt motor, K, V, R_a, and ϕ are all substantially constant. Therefore, the only variable is I_a. As the load on the motor increases, I_a increases and the numerator of this equation decreases. As a rule the denominator changes only a small amount. The speed of the motor will then drop with increase of load, as shown in Fig. 293. As I_aR_a is ordinarily from 2 to 6 per cent. of V, the percentage drop in speed of the motor is of this order of magnitude. For this reason the shunt motor is considered a constant speed motor, even though its speed does drop slightly with increase of load.

Owing to armature reaction, ϕ ordinarily decreases slightly with increase of load and this tends to maintain the speed constant. Occasionally the armature reaction is sufficiently great to give a rising speed characteristic with increase of load.

Speed Regulation.—The speed regulation of a shunt motor is almost identical with the voltage regulation of a shunt generator. It is defined in the A. I. E. E. Standardization rules as being the difference in the no-load and the rated-load speed divided by

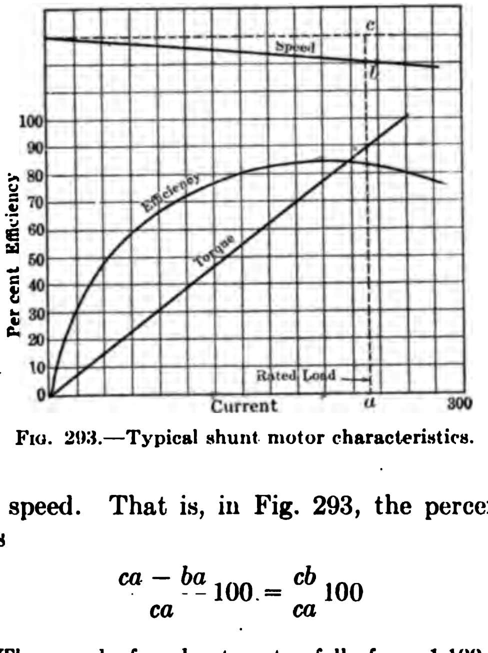


FIG. 293.—Typical shunt motor characteristics.

the no-load speed. That is, in Fig. 293, the percentage speed regulation is

$$\frac{ca - ba}{ca}100 = \frac{cb}{ca}100$$

Example.—The speed of a shunt motor falls from 1,100 r.p.m. at no load to 1,050 r.p.m. at rated load. What is its percentage speed regulation?

$$\text{Regulation} = \frac{1,100 - 1,050}{1,100}100 = 4.54 \text{ per cent.} \quad \textit{Ans.}$$

The speed regulation is a measure of a motor's ability to maintain its speed when load is applied.

Fig. 293 shows the three essential characteristics of a shunt motor, the torque, the speed, and the efficiency, each plotted against current. The effect of the machine losses upon the effi-

ciency will be discussed in the next chapter. It will be noted that the shunt motor has a definite no-load speed. Therefore it does not run away when the load is removed, provided the field circuit remains intact.

Shunt motors are used where a substantially constant speed is required, as in machine shop drives, spinning frames, blowers, etc.

There is an erroneous impression that shunt motors have a low starting torque and therefore should not be started under load. Starting boxes are usually designed to allow 125 per cent. of full-load current to flow through the armature on the first notch. Therefore, the motor develops 125 per cent. of full-load torque at starting. By decreasing the starting resistance, the motor could be made to develop 150 per cent. of full-load torque without trouble.

217. The Series Motor.—In the series motor the field is connected in series with the armature, as shown in Fig. 294. The field has comparatively few turns of wire and this wire must be of sufficient cross-section to carry the rated armature current of the motor.

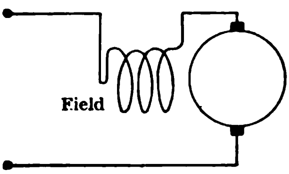


FIG. 294.—Connections of a series motor.

In the series motor the flux, ϕ, depends entirely on the armature current. If the iron of the motor is operated at moderate saturation, the flux will be almost directly proportional to the armature current. Therefore, in the expression for torque,

$$T = K_t I \phi$$

if ϕ is assumed to be proportional to I, the expression becomes

$$T = K'_t I^2 \tag{113}$$

when K'_t is a constant.

The torque is proportional to the *square* of the armature current, as shown in Fig. 292. When the current is 30 amp. the torque is 20 lb.-ft.; at 60 amp. the torque is 80 lb.-ft. That is, the doubling of the armature current results in the quadrupling of the torque. It will be noted that as the current increases above 60 amp., the torque rises very rapidly. This characteristic of the series motor makes its use desirable where large increases of torque are desired with moderate increases in cur-

rent. In practice, saturation and armature reaction both tend to prevent the torque increasing as rapidly as the square of the current.

When equation (112) is applied to the series motor, the speed

$$S = K\frac{V - I_a(R_a + R_s)}{\phi} \tag{114}$$

where K is a constant, V the terminal voltage, I_a the motor current, R_a the armature resistance including brushes, R_s the series field resistance and ϕ the flux entering the armature from a north pole. R_s, the resistance of the series field, is now added to the armature resistance in order to obtain the total motor resistance. Both I_a and ϕ vary with the load.

As the load increases, the voltage drop in the field resistance and the armature resistance increases because this voltage drop is proportional to the current. Therefore, the back emf. becomes less, which causes the motor to run more slowly, although this effect is only of the magnitude of a few per cent. The flux ϕ, however, increases almost directly with the load. Therefore the speed must drop, in order that the back emf. be of the proper value, which is usually a few per cent. less than the terminal voltage. Both effects tend to slow down the motor. The resistance drop is ordinarily from 2 to 6 per cent. of the terminal voltage V so its effect on the speed is only of this magnitude. The speed is, however, inversely proportional to the flux ϕ and a given percentage change in ϕ produces the same percentage change in the speed.

When the load is decreased, the flux ϕ correspondingly decreases and the armature must speed up in order to develop the required back emf. If the load be removed altogether, ϕ becomes extremely small, resulting in a very high speed. It is dangerous to remove the load from series motors, as their armatures are almost certain to reach speeds where centrifugal action will wreck them.

Fig. 295 shows the characteristic curves of a series motor plotted with current as abscissæ. The torque curve concaves upward for the reasons which have just been stated. The speed is practically inversely as the current, that is, at large values of current the speed is low and at small values of current the

speed is high. The characteristics cannot be determined for small values of current because the speed becomes dangerously high.

The efficiency increases rapidly at first, reaches a maximum at about half load and then decreases. This is due to the fact that at light loads the friction and iron losses are large as compared with the load. The effect of these becomes less as the load increases. The field and armature loss varies as the *square* of the current (I^2R), so that these losses incerase rapidly with the load. The maximum efficiency occurs when the friction and iron

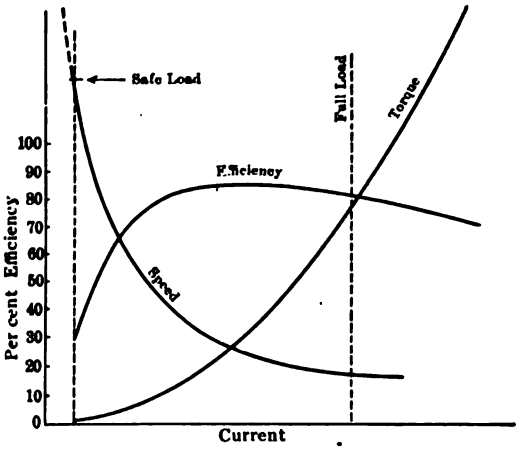


FIG. 295.—Typical series motor characteristics.

losses are practically equal to the copper losses. These curves should be carefully compared with the corresponding characteristic curves of the shunt motor, Fig. 293.

Series motors are used for work which demands large starting torque, such as street cars, locomotives, cranes, etc. In addition to the large starting torque, there is another characteristic of series motors which makes them especially desirable for traction purposes. Assume that a shunt motor is used to drive a street car. When the car ascends a grade, the shunt motor maintains the speed of the car at approximately the same value that it has when the car is running on level ground. The motor therefore tends to take an excessive current. A series motor, on the other hand, automatically slows down

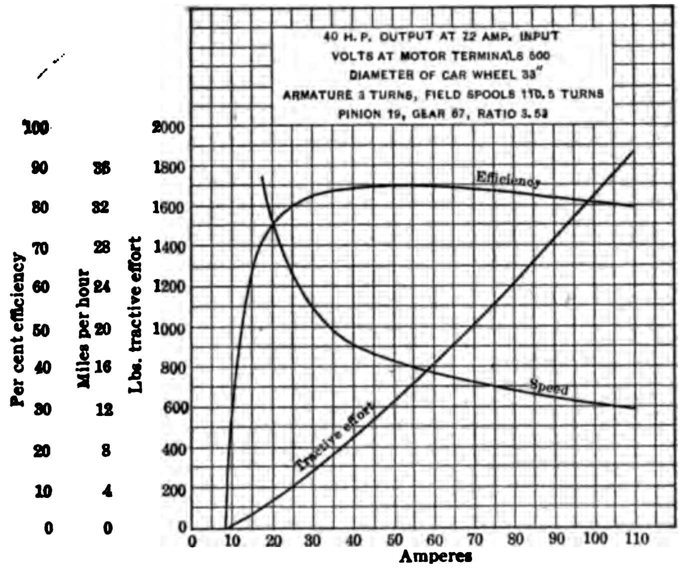


FIG. 296.—Typical railway motor characteristics.

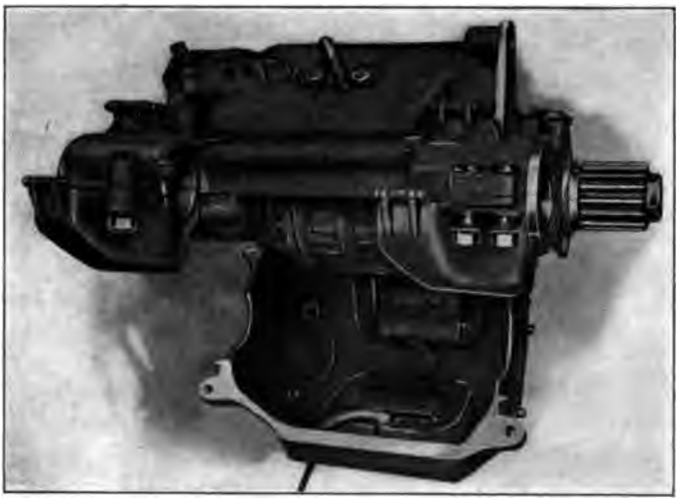


FIG. 297.—Railway motor with frame lowered for inspection.

upon reaching such a grade, because of the increased current. It therefore develops more torque at reduced speed. The drop in speed allows the motor to develop a large torque with but a moderate increase of power. Hence, a series motor could be smaller than a shunt motor operating under the same conditions.

When the characteristics of railway motors are plotted, the curves refer to the output at the track and not at the motor shaft. Fig. 296 gives such characteristics for a 500-volt, 40-hp., General Electric railway motor. It will be noted that tractive effort is plotted rather than torque. The speed of the car in miles per hour is given rather than the r.p.m. of the motor armature. These curves differ from the curves of torque and r.p.m. respectively by a constant quantity, determined by the gear ratio and by the diameter of the driving wheels. The efficiency curve is also the efficiency at the rails. These curves resemble closely the characteristic curves of Fig. 295. Fig. 297 shows a typical railway motor with half of the casing lowered.

218. The Compound Motor.—A shunt motor may have an additional series winding in the same manner as a shunt generator. This winding may be connected so that it aids the shunt winding, in which case the motor is said to be *cumulative compound;* or the series winding may oppose the shunt winding, in which case the motor is said to be *differential compound.*

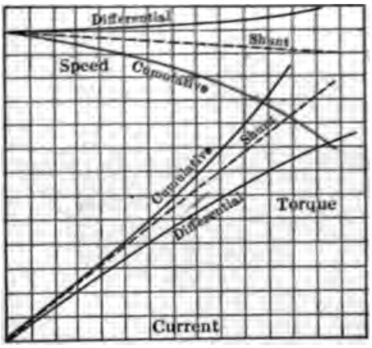


FIG. 298.—Torque and speed characteristics of shunt and compound motors.

The characteristics of the cumulative compound motor are a combination of the shunt and series characteristics. As the load is applied the series turns increase the flux, causing the torque for any given current to be greater than it would be for the simple shunt motor. On the other hand, this increase of flux causes the speed to decrease more rapidly than it does in the shunt motor. These characteristics are shown in Fig. 298. The cumulative compound motor develops a high torque with

sudden increase of load. It also has a definite no-load speed, so does not run away when the load is removed.

Its field of application lies principally in driving machines which are subject to sudden applications of heavy load, such as occur in rolling mills, shears, punches, etc. This type of motor is used also where a large starting torque is desirable but where a straight series motor cannot be conveniently used. Cranes and elevators are representative of such loads. In elevators the series turns are usually short-circuited when the motor reaches speed.

In the *differential* compound motor, the series field opposes the shunt field so that the flux is decreased as the load is applied. This results in the speed remaining substantially constant or even increasing with increase of load. This speed characteristic is obtained with a corresponding decrease in the rate at which the torque increases with load. Such motors are used where a very constant speed is desired. Because of the substantially constant speed of the shunt motor there is little occasion to use the differential motor. In starting a differential compound motor the series field should be short-circuited, as the large starting current passing through the series field may be sufficiently large to overbalance the shunt field ampere-turns and cause the motor to start in the wrong direction. Typical torque and speed curves of the differential compound motor are also shown in Fig. 298.

To reverse the direction of rotation in any motor, either the armature alone or the field alone must be reversed. If both are reversed the direction of rotation remains unchanged. Therefore, in so far as the direction of rotation of the motor is concerned, it is immaterial which line is positive.

219. Motor Starters.—It was shown in Par. 214 that if a 10-hp., 110-volt motor were connected directly across 110-volt mains, the resulting current would be $\frac{110}{0.05}$ or 2,200 amp. Such a current would not be permissible under commercial conditions. Hence, resistance should be connected in series with the motor *armature* when starting. This resistance may be gradually cut out as the armature comes up to speed and develops a back electromotive force.

Fig. 299 shows the use of a simple resistance R for starting a motor. It will be noted that this resistance is in the *armature* circuit and that the field is connected directly across the line and outside the resistance. If the field were connected across the armature terminals, putting the resistance R in series with the whole motor, there would be little or no voltage across the field at starting. There would be little torque developed and difficulty in starting would be experienced.

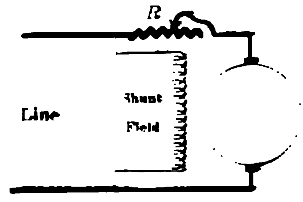


FIG. 299.—Resistance used for starting purposes.

Fig. 300 shows a 3-point starter. This does not differ fundamentally from the connections shown in Fig. 299. One line connects directly to an armature and a field terminal tied together. It makes no connection whatever with the starting box. The other line goes to the line terminal of the starting box which is connected directly to the starting arm. The starting arm moves over contacts set in the slate front of the starting box. These contacts connect with taps distributed along the starting resistance. The armature terminal of the starting box, which is the right-hand end of the starting resistance, is connected to the other armature terminal of the motor. The field connection in the starting box is connected from the first starting contact, through the hold-up magnet, to the field terminal of the box. This field terminal is connected directly to the other terminal of the shunt field.

When the starting arm makes connection with the first contact, the field is put directly across the line and at the same time all the starting resistance is in series with the armature. As this arm is moved the starting resistance is gradually cut out. When the arm reaches the running position, the starting resistance is all cut out and, to insure good contact, the line and armature conductors frequently are connected directly by a laminated copper brush, shown in Fig. 300. The field current now feeds back through the starting resistance. This resistance is so low compared with the resistance of the field itself that it has no material effect upon the value of the field current. A spring tends to pull the starting arm back to the starting position.

When the arm reaches the running position, it is held against the action of this spring by a soft-iron magnet (hold-up magnet), connected in series with the shunt field. (A soft-iron armature is often attached to the starting arm as shown in the figure.) If for any reason the line is without voltage, the starting arm will

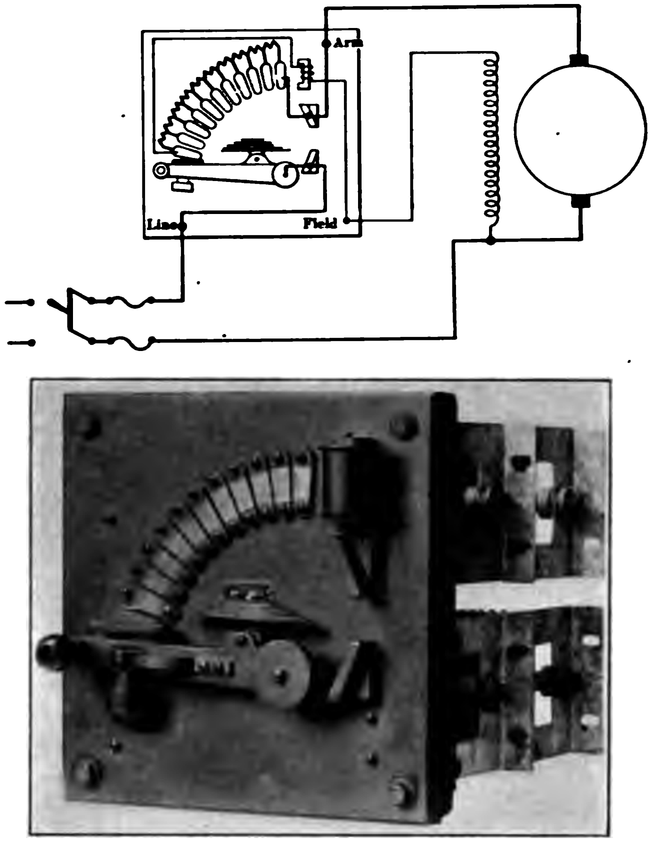


FIG. 300.—Three-point starting box.

spring back to the starting position. Otherwise, if the voltage again came on the line after a temporary shut-down, the stationary motor armature would be thrown directly across the line and a short-circuit would result.

The advantage of connecting the hold-up coil in series with the field is that, should the field circuit become opened, the arm

springs back to the starting position and so prevents the motor running away.

The 3-point starting box cannot be used to advantage upon variable speed motors having field control. Such motors frequently have a speed variation of five to one. This results in the field current having approximately this same range. The

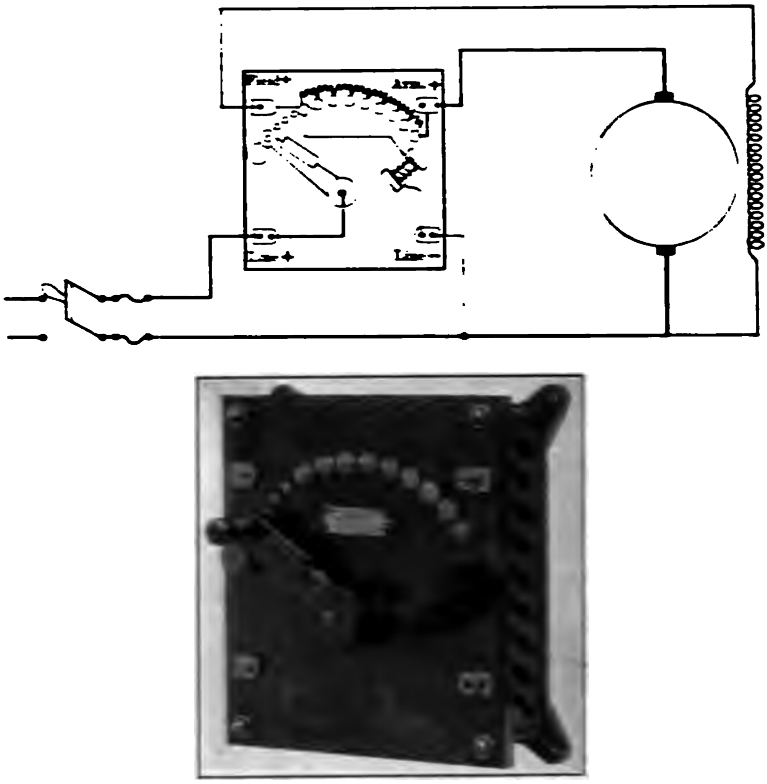


Fig. 301.—Connections for a 4-point starting box.

hold-up magnet may be too strong, therefore, at the higher values of field current and too weak at the lower values. To obviate this difficulty a 4-point box is used. Fig. 301. It is similar to the box shown in Fig. 300, except that the hold-up coil is of high resistance and is connected *directly across the line*. The only difference in the connection is that the "line terminal" must be connected to the side of the line which runs directly to the com-

mon armature and field terminals. When the voltage leaves the line, the hold-up coil becomes dead and allows the arm to spring back to the starting position.

Sometimes the field resistance is contained within the starting box. The box then has two arms, as shown in Fig. 302. The

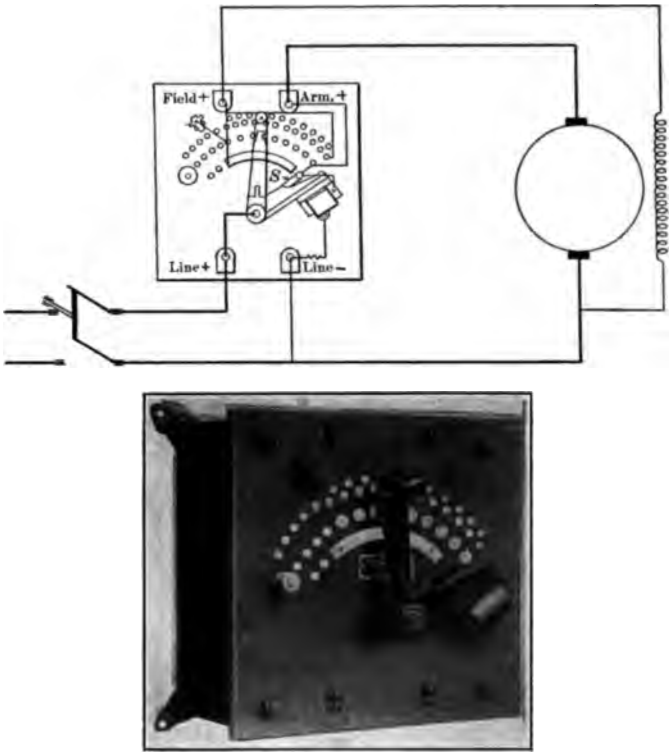
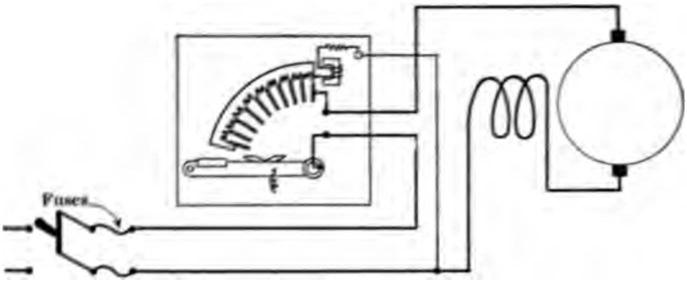


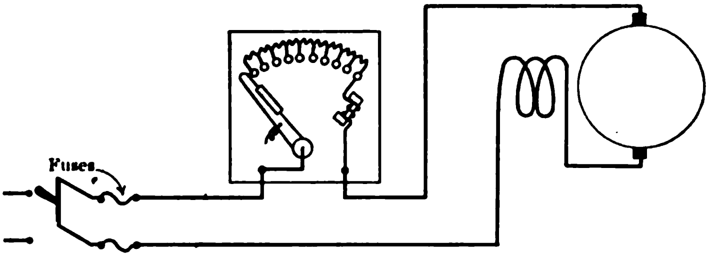
FIG. 302.—Westinghouse starting and speed adjusting rheostat.

shorter arm is pushed up by the longer arm and cuts out the armature resistance in the ordinary manner. During the starting period the field rheostat is short-circuited by the finger *S*, Fig. 302. When the starting resistance is all cut out, the shorter arm is held by the magnet and the short circuit of the field resistance is removed by this arm pushing *S* to the right. The longer arm, which has no spring, inserts resistance into the field circuit when moved backward. When the voltage goes off, the shorter arm springs back carrying the longer one with it.

In stopping a motor, the line switch should always be opened rather than throwing back the starting arm. With shunt motors, the line switch can be opened with no appreciable arc, since the motor has a back electromotive force and the field can discharge gradually through the armature. On the other hand, if the starting arm is thrown back, the field circuit is broken at the last contact button. Owing to the inductive nature of the field, this results in a hot arc which burns the contact. To prevent



(*a*) Series starter, no-voltage release



(*b*) Series starter, no-load release

Fig. 303.—Series motor starters.

the contact from being burned, a small finger breaks the arc, Fig. 302.

The series motor starter needs no shunt field connection. There are two principal types, one having a no-load voltage release, shown in Fig. 303 (*a*), and one having a no-load release, shown in Fig. 303 (*b*). In the former type, the hold-up coil is connected directly across the line and releases the arm when the voltage goes off the line. In the latter type, the hold-up coil consists of a few turns in series with the motor. When the motor current falls below the desired value, the starting arm is released. This

last type is particularly adapted to series motors where there is a possibility of the load dropping to such a low value that the motor speed may become dangerous.

Controllers are used where the operation of the motor is continually under the direct control of an operator, as in street car, crane and elevator motors. The controller must be more rugged than the starting box, since the controller is used for constant starting. stopping and reversing the motor while operating. Such controllers usually have an external resistance which is cut in and out by fingers in the controller. A shunt motor field rheostat may also be incorporated in the controller. Controllers are usually fitted with a "reverse," so that the motor may be run in either direction.

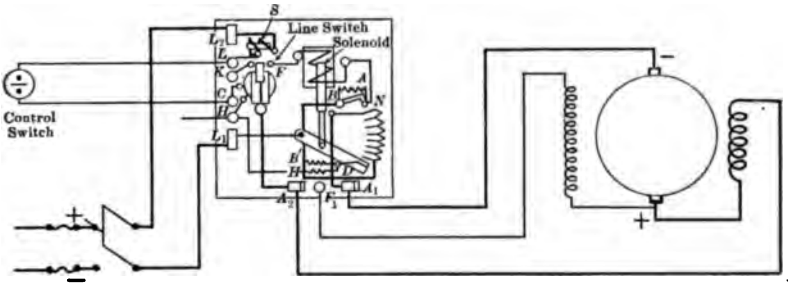


FIG. 304.—Cutler-Hammer automatic starter—dash-pot type.

Automatic starters are often used in practice. They have many advantages over the hand-operated starter. They cut out the starting resistance at a definite rate, so that the blowing of fuses and the opening of circuit breakers, due to too rapid acceleration, are avoided. In many installations where a motor is used intermittently, it may be started and stopped by merely turning a snap switch. Employees will be more likely to shut the motor down when the power is not being used, because of the ease with which starting and stopping are effected. In the larger sizes of motors, especially when extremely rapid operation is necessary as in rolling mills, automatic starters alone can give satisfactory results.

Fig. 304 shows an automatic starter of the sliding contact type, with remote control. When the control switch is closed, the solenoid S becomes energized through L_2, L, the control switch,

C, H, D, to L_1. This closes the line switch and energizes the solenoid through the auxiliary contact F, to A, B, to L_1. The energizing of this solenoid pulls up the starting arm against the action of a dash pot. The armature current flows from L_2, the switch, A_2, through the series field and armature of the motor back to A_1, to N, through the starting resistance to H, B', and L_1. As soon as the arm starts to move, the line switch solenoid circuit becomes L_2, L, the control switch, C, H, D, B', L_1. This inserts the additional resistance DB' into this circuit, reducing its current and therefore its power consumption. When

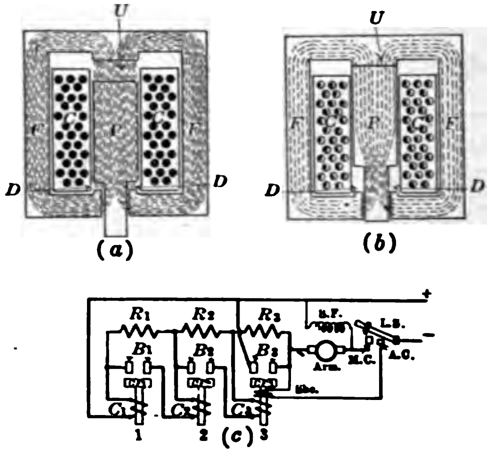


FIG. 305.—Electric Controller and Mfg. Co. automatic starter.

the starting resistance is all cut out, a brush on the arm makes contact with B, making direct connection between L_1 and A_1. At the same time the contact short-circuiting resistance A-B is opened. This reduces the current in the solenoid to a value sufficient to hold up the starting arm. The motor is stopped by opening the control switch.

By using 3- and 4-way switches, this type of controller may be operated from widely separated points. Instead of a simple snap switch, the motor may be controlled by a float switch, a pressure switch, or any other automatically operated switch.

Fig. 305 shows a simple and ingenious type of starter of the contactor type. The contactors themselves operate as follows:

Fig. 305 (*a*) and (*b*) represent a rectangular iron frame, *FF*, and plunger, *P*. The plunger, *P*, is narrower at the bottom than at the top and the narrow part of it fits loosely in an opening in the bottom of the frame, *FF*. There are two air gaps, *DD*, between the plunger, *P*, and the bottom of the frame, *FF*, and one air gap, *U*, between the plunger, *P*, and the top of the frame, *FF*. A coil is placed around the plunger, *P*, as shown in the figure, where the black circles represent the cross-sections of the wires of the coil, *CC*. If a heavy current flows through the coil, magnetic lines will stream through the plunger, *P*, across the air gap, *U*, back through the frame, *FF*, and through the narrow part of the plunger *P*, and also across the air gaps, *DD* Fig. 305 (*a*). The reason that some of the lines go through the air gaps, *DD*, is that the narrow part of the plunger, *P*, is saturated, or, in other words, it cannot easily carry any more magnetic lines. These lines, therefore, are forced to pass through the air gaps, *DD*, when a large current flows through the coil. The magnetic lines in the air gap, *U*, cause an upward pull on the plunger, but the weight of the plunger and the downward pull of the magnetic lines in the air gaps, *DD*, hold the plunger down. In Fig. 305 (*b*) everything is the same except that less current flows through the coil *CC*, with the result that there are not so many lines existing through the plunger *P*, the air gap *U*, and the frame *FF*. Most of these lines now pass through the narrow part of the plunger, but there are still a few in the air gaps, *DD*. The downward pull, due to the lines passing through the gap, *DD*, is now small and the pull in the gap, *U*, is sufficient to raise the plunger.

The operation of the switch is shown in Fig. 305 (*c*). When the line switch is closed, the current flows from the positive main through the coil C_1 of contact 1, the resistances R_1, R_2, and R_3 in series and the motor armature to the negative main. A shunt coil, *Shc*, on contactor CC_3 is also put across the line but it is not strong enough to raise the plunger of 3.

When the current falls to a sufficiently low value, the plunger *CP*1 rises, as already described, closing the contact points *B*1, which short-circuits R_1. This causes an increase of current which now passes through the coil C_2. When the current drops again, due to the motor coming up to speed, contactor *CP*2 operates, short-circuiting R_2 and causing the current to feed

through C_3. When the current drops again, C_3 operates and short-circuits all the resistances and coils so that the plungers of 1 and 2 fall back. 3 is held up by the shunt coil *Shc*.

220. Magnetic Blow-outs.—Controllers and circuit breakers are often equipped with magnetic blow-outs. Their function is to extinguish the arc, resulting from opening a circuit, so that the arc does not persist and so burn the contacts. The principle of blow-outs is as follows: The contacts between which the arc is to be broken are placed between the poles of a magnet, as shown in Fig. 306. When the contacts open, the current tends to per-

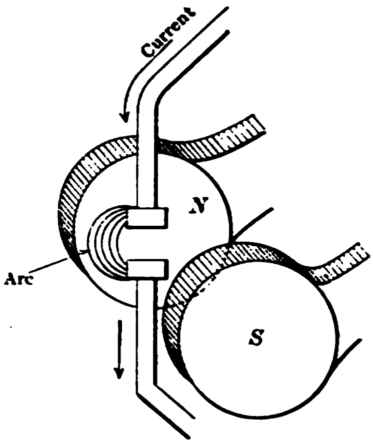


Fig. 306.—Magnetic blow-out.

sist in the form of an arc. This arc finds itself in a magnetic field so that motor action immediately follows. The arc starts to move across the field according to Fleming's left-hand rule. In doing so it draws itself out to such an extent that it is broken.

221. Resistance Units.—Starting boxes are usually designed for starting duty only. They can carry the starting current of the motor safely for the short period of starting, but they cannot carry such a current continuously. The box resistance units are usually of the type shown in Fig. 307. In the smaller types the wire is wound in the form of a helix. It may be self supporting or it may be wound on asbestos or porcelain forms, as shown in Fig. 307. In the larger types, cast-iron grids are used.

These grids are bolted together. Current lugs are clamped on at suitable points so that the desired ranges of resistance are readily obtainable.

Some types of starter are built in the form of controllers. The resistance, usually of the grid type, is designed to carry the rated current of the motor continuously so that it may be used to secure speed control.

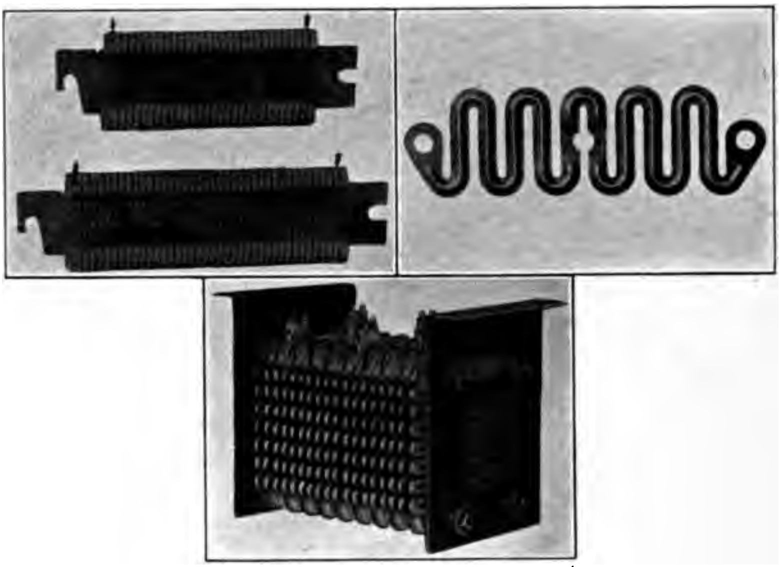


Fig. 307.—Starting box resistance units.

222. Speed Control.—In the equation for motor speed, $S = KE/\phi$, there are but two factors that can be changed to secure speed control without making changes in the motor construction. These factors are the back electromotive force E and the flux ϕ.

Armature Resistance Control.—In this method the speed control is obtained by connecting a resistance directly in series with the motor *armature*, keeping the field across the full line potential, as shown in Fig. 308 (*a*). A wide range of speed can be obtained by this method and at the same time the motor will develop any desired torque over its working range, for the torque depends only upon the flux and armature *current*.

The principal objections to this method of speed control are that an excessive amount of power is lost in the armature series resistance and the speed regulation is very poor. In Fig. 306 (b) there is shown for comparison the speed–load curves of a shunt motor with and without resistance in series with the armature. The speed–load curve with series armature resistance shows that half speed is obtained at rated load. It will be

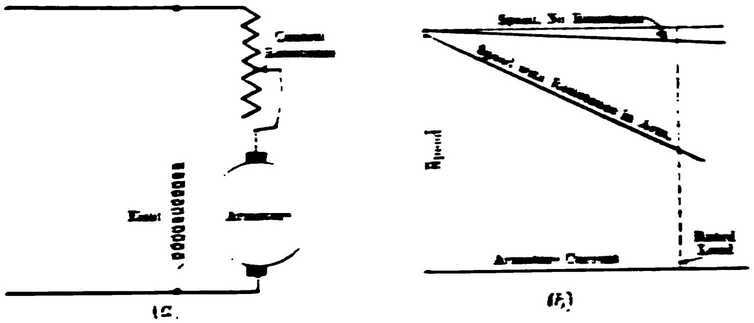


Fig. 306.—Speed control and regulation—armature-resistance method.

observed that the speed at no load rises to a value which is practically equal to the speed of the motor when there is no series armature resistance. The speed regulation with resistance is about 50 per cent. and about 50 per cent. of the power supplied to the armature is lost in the series resistance. Without series resistance the speed regulation is the usual 3 or 4 per cent.

Example.—A 220-volt, 7-hp. motor has an armature resistance of 0.25 ohm. When running without load at 1,200 r.p.m. the armature takes 6 amp. (a) What resistance should be connected in series with the armature to reduce the speed of the motor to 600 r.p.m. at its rated load of 30 amp.? (b) How much power is lost in the resistance? (c) What percentage of the power delivered to the armature circuit is delivered at the armature terminals? (d) What is the speed regulation of the armature? Neglect armature reaction.

(a) $$E_1 \text{ at no load} = 220 - 6 \times 0.25 = 218.5 \text{ volts.}$$

$$E_2 \text{ at 600 r.p.m.} = \frac{600}{1{,}200}\,218.5 = 109.3 \text{ volts.}$$

$$\text{Total } R + R_a = \frac{220 - 109.3}{30} = \frac{110.7}{30} = 3.69 \text{ ohms.}$$

Subtracting the armature resistance

$$R = 3.69 - 0.25 = 3.44 \text{ ohms.} \quad \textit{Ans.}$$

(*b*) Power lost in the series resistance

$$P_1 = (30)^2 \times 3.44 = 3{,}096 \text{ watts.} \quad \textit{Ans.}$$

(*c*) Power delivered to armature circuit

$$P_2 = 220 \times 30 = 6{,}600 \text{ watts.}$$

Power delivered to armature

$$P_3 = 6{,}600 - 3{,}096 = 3{,}504 \text{ watts.}$$

Percentage power delivered to armature

$$= \frac{3{,}504}{6{,}600} = 53.1 \text{ per cent.} \quad \textit{Ans.}$$

(*d*) Speed regulation

$$\frac{1{,}200 - 600}{1{,}200} = 50 \text{ per cent.} \quad \textit{Ans.}$$

Multi-voltage System.—In this system several different voltages are available at the armature terminals of the motor. These voltages are often supplied by a balancer set, Fig. 309.

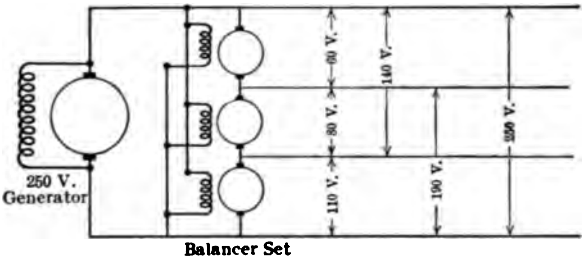


Fig. 309.—Multi-voltage speed control.

The shunt field of the motor is connected permanently across a fixed voltage and, with the 4-wire system shown, six voltages are available for the armature. Intermediate speed adjustments can be made with a limited field control. Owing to the necessity of having a balancer set, or its equivalent, and due to the large number of wires necessary, this system is little used in this country.

Ward Leonard System.—In this system, shown in Fig. 310, variable motor voltage is obtained by means of a separate generator, G, driven by a motor, M_1. By varying the field of the generator, the desired voltage across the motor terminals, M_2, is obtained. The motor field is connected across the supply mains in parallel with the fields of the other two machines.

In Fig. 310, M_1 is a motor driving generator G. G in turn supplies variable voltage to the armature of motor M_2 whose speed is to be varied. This system is very flexible and gives close adjustment of speed. The chief disadvantages are the necessity of having the two extra machines and the low over-all efficiency of the system, especially at light loads. This system has been

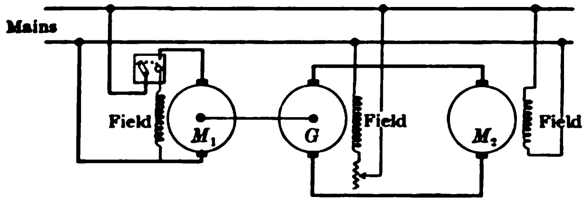


FIG. 310.—Ward Leonard system of speed control.

used extensively for turning the turrets of battleships, but is now superseded for this purpose.

Field Control.—In the foregoing methods of speed control, the armature volts have been varied. A change of speed may also be obtained by varying the flux, ϕ, by means of a field rheostat. This method is very efficient so far as power is concerned and for

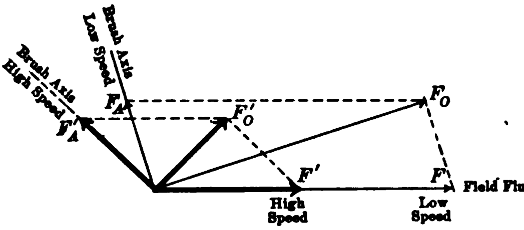


FIG. 311.—Effect of a weak field upon brush position.

any particular speed adjustment the speed regulation from no load to full load is excellent. The range of speed obtainable by this method with the ordinary motor is limited by commutation difficulties. Referring to Fig. 311, F is the field flux at low speed and F_A is the corresponding armature flux. The resultant flux is F_o. If it be attempted to double the speed of the motor by weakening its field, the new field flux will be F'. The brushes will now have to be moved farther backward so that the armature

flux will be at the position shown at F'_A. The resultant field is F'_o.

It is evident that the neutral plane has been moved backward to a considerable extent and that the armature flux is about equal to the field flux. In addition to severe sparking at the commutator, the strong armature field may so weaken the main field that the motor tends to run away. In order to eliminate the demagnetizing action due to the moving of the brushes, commutating pole motors only should be used where the speed range is large. A range of 5 to 1 in speed variation is obtainable with properly designed machines having commutating poles.

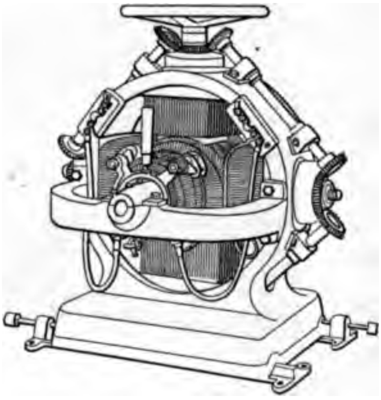


Fig. 312.—The Stow motor.

The Stow Motor.—In this type of motor, shown in Fig. 312, the field cores slide in and out of the yoke and are actuated by a hand wheel through a rod and bevel gear mechanism. By varying the length of the air gap, the flux, and therefore the speed of the motor, may be varied. As the armature reaction is reduced at the higher speeds with the increased air gaps, there is little difficulty with commutation. In other words, the ratio of field ampere-turns to armature ampere-turns does not change.

The Lincoln Motor.—In the Lincoln motor, made by the Reliance Electric and Engineering Company, the flux entering the armature is varied by moving the rotating armature in and out of the field structure, as shown in Fig. 313. As the armature is moved out of the field the length of armature conductor cutting flux is reduced. Therefore the armature must rotate faster in order to develop the requisite electromotive force. This gives a finely graduated speed control over wide ranges, ratios as high as 10 to 1 being obtained. These motors are provided with commutating poles.

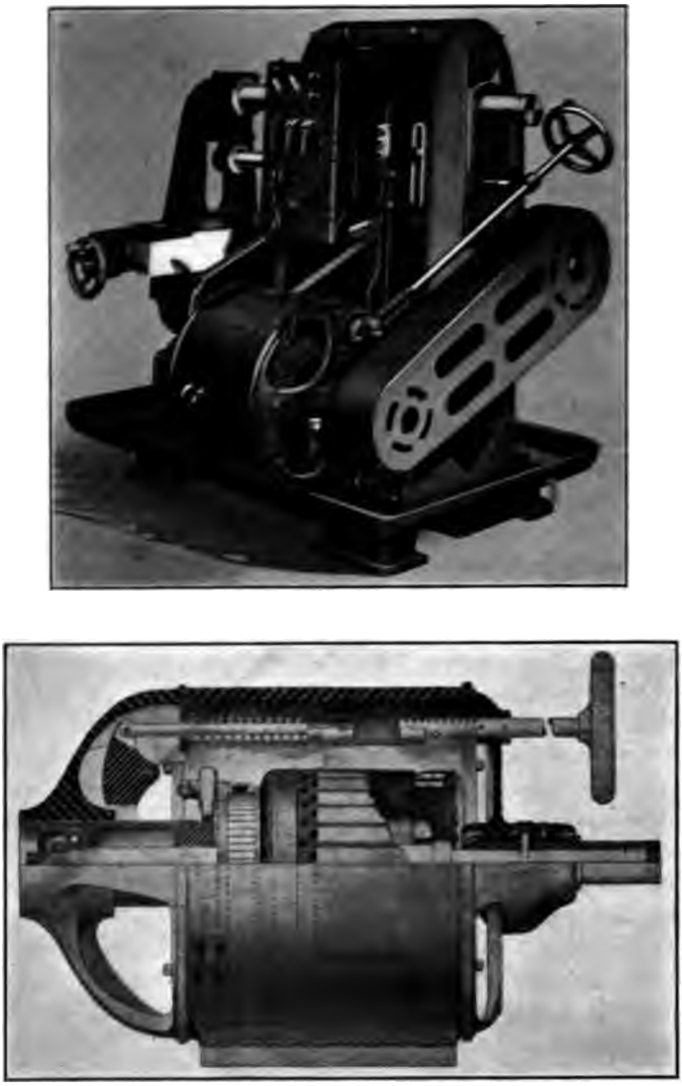
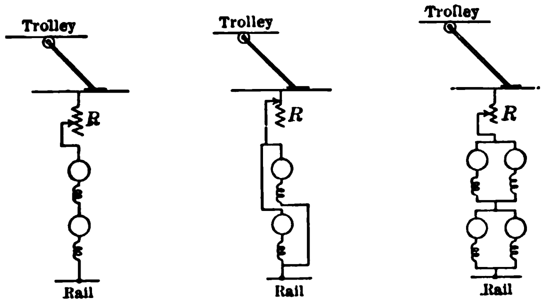


FIG. 313.—Lincoln adjustable speed motor.

223. Railway Motor Control.—In a 2-motor trolley car, two different speeds can be efficiently obtained. The motors are first connected in series through a starting resistance R as shown in Fig. 314 (a). This resistance is gradually cut out by the controller as the car comes up to speed and then each motor receives one-half the line voltage. This is the first running position. For any given value of armature current each motor will run at half its rated speed. As there is no external resistance in the circuit, the motors are operating at an efficiency very nearly equal to that obtainable with full-line voltage across the terminals of each.



(a) Starting (b) Running with Motors in Parallel (c) Starting, 4 Motors

FIG. 314.—Series-parallel control of series motors.

When it is desired to increase the speed of the car, the two motors are thrown in parallel with each other and in series with a portion of the resistance R. This resistance is gradually cut out and when the running position is reached, each motor receives full-line voltage, as shown in Fig. 314 (b).

In a 4-motor car, the motors are usually divided into two groups, each group consisting of two motors which are always in parallel with each other. In starting, these two groups are connected in series, each group taking the place of the single motor of a 2-motor car. This starting condition is shown in Fig. 314 (c). When the full-speed running position is reached, both groups are connected in parallel across the line. Each motor then receives full-line voltage.

Multiple Unit Control.—In the heavier electric cars and locomotives, the currents become so large that direct platform control is out of the question from the standpoint of the size of controller,

safety, and expense. Moreover, when cars are operated in trains, it is necessary that the motors on all the cars shall be under a single control and that they shall operate simultaneously.

In the multiple-unit system, all the heavy current switching is done by solenoid-operated contactors located beneath the car. These contactors in turn are operated by an auxiliary circuit called the train line, which runs the entire length of the train (Fig. 315). The train line is made continuous through plug and socket

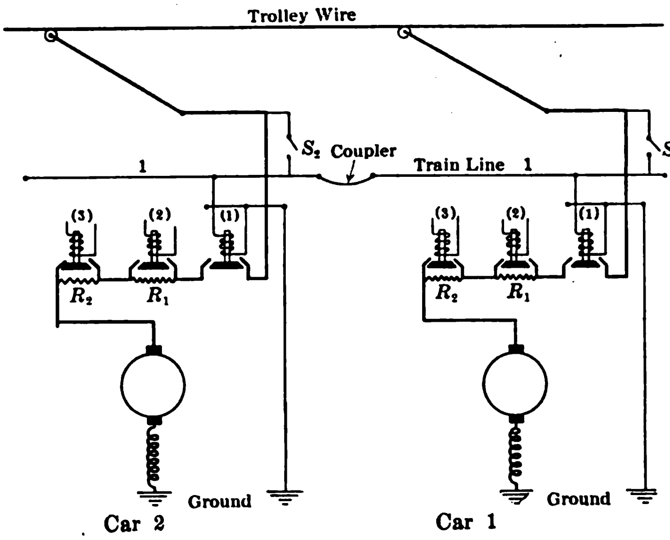


FIG. 315.—Principle of multiple-unit control.

connectors located in the car couplers. The wires of this train line receive their power through the master controller operated by the motorman. As this train line current is only of the magnitude of 2.5 amp., a small platform controller can be used. Another distinct advantage of this system is that the rate of cutting out the starting resistance during the acceleration periods is outside the control of the motorman, being accomplished by automatically operated contactors which close in sequence at the proper times. This insures uniform acceleration and eliminates the opening of the car circuit breakers and the shocks to the equipment caused by too rapid acceleration when manual operation is used.

Fig. 315 shows the underlying principle of the system, no attempt being made to give the many details which must necessarily accompany such a system. Each car has its own trolley or third rail shoe for collecting the current. A train line of small wires runs the entire length of the train, by connections being made by the use of couplers between cars. This line usually consists of six wires. Solenoids, operating contactors, are connected across the train lines. Some of the contactors are operated directly by the controller in the hands of the motorman and others operate automatically after the controller has been turned to the desired position. For example, in Fig. 315 are shown two motors, one in each car. One line of the train line is shown running betwen cars and connected by the coupler. It is assumed that the train is to be operated from car 1. If the switch S_1 in the controller of car 1 be closed, train line 1–1 becomes alive. This energizes relay (1) (1) in each car and both relays simultaneously close the motor circuits, the starting resistances R_1, R_2 being in series with each motor respectively. As the motors "pick up," the current drops and relay (2), become automatically energized and some of the starting resistance R_1, R_1 is cut out in each car. The next set of relays become energized in a similar manner, until all the starting resistance is cut out and the motors are across the line.

The above is merely an abbreviated description of the system. In the complete system there are six train lines, some of which reverse, change from series to parallel, etc. The great advantage of this sytem is that every motor on the train can be operated from either controller on any one car, that all the motors act simultaneously, the acceleration cannot exceed a certain value irrespective of the motorman, and as there are driving wheels on every car, high accelerations can be obtained. This system is also used extensively on single cars.

224. Dynamic Braking.—It is often desirable to brake a motor when it is being driven by its load, as in the case of descending elevators, cranes, etc. This is often done by using a controller which leaves the field connected across the line and at the same time puts a resistance load across the armature terminals. This produces generator action and therefore retards the armature. If series motors are used, their fields must be connected across

the line in series with a resistance. Such braking is not effective for completely stopping the motor armature, as the braking action ceases when the armature is stationary.

Dynamic braking for a series motor is shown in Fig. 316. In (*a*), which shows the holding or "off" position, the motor is totally disconnected from the line. The solenoid of the mechanical brake becomes de-energized, resulting in the brake being set. (See Fig. 33, page 23.) In (*b*), the brake solenoid and the series field are connected across the line in series with a resistance.

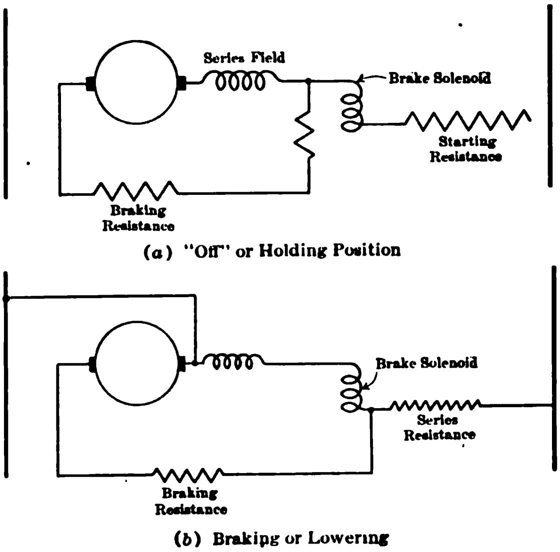


FIG. 316.—Dynamic braking.

The armature has a resistance connected across its terminals through the brake solenoid and series field on one side. The brake is released, the armature acts as a generator sending current through the braking resistance and so is retarded.

Regenerative braking is based on this same principle, except that the power is returned to the line rather than wasted in resistance. Such a system is used on the electric locomotives of the Chicago, Milwaukee and St. Paul Railroad.

225. Motor Testing—Prony Brake.—It is often necessary to determine the efficiency of a motor at certain definite loads and

frequently over its entire range of operation. A knowledge of the efficiency may be necessary, as in the case of an acceptance test; further, the motor may be used as a power-measuring device for determining the power taken by some machine, such as a generator, pump, blower, etc. Knowing the motor input, which can be measured with an ammeter and a voltmeter, and also

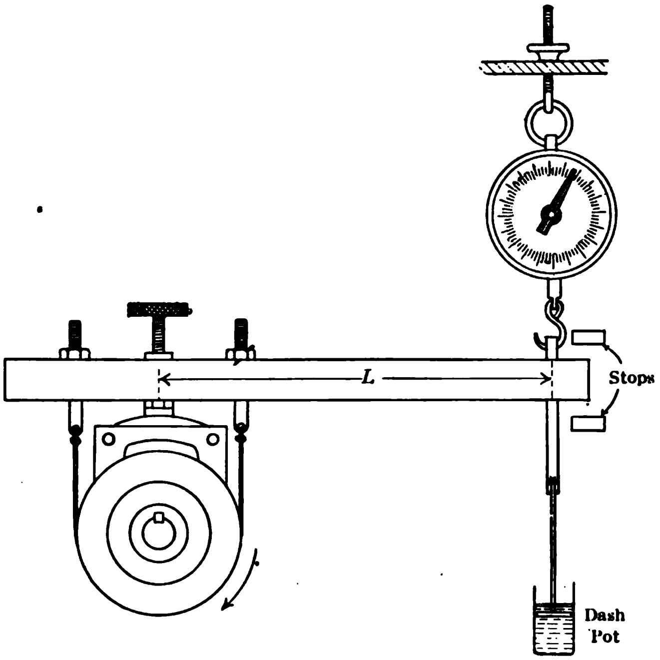


FIG. 317.—Typical prony brake.

knowing the motor efficiency, the output for any given input can be computed. This output will be the power delivered to the generator, the pump, etc.

The most common method of making direct measurements of efficiency in motors up to about 50 hp. is to use a prony brake. Such brakes are made in various forms. One typical form is shown in Fig. 317. It consists of a wooden arm of the proper length, a canvas brake band and a hand wheel for applying tension to the brake band. By means of this hand wheel the motor

load can be controlled. An oil dash pot is advisable, to prevent vibrations of the brake arm.

The balance measures the pull on the arm due to the rotation of the drum, plus the dead weight of the arm. By multiplying the net balance reading by the distance *L*, the torque of the motor can be determined.

There are two simple methods for determining the dead weight of the brake arm. The brake band is loosened and some sort of knife edge, such as a pencil, is placed between the top of the drum and the brake carriage. This acts as a substantially frictionless fulcrum, so that the balance registers the dead weight of the arm alone. Another and easier way is to turn the drum toward the balance by hand, stop and read the balance. In this case the friction of the brake causes the balance to read too high. If this operation be repeated by rotating the drum in the opposite direction, the balance reading will be too low, due to the same friction. The average of these two balance readings will give very nearly the correct value for the dead weight of the arm.

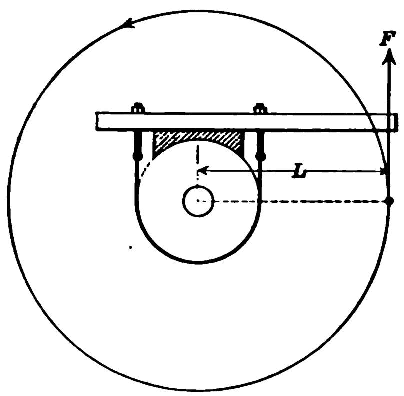


Fig. 318.—Work developed by a prony brake.

Brakes of this type are cooled ordinarily by pouring water into the hollow brake drum. This water prevents the drum from becoming excessively hot. As the maximum temperature which water can reach in the open air is 100° C., the drum temperature cannot much exceed this. The heat developed in the drum is utilized in converting the water into steam. As a considerable number of heat units are required to convert a small amount of water into steam, a moderate amount of water will keep the drum comparatively cool.

To determine the equation for the horse power developed by such a brake, consider Fig. 318. Let *F* be the net force in pounds acting at a perpendicular distance *L* from the center of the drum.

First assume that the drum is stationary and that the arm is pulled around the drum by means of the force F. The distance per revolution through which the force F acts is $2\pi L$. The work done in one revolution of this arm around the drum is the force times the distance $= F(2\pi L)$.

The work done is S revolutions $= F(2\pi L)S$.

If S is the revolutions per minute, the horse power

$$\text{Hp.} = \frac{2\pi(FL)S}{33,000}$$

but FL is the torque T, therefore

$$\text{Hp.} = \frac{2\pi TS}{33,000}$$

$$\frac{2\pi}{33,000} = 0.00019$$

Therefore $$\text{Hp.} = 0.00019\ TS \qquad (115)$$

Obviously, the same amount of work is done on the brake surface whether the drum is stationary and the arm rotates or the arm is stationary and the drum rotates. Therefore, equation (115) applies to brakes of the type shown in Figs. 317 and 318. It will be noted that the horse power is independent of the diameter of the drum.

Example.—In a brake test of a shunt motor, the ammeter and voltmeter measuring the input read 34 amp., 220 volts. The speed of the motor is found to be 910 r.p.m. and the balance on a 2-ft. brake arm reads 26.2 lb. The dead weight of the arm is found to be +2.4 lb. (*a*) What is the output of the motor? (*b*) What is its efficiency at this particular load?

(*a*) Net reading of balance $= 26.2 - 2.4 = 23.8$ lb.

The torque $T = 23.8 \times 2 = 47.6$ lb.-ft.

Hp. output $= 0.00019 \times 47.6 \times 910 = 8.23$ hp. *Ans.*

(*b*) Output $= 8.23 \times 746 = 6,140$ watts.

Input $= 220 \times 34 = 7,480$ watts.

$$\text{Efficiency } \eta = \frac{6,140}{7,480} 100 = 82.1 \text{ per cent. } \textit{Ans.}$$

In brakes of this type, the brake arm should be kept approximately level.

Another simple type of brake is the rope brake shown in Fig. 319. A rope is given a turn and a half around a drum and the two free ends are each held by a spring balance. The larger bal-

ance is on the end of the rope which is being pulled downward by the rotation of the drum. Let F_1 be the reading of the larger balance and F_2 that of the smaller balance. As F_1 and F_2 pull in opposite directions with respect to the rotation of the drum, the net pull at the drum periphery is $F_1 - F_2$.

The torque in lb.-ft. is

$$T = (F_1 - F_2)\,R$$

where R is the radius of the pulley in *feet*.

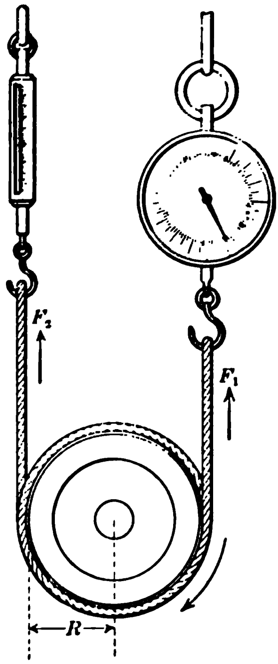


FIG. 319. Rope brake.

FIG. 320.—Jagabi Tachoscope.

Example.—In a rope brake of the type shown in Fig. 319, F_1 = 32.4 lb. and F_2 = 8.2 lb. The drum is 10 in. in diameter. If the motor speed is 1,400 r.p.m., what horse power does the motor develop?

The torque

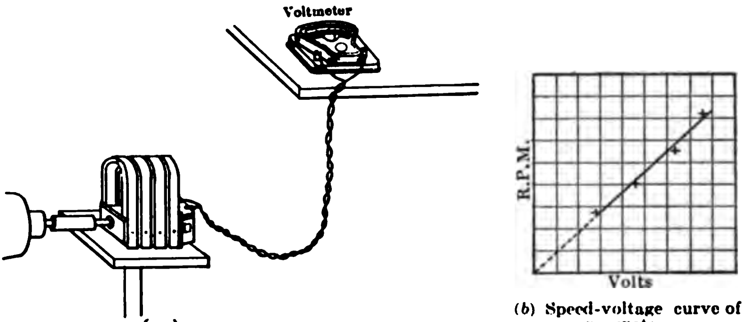
$$T = (32.4 - 8.2)\,\frac{5}{12} = 24.2 \times 5/12 = 10.08\text{ lb.}$$

The horse power

$$\text{Hp.} = 0.00019 \times 10.08 \times 1{,}400 = 2.68. \quad \textit{Ans.}$$

226. Measurement of Speed.—The measurement of the speed of machines is as a rule much simpler than the measurement of torque. The most common method is to use a simple revolution counter having a conical rubber tip which fits into the counter-sink of the shaft. The Veeder type is a convenient form of revolution counter. The revolutions are recorded directly on the counter. As this counter cannot be set to zero, the actual speed must be found by subtracting the counter reading before from that after the measurement.

The Jagabi tachoscope, Fig. 320, is a combination of speed counter and stop watch. The spindle may be inserted in the counter-sink of the shaft without recording. A little pressure,



(a)

(b) Speed-voltage curve of magneto.

FIG. 321.—Speed measurement with magneto and voltmeter.

however, causes the counter and stop watch to start simultaneously. They also stop simultaneously when the pressure on the tachoscope is removed. Measurements made with this type of instrument are free from personal error.

Tachometers indicate the instantaneous value of speed. There are mechanical tachometers, where the indicator is actuated by centrifugal action. This type should be carefully checked at each occasion of use, as it is especially subject to error after having been in service for some time.

A simple and convenient type of tachometer is the combination of a direct current magneto and a voltmeter, as shown in Fig. 321 (*a*). In the magneto the flux is produced by permanent magnets and so is constant. Therefore, the voltage induced in the magneto armature is directly proportional to the speed. If this voltage

23

be measured with a voltmeter, the voltmeter reading multiplied by a constant gives the speed directly. The relation of speed to volts may be plotted as shown in Fig. 321(*b*) and the speed read directly from the plot. This plot is ordinarily a straight line through the origin, which makes one point accurately determined. It is convenient to attach the magneto to the shaft of the machine whose speed is being measured, by a piece of rubber tubing. It is usually necessary to thread a small stud into the end of the shaft whose speed is to be measured, as shown in Fig. 321 (*a*).

CHAPTER XIII

LOSSES; EFFICIENCY; OPERATION

228. Dynamo Losses.—A certain portion of the energy delivered to any motor or generator is lost within the machine itself, being converted into heat, and therefore wasted. This represents not only energy lost, but has the further objection that it heats the machine and so limits its output. If the energy loss in the machine becomes excessive, the resulting temperature rise may injure the insulation by carbonizing it.

As a motor and a generator are similar, they have the same types of losses throughout. Therefore, the following applies to either a motor or a generator.

COPPER LOSSES

Armature.—The armature windings have a certain resistance and when current flows through them a certain amount of

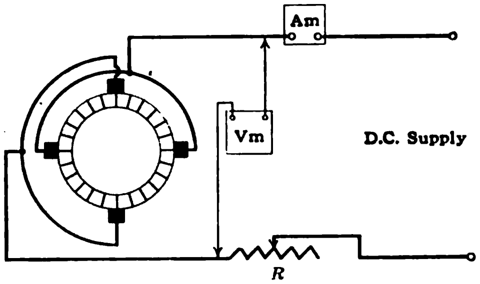


FIG. 322.—Measurement of armature resistance.

power must be lost. In addition to the loss in the armature copper, there is an electrical loss in the brushes and in the commutator. Let this total power loss be P_a. Then,

$$P_a = I_a^2 R_a \tag{116}$$

where I_a is the armature current and R_a is the armature resistance measured between the terminals of the machine and including, therefore, the brushes and their contact resistance. This contact resistance is not exactly constant, but little error is made in assuming it to be so. (See Par. 196.) The resistance measurement is often made by the connections shown in Fig. 322. The

resistance R is inserted to limit the current flowing through the stationary armature. (See Par. 118. The measurement should be made with the armature in three or four different positions in order to obtain an average value of resistance. As the low reading scale of the voltmeter is ordinarily used in making this measurement, the instrument may be injured on opening the circuit by the rise of voltage due to the self-inductance of the armature. Therefore, the voltmeter should be disconnected when the circuit is being opened or closed and when the armature is being turned.

Shunt Field.—The field takes a current I_f at the terminal voltage V of the generator or motor. Therefore, the power lost in the field is

$$P_f = VI_f \tag{117}$$

This includes the power lost in the field rheostat as this is chargeable to the field circuit.

Series Field.—The series field loss is

$$P_s = I_s^2R_s \tag{118}$$

where I_s is the series field current, which may or may not be equal to the armature current, depending on whether the machine is long or short shunt.

R_s is the series field resistance. If a series field shunt or diverter is used, R_s is the equivalent parallel resistance of this diverter and the series field and I_s is the current of the series field plus that of the diverter.

The losses in the commutating pole circuit are determined in the same way as are those of the series field.

The foregoing losses are all copper losses and can be either measured directly or calculated with a high degree of precision from instrument readings.

IRON LOSSES

Eddy Currents.—As the armature iron rotates in the same magnetic field as the copper conductors, voltages are also induced in this iron. As the iron is a good conductor of electricity and the current paths are short and of large cross-section, large currents would be set up in the armature iron were it a solid mass as shown in Fig. 323 (a). These currents represent an excessive

power loss which could not be tolerated in a commercial machine. By laminating the armature iron in the manner indicated in Fig. 323(*b*), the paths of these currents are broken up and their magnitude is reduced to a very low value. Laminating does not entirely eliminate these eddy current losses, but it does reduce them to a small value. It will be noted that although the laminations

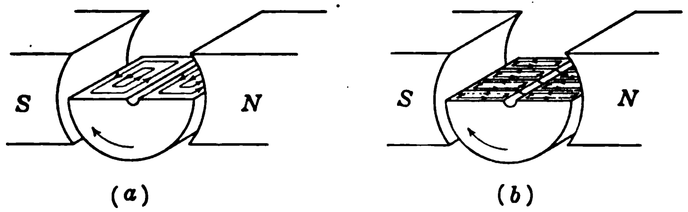


FIG. 323.—Eddy currents in armature iron without and with laminations.

break up the eddy current paths, they do not interpose reluctance in the magnetic circuit, since they are parallel to the direction of the magnetic flux.

These eddy currents are proportional to both the speed and the flux. As the loss varies as the square of the current (I^2R), the eddy current *loss varies* as the *square* of both the speed and the flux.

Example.—The eddy current loss in a certain machine is 600 watts when the total flux is 2,000,000 lines per pole and the speed is 800 r.p.m. What is the loss when the flux is increased to 2,500,000 lines and the speed increased to 1,200 r.p.m.?

$$P_e = 600 \times \left(\frac{2,500,000}{2,000,000}\right)^2 \times \left(\frac{1,200}{800}\right)^2 = 2,100 \text{ watts. } Ans.$$

Hysteresis.—It was shown in Chapter VIII that when iron is carried through a cycle of magnetization (Par. 143) there results an energy loss proportional to the area of the hysteresis loop. The iron in an armature undergoes a similar cyclic change of magnetization when the armature rotates. Consider the small section of the armature iron at (*a*), Fig. 324, when it happens to be under a north pole. This small section has a north and a south pole at its ends. When the section reaches position (*b*) its poles have become reversed, as shown. Obviously, nearly all the armature

iron is continually going through similar cycles of magnetic reversals. Therefore, there results a hysteresis loss in the armature iron as the armature rotates. This loss is directly proportional to the speed and is proportional to the 1.6 power of the maximum flux density, by the Steinmetz formula, (equation 72, page 183). Laminating the iron does *not* affect the hysteresis loss.

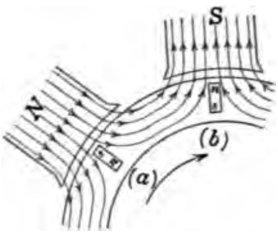


FIG. 324.—Reversal of magnetic flux in armature iron.

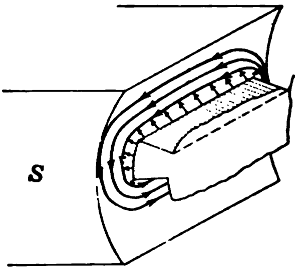


FIG. 325.—Pole face loss due to tufts of flux from teeth.

Pole Face Loss.—The flux enters and leaves the armature in tufts through the teeth as has already been shown (Fig. 40, Chap. II). As these tufts of flux pass across the pole face, they produce flux pulsations in the pole face. These pulsations set up eddy currents in the pole face, as shown in Fig. 325. This results in a power loss. A hysteresis loss also accompanies these flux pulsations. These combined losses are some function of the flux and of the speed. They are reduced, being in part due to eddy currents, by laminating the pole faces. (See Fig. 221.)

FRICTION LOSSES

These losses consist of bearing friction, brush friction and windage, and all are functions of the speed.

SUMMARY

The foregoing losses may be summarized as follows:

Copper losses:

Armature $I_a^2R_a$
Shunt field VI_f
Series field $I_s^2R_s$

Stray power
- Iron losses (armature and pole face):
  - Eddy current—function of flux and speed.
  - Hysteresis—function of flux and speed.
- Friction losses (bearings, brushes, windage)—function of speed.

The copper losses can be accurately measured or can be calculated. The iron and friction losses can neither be so accurately calculated nor so readily measured as separate losses. Moreover, since they are all some function of the flux, or speed, or both, these losses are combined and are called *stray losses;* the power that they represent being called *stray power.*

As stray power is a function of the speed and the flux only, it will be constant in a given machine provided the flux and the speed be kept constant. Therefore, no matter what the load is, the stray power does not change unless either the flux or the speed changes.

In distinction to the copper losses the stray power is all supplied *mechanically.* For instance, in a motor, a mechanical torque is required to supply these losses, making the torque available at the pulley less than that developed by the armature. In a generator these losses are supplied by the prime mover and not by the generator itself. On the other hand, the electrical losses are supplied by the generator itself.

229. Efficiency.—The efficiency of a machine is the ratio of output to input. Thus:

$$\text{Eff.} = \frac{\text{output}}{\text{input}}$$

This may also be written in either of the following ways:

$$\text{Eff.} = \frac{\text{output}}{\text{output} + \text{losses}} \tag{119}$$

$$\text{Eff.} = \frac{\text{input} - \text{losses}}{\text{input}} \tag{120}$$

Therefore, if the losses in a machine be known, the efficiency may be found for any given input or output.

Example.—A shunt motor takes 40 amp. at 220 volts. The total motor losses are 1,800 watts. What is the motor efficiency?

Using equation (120)

$$\text{Eff.} = \frac{(220 \times 40) - 1{,}800}{220 \times 40} = 79.6 \text{ per cent.} \quad \textit{Ans.}$$

As electrical units rather than mechanical quantities are ordinarily used in efficiency determinations, equation (119) is used for generators (output is electrical) and equation (120) for motors (input is electrical).

230. Efficiencies of Motors and Generators.—The efficiency of electrical apparatus is high as a rule. For instance, a 1-hp. motor has an efficiency of about 65 per cent.; a 5-hp. 75 per cent.; a 10-hp. 82 per cent., and a 20-hp. 88 or 89 per cent. A 500-kw. machine may have an efficiency of 94 per cent.

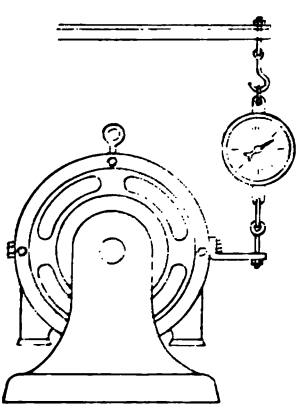


FIG. 326.—Cradle dynamometer.

The efficiency of a motor may be determined from simultaneous measurements of its input and its output as was shown in Par. 225, where a prony brake was used.

Theoretically, the efficiency of a generator may be determined in a similar manner by measurements of its input and output. The output is readily measured with an ammeter and a voltmeter. The input, however, is very difficult to measure. The difficulty lies in the measurement of the torque transmitted to the generator. Torsion dynamometers have been devised but they are unsatisfactory as a rule. The generator may be suspended in a "cradle," as shown in Fig. 326. The ends of the generator shaft are supported in bearings, so that the frame is free to turn. The torque is determined by measuring the torque necessary to prevent the frame's turning. Such a cradle is expensive, is not readily adaptable to all generators and necessitates the generator shafts' protruding beyond both generator bearings.

In any direct measurement of efficiency any percentage error in the measurement of either output or input introduces the same percentage error into the efficiency.

In the direct measurement of efficiency the power necessary for the test must be equal to the rating of the machine. In addition to *supplying* this power there must be means for *ab-*

sorbing it. This is not a serious matter with small machines, but when large machines are tested, supplying and absorbing the necessary power may be difficult, if not quite impossible.

Because of the foregoing reasons, it is often desirable and even necessary to obtain the efficiency by determining the losses.

Example.—A 250-kw. 230-volt d.c. generator is delivering 800 amp. at 230 volts. The field current is 20 amp. The armature resistance is 0.005 ohm and the series field resistance is 0.002 ohm. The stray power at this load is 2,500 watts. The generator is connected long shunt. What is the generator efficiency at this load?

Output = 230 × 800 = 184,000 watts.

Sh. field loss	= 230 × 20	= 4,600 watts
Armature loss	= $820^2 \times 0.005$	= 3,360 watts
Ser. field loss	= $820^2 \times 0.002$	= 1,340 watts
Stray power		= 2,500 watts
	Total loss	= 11,800 watts

$$\text{Eff.} = \frac{184{,}000}{184{,}000 + 11{,}800} = \frac{184{,}000}{195{,}800} = 94 \text{ per cent. } \mathit{Ans.}$$

231. Measurement of Stray Power.—It is necessary merely to duplicate the flux and the speed in a motor or a generator in order to duplicate the stray power loss. As the speed from equation (111) is $S = KE/\phi$, it is only necessary to duplicate the speed S and the electromotive force E in order to obtain the proper value of ϕ.

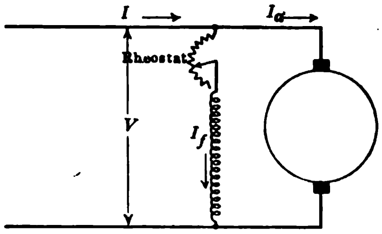


FIG. 327.—Determination of stray power in a dynamo.

To measure stray power, the machine, whether it be a motor or a generator, is run light (without load) as a motor, as shown in Fig. 327. The field is connected across the line in series with a rheostat.

The total power input to the machine is:

$$VI = V(I_a + I_f) = VI_a + VI_f$$

This power is distributed as follows: Some goes to supply the field loss, some supplies the armature $I_a^2R_a$ loss and the remainder is the stray power, S.P., the output being zero. Therefore

$$VI_a + VI_f = VI_f + I_a^2R_a + \text{S.P.}$$

$$\text{S.P.} = VI_a - I_a^2R_a \qquad (121)$$

The *stray power* is equal to the total input to the *armature* minus the armature resistance loss.

Example. A shunt generator when running light as a motor takes 12 amp. from 115-volt mains. The field current is 7 amp. and the armature resistance is 0.05 ohm. What is the stray power loss of the machine at this particular value of flux and speed?

The armature current $I_a = 12 - 7 = 5$ amp.

The stray power, $S.P. = 115 \times 5 - 5^2 \, 0.05 =$

$575 - 1.25 = 574$ watts. *Ans.*

It will be observed that the armature $I_a^2R_a$ is negligible in this instance.

Assume that the above generator is delivering 100 amp. at 110 volts at 1,000 r.p.m. The field current is 7 amp. It is desired to determine the value of its stray power under these conditions.

If the full-load electromotive force E and speed S be duplicated when the generator is running light, the stray power will be the same in both cases. When the machine is running light as a motor the stray power is readily measured as follows: When carrying the above load, the induced emf.

$$E = 110 + (107 \times 0.03) = 113.2 \text{ volts}$$
$$S = 1{,}000 \text{ r.p.m.}$$

To make these adjustments of E and S, the generator is run as a motor, connected as shown in Fig. 328. A rheostat R and an ammeter are connected directly in the armature circuit and a voltmeter is connected directly across the armature terminals. The rheostat R is first adjusted so that V_1 = 113.2 volts, the small armature drop at this load being negligible. The field rheostat is then adjusted to give a speed of 1,000 r.p.m. The machine is now operating at the same value of speed and flux as it did under load. Therefore, the stray power is the same in the two cases and is equal to $V_1I_a - I_a^2R_a$.

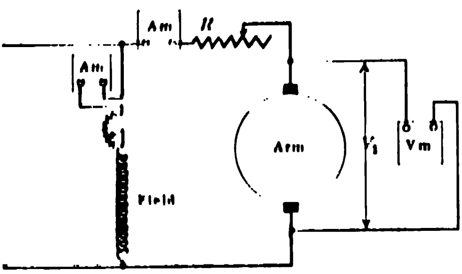


Fig. 328. Connections for stray power measurement.

As an example, assume that the current I_a is 4.8 amp. and V_1 = 113.2 volts. (This neglects the small drop in the armature, 4.8 × 0.03.) The stray power

$$\text{S.P.} = 113.2 \times 4.8 - (4.8)^2\,0.03 = 543 \text{ watts.} \quad \textit{Ans.}$$

The efficiency of the generator can now be determined.

$$\text{Output under load} = 110 \times 100 = 11{,}000 \text{ watts}$$

$$\begin{aligned} I_a{}^2R_a &= (100 + 7)^2\,0.03 = 344 \text{ watts} \\ VI_f &= 110 \times 7 = 770 \text{ watts} \\ \text{S.P.} &= \ 543 \text{ watts} \\ &\text{Total loss} = 1{,}657 \text{ watts} \end{aligned}$$

$$\text{Eff.} = \frac{11{,}000}{11{,}000 + 1{,}657} = \frac{11{,}000}{12{,}660} = 86.8 \text{ per cent.} \quad \textit{Ans.}$$

232. Stray Power Curves.—It is sometimes desired to determine the stray power of a machine over a considerable range, in order to have sufficient data for obtaining the stray power under various operating conditions. Stray power is a function of two variables, flux and speed, and a single curve cannot express the relationship under all conditions. To plot the relation, one quantity, either flux or speed, is held constant and the other is varied. Because it is more convenient, the flux is usually held constant and the speed is varied, the connections being shown in Fig. 328. The flux is held constant by means of the field rheostat and the speed is varied by means of the rheostat R in the armature circuit.

Since the induced voltage in a machine is $E = K\phi S$, the flux

$$\phi = \frac{1}{K}\frac{E}{S} \tag{122}$$

That is, the flux is equal to a constant multiplied by the ratio of voltage to speed. The field current can be used without great error in determining the flux. If this is done, the errors introduced are that the flux under light and under full load may be different for the same value of field current, owing to armature reaction; and the flux for a given value of field current may vary due to hysteresis. Therefore, if the field current instead of the flux is used to determine stray power, the stray power may be too large with the machine running light owing to armature reaction. This is in part compensated by the fact that the flux

"peaks" under load so that the loss for any value of total flux is increased, even though the average flux be the same. (See Fig. 244.)

In a stray power run, the field current may be held at a definite value and the speed varied over the probable working range of the machine. The field current may then be adjusted to another value and the run repeated. At least three values

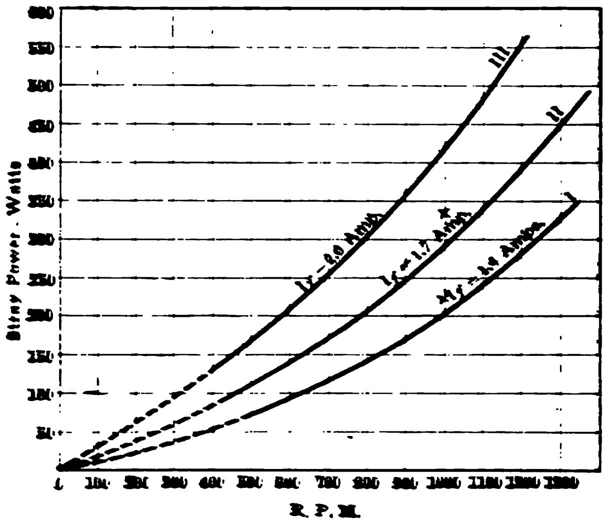


Fig. 329.—Typical stray power curves.

of field current should be used, the maximum and the minimum value under which the machine is likely to operate and an intermediate value. Curves similar to those shown in Fig. 329 are obtained in this manner.

Example.—The curves of Fig. 329 were obtained from a 10-kw. 230-volt generator by the method just described, the generator being run as a motor when these curves were obtained. The rated current of this machine is [illegible] amp. and its armature resistance is 0.14 ohm.

Determine its efficiency as a generator at half load and at rated load, the voltage being the same in each case, the respective values of field current being [illegible] and [illegible] amp. The speed is constant at 1,000 r.p.m.

At half load.

$$I = [illegible] = 21.8 \text{ amp.}$$

$$I_a = [illegible] = 23.5 \text{ amp.}$$

$$I_a^2 R_a = [illegible] = 77 \text{ watts}$$

$$[illegible] \; 230 \times [illegible] = [illegible] \text{ watts}$$

From Fig. 329, on the 1,000 r.p.m. ordinate, one-third the distance from curve I to curve II (= 1.5 amp.), the stray power is found to be 230 watts. The efficiency at this load is:

$$\text{Eff.} = \frac{230 \times 21.8}{230 \times 21.8 + 76 + 345 + 230} = \frac{5,000}{5,650} = 88.5 \text{ per cent.} \quad \textit{Ans.}$$

At rated load,

$$I = 43.5$$
$$I_a = 43.5 + 1.8 = 45.3 \text{ amp.}$$
$$I_a{}^2R_a = (45.3)^2\, 0.14 = 287 \text{ watts}$$
$$VI_f = 230 \times 1.8 = 414 \text{ watts.}$$

In Fig. 329, on the 1,000 r.p.m. ordinate, one-third the distance from curve II to curve III, corresponding to 1.8 amp., the stray power is found to be 330 watts.

$$\text{Eff.} = \frac{230 \times 43.5}{230 \times 43.5 + 287 + 414 + 330} = \frac{10,000}{11,030} = 90.7 \text{ per cent.} \quad \textit{Ans.}$$

Assume that it is desired to determine the efficiency of this machine when running as a motor at 900 r.p.m. and taking 45 amp. at 230 volts from the line. Under these conditions the field current is found to be 1.6 amp.

$$I_a = 45 - 1.6 = 43.4 \text{ amp.}$$
$$I_a{}^2R_a = (43.4)^2\, 0.14 = 264 \text{ watts}$$
$$VI_f = 230 \times 1.6 = 368 \text{ watts}$$

On the 900 r.p.m. ordinate, Fig. 329, two-thirds the distance from curve I to curve II (= 1.6 amp.), the stray power is found to be 225 watts.

$$\text{Eff.} = \frac{230 \times 45 - 264 - 368 - 225}{230 \times 45} = \frac{9,490}{10,350} = 91.7 \text{ per cent.} \quad \textit{Ans.}$$

It is also possible to determine the stray power of a machine by driving it without load by means of a smaller machine whose efficiency is known. In using this method it is possible to separate the friction and windage losses from the core loss by measuring the power delivered to the machine when the field circuit is closed and again when it is opened.

233. Opposition Test—Kapp Method.—The objection to the foregoing stray power method of measuring losses is that the machine is not under load when the losses are being measured, so their values may be in error. If two similar machines are available, their losses may be determined when both machines are loaded, and yet the line supplies only the *losses* of the two machines. The connections for making such a test are shown in Fig. 330.

The two similar machines are coupled together mechanically and are then connected to the line, as shown. The motor should have a starting box. Five ammeters are used, one in each field, one in each armature circuit and one in the line supplying the two armatures. The fields are connected directly to the line so that their currents are not indicated by the ammeter A_1.

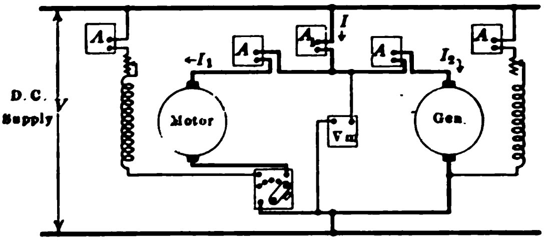


FIG. 330.—Kapp opposition method for determining losses.

The operation of the set is as follows: The motor supplies mechanical power to the generator. This in turn supplies electrical power to the motor. The power delivered by the generator is less than that required by the motor, owing to the losses in the two machines. Therefore, this deficit must be made up by the line which supplies the current I.

The total input to the two armatures is VI.

This power is distributed as follows:

Motor armature loss $= I_1^2R_1$
Generator armature loss $= I_2^2R_2$
Motor stray power
Generator stray power

where R_1 and R_2 are the motor and generator armature resistances.

As the generator field is necessarily stronger than that of the motor, because it requires the higher internal voltage, its stray power will be greater than that of the motor, as stray power increases with increase of flux. As a close approximation, the total stray power loss may be divided between the two machines in proportion to their induced voltages.

Let E_1 equal the motor induced volts and E_2 the generator induced volts.

$$E_1 = V - I_1R_1$$
$$E_2 = V + I_2R_2$$

Let P_1 and P_2 be the values of stray power in the two machines. Then:

$$\frac{P_1}{P_2} = \frac{E_1}{E_2} \qquad (123)$$

The total input to the two machines goes to supply their armature and stray power losses, because the output of the system is zero and the field power is supplied separately. By subtracting the armature losses from the input, the total stray power ($P_1 + P_2$) remains.

That is:

$$P_1 + P_2 = VI - I_1^2R_1 - I_2^2R_2$$

The field losses are measured directly by the ammeter in each field circuit.

The advantages of this method are that each machine is operating under load conditions; the regulation of each machine may be determined; the line need supply only the losses.

The principal disadvantage is that it requires two similar machines. The assumptions made in regard to the stray power distribution may be slightly in error.

The machines are brought into operation by first starting the motor with the starting box. The generator voltage is then made equal to the motor terminal voltage and the generator terminals are then connected directly across the motor terminals, just as generators are connected in parallel. Care should be taken that the correct polarity is observed. The generator field is then strengthened and the motor field weakened until the desired conditions of load and speed are obtained.

Example.—Two similar 120-volt, 7.5-hp. motors are connected in the manner shown in Fig. 330. The armature resistance of each is 0.12 ohm. The fields are so adjusted that the motor current I_1 is 57 amp., and the generator current I_2 is 45 amp. Under these conditions the line is supplying a current I of 12 amp. at 120 volts. Find the stray power of each machine under these conditions of load.

The power supplied by the line

$$\begin{aligned} P &= 120 \times 12 = 1{,}440 \text{ watts} \\ I_1^2R_1 &= 57^2 \times 0.12 = 390 \text{ watts} \\ I_2^2R_2 &= 45^2 \times 0.12 = 243 \text{ watts} \\ &\text{Total} \quad = 633 \text{ watts.} \end{aligned}$$

Total stray power = 1,440 − 633 = 807 watts.

$$E_1 = 120 - (57 \times 0.12) = 113.2 \text{ volts}$$
$$E_2 = 120 + (45 \times 0.12) = 125.4 \text{ volts.}$$

The motor stray power

$$P_1 = \frac{113.2}{113.2 + 125.4} \cdot 807 = 383 \text{ watts.}$$

The generator stray power

$$P_2 = \frac{125.4}{113.2 + 125.4} \cdot 807 = 424 \text{ watts.}$$

Knowing the stray power, and the armature and field losses, the efficiency is readily calculated.

234. Ratings and Heating.—Practically all power apparatus, whether it be steam engines, gas engines or dynamos, has definite power ratings. These ratings are determined by the manufacturer and are supposed to give the power which the apparatus can safely or efficiently deliver. It is interesting to consider what, in general, determines the rating of various power devices.

Both a steam engine and a steam turbine are usually rated at the load for which their *efficiency* is a maximum. These two types of prime mover can carry a high overload without difficulty. Ordinarily, they can carry at least 100 per cent. overload easily, but at reduced efficiency.

Owing to their excessive weights and costs, large gas engines are usually rated as high as possible, which is near the point at which they cease to operate. Their thermal efficiency is ordinarily so much greater than that of the steam engine or turbine that the question of weight is more important than the question of efficiency.

Electrical apparatus is usually rated at the load which it can safely carry without overheating. (Commutation may at times limit the output of direct current machines.)

If the temperature of electrical apparatus becomes too high, the cotton insulation upon the armature and the field conductors, and the insulating varnishes, become carbonized and brittle. This may result ultimately in grounds and short circuits within the machine. The A. I. E. E. Standardization Rules specify safe temperature limits as follows:

(*A*)	Cotton, silk, paper, all impregnated; enameled wire	105° C.
	Above untreated	95° C.
(*B*)	Mica, asbestos	125° C.
(*C*)	Pure mica, quartz, etc.	No limits specified.

It is very important, therefore, to be able to test a machine in order to determine whether it is operating within safe temperature limits. The difficulty in making such tests lies in the fact that the highest temperatures are within the coils, at points which are not easily accessible. The highest temperature within the machine is called the "hot spot" temperature.

The temperature at the surface of the winding may be measured by placing a thermometer bulb against the surface and covering it with a small pad of cotton. It has been found that 15° C. added to this reading will give an approximate value of the hot spot temperature.

It has already been shown that the resistance of copper conductors changes with the temperature. By utilizing this principle, an idea of the average temperature within a winding may be obtained. The increase of resistance per degree rise of temperature may be obtained from the formula $1/(234.5 + t)$,[1] where t is the surrounding or ambient temperature. For example, at an ambient or room temperature of 30° C., the increase of resistance per degree rise is $1/264.5 = 0.00378$.

Example.—With an ambient temperature of 30° C. the resistance of the field of a shunt generator increases from 104 to 112 ohms. What is its temperature rise?

$$\text{The fractional change in resistance is } \frac{112 - 104}{104} = 0.077$$

$$\text{Temperature } rise = 0.077/0.00378 = 20°.4\text{ C.} \quad \textit{Ans.}$$

Owing to the long time required to reach a constant temperature, motors and generators should be run from 6 to 18 hours, in order that an accurate test of their temperature may be made. As such a long time is usually prohibitive, the heating is often accelerated by running overload for an hour or so and then dropping back to rated load. By this procedure a very good idea of the ultimate temperature may often be obtained in a run of 2 or 3 hours.

To get an idea as to how close a machine is to its ultimate temperature, it is often desirable to plot a curve of temperature

[1] See Par. 48, page 43.

rise during the test. A typical curve of this type for a shunt field is shown in Fig. 331. At the beginning of the test, there is but a slight difference of temperature between the field coils and the room. Therefore, but a small amount of heat is given out by the coils and as a result the temperature rises rapidly. As the difference between the coil temperature and the room temperature increases, more and more heat is given out by the coils, and the temperature rises less rapidly. Therefore, the rate of temperature increase becomes less as the time increases. This is illustrated by the curve of Fig. 331. When the curve becomes practically horizontal, the total heat developed in the coils is equal to the heat dissipated by the coils and the coils have reached a constant temperature. Similar curves would hold for other parts of the machine.

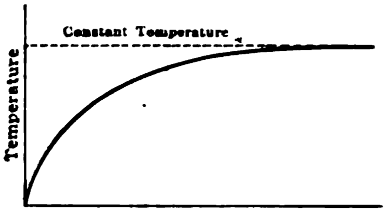


FIG. 331.—Curve of temperature rise with time, for a dynamo.

Care must be taken in measuring the armature resistance when determining temperature rise. The object of this measurement is not to determine the resistance with the idea of calculating the loss, but to determine the *change of resistance in the armature copper, due to change of temperature*. Therefore, it is essential that the resistance of the *copper alone* be measured and that the current path through the copper be the same in every measurement. To exclude all resistance except that of the copper, the brush and contact resistances must not be included in the measurement. Therefore, the voltmeter leads must be held on the commutator segments inside the brushes, as shown in Fig. 332 (*a*). Moreover, these segments should be marked and in every subsequent measurement they should be directly under the same brushes. This insures the same conducting path for each measurement.

When a multi-polar armature is so measured, the *division* of current in the various paths is determined in part by the brush contact resistance. Thus in Fig. 332 (*b*), the current from brush *a* to brush *b* is I_1 and that from brush *a* to brush *c* is I_2. The total current entering the brush *a* is their sum, *I* amp.

The division of the current I between brushes b and c is in part determined by the contact resistance at these two brushes. As contact resistance is a variable quantity, the current *division* in the armature may change considerably with different measurements. To keep the current in definite paths, two brushes may be insulated as shown in Fig. 332 (c). In this case the current paths are not symmetrical, but the division of current is deter-

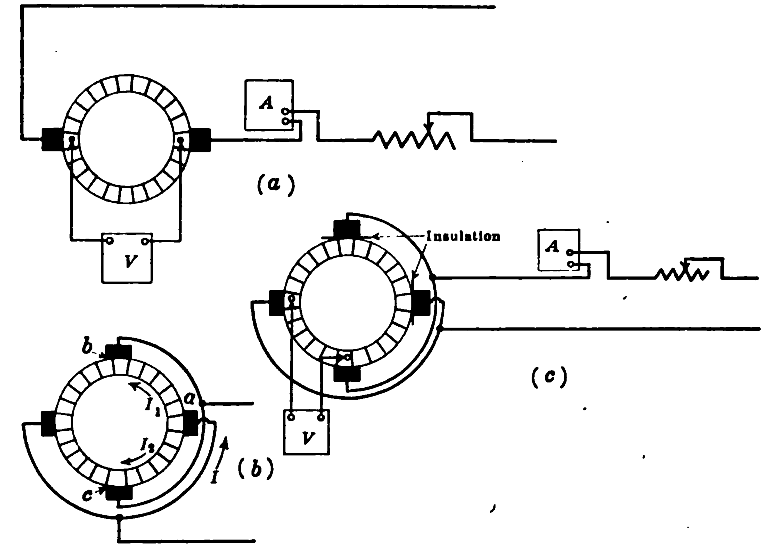


Fig. 332.—Measurement of armature resistance for temperature test.

mined not by brush contact resistance but by the copper resistance itself.

In measuring the shunt field resistance, the voltmeter should be connected directly across the winding so as to exclude the drop in the rheostat.

This resistance method gives an average value of the temperature of the windings. To find the hot spot temperature, 10° C. should be added according to the A. I. E. E. rules.

In addition to measuring the temperature of the windings, the rise of temperature of bearings and of commutator should be measured with a thermometer.

In modern machines of large size, thermo-couples are inserted in the windings and are connected to milli-voltmeters on the switchboard, so that the operator can determine the "hot spot"

temperatures at any time. If the thermo-couples are located between coil sides or between coil sides and core in a 2-layer winding 5° C. are added; if the thermo-couples are placed between coil sides and core or between coil sides and wedge in a single-layer winding 10° C. are added, and 1° C. for every 1000 volts above 5000 volts terminal pressure. The hottest spot is the highest value by either method after corrections have been applied.

235. Parallel Running of Shunt Generators.—In most power plants it is necessary and desirable that the power be supplied by several small units rather than by a single large unit.

(*a*) Several small units are more reliable than a single large unit, for if a unit is disabled the entire power supply is not cut off. (*b*) The units may be connected in service and taken out of service to correspond with the load on the station. This keeps the units loaded up to their rated capacity which increases the efficiency of operation. (*c*) Units may be repaired more readily if there are several in the station. (*d*) Additional units may be installed to correspond with the growth of station load. (*e*) The station load may exceed the capacity of any single available unit.

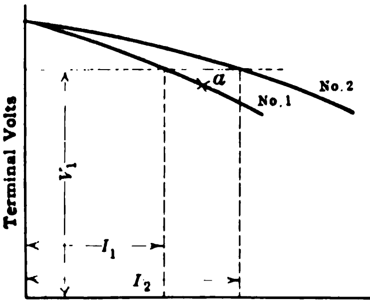


FIG. 333.—Characteristics of shunt generators in parallel.

Shunt generators, because of their drooping characteristic, are particularly well suited for parallel operation. In Fig. 333 are shown the characteristics of two shunt generators which will be designated as No. 1 and No. 2 respectively. It will be noted that generator No. 1 has the more drooping characteristic. If the two generators are connected in parallel, Fig. 334, their terminal voltages must be the same, neglecting any very small voltage drop in the connecting leads. Therefore, for a common terminal voltage, V_1, Fig. 333, generator No. 1 delivers I_1 amp. and generator No. 2 delivers I_2 amp. That is, the machine with the more drooping characteristic carries the smaller load.

Assume that some condition arises which temporarily causes

generator No. 1 to take more than its share of the load. This condition might arise from a temporary increase in the speed of its prime mover, or it might be occasioned by change of load on the system. Generator No. 1 would immediately tend to operate at some point *a* on its characteristic. This results in a drop in its terminal voltage, which tends to make it take *less* load. Therefore, any tendency of one machine to take more than its share of the load results in a change of voltage which opposes this tendency. Therefore, shunt generators in parallel

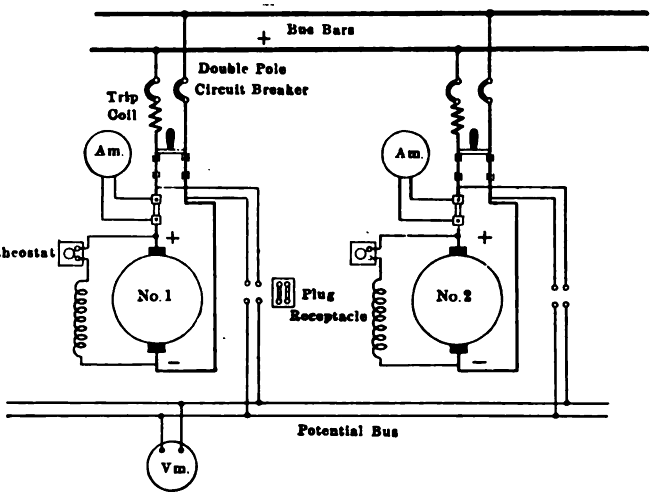


FIG. 334.—Connections for the parallel operation of shunt generators.

may be said to be in *stable* equilibrium. The reactions of the system are such as to hold the generators in parallel. Moreover, if any *change* of load on the system occurs, each machine must carry some of the increase or decrease of load.

The connections for operating shunt generators in parallel are shown in Fig. 334. Each generator should have its own ammeter. A common voltmeter is sufficient for all the machines. The individual machines can be connected to the voltmeter or potential bus through suitable plug connectors or selective switches. Assume that No. 2 is out of service and that No. 1 is supplying all the load. It is desired to put No. 2 in service. The prime mover of No. 2 is started and No. 2 is brought up to speed.

Its field is then adjusted so that its voltage is just equal to that of the bus-bars, which condition may be determined by the voltmeter. The breaker and switch are now closed and No. 2 is connected to the system. Under these conditions, however, it is not taking any load, as its *induced* voltage is just equal to the bus-bar voltage and no current will flow between points at the same potential. Its induced voltage must be greater than that of the bus-bars in order that it may deliver current. Therefore, the field of No. 2 is strengthened until the generator takes its share of the load. It may be necessary to weaken the field of No. 1 simultaneously in order to keep the bus-bar voltage constant.

To take a machine out of service, its field is weakened and that of the other machine is strengthened until the load of the first

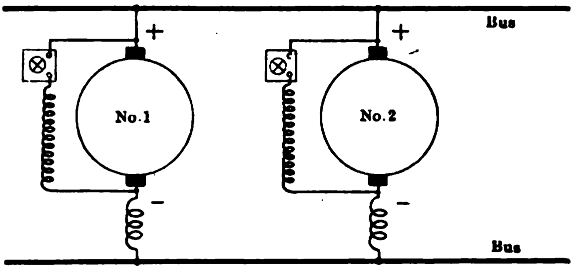


FIG. 335.—Compound generators in parallel.

machine is zero. The breaker and then the switch are opened, clearing the machine. Connecting in and removing a machine from service in this manner prevent any shocks or disturbance to the prime mover or to the system.

If the field of one generator be weakened too much, current will be delivered to this generator, which will run as a motor and tend to drive its prime mover.

It is evident that if shunt generators are to divide the load properly at all points, *their characteristics should be similar*, that is, *each should have the same voltage drop from no load to full load.*

236. Parallel Running of Compound Generators.—Fig. 335 shows two over-compounded generators connected to the bus-bars, positive and negative terminals being properly connected as regards polarity. Each generator is taking its proper share of the load.

Assume that for some reason generator No. 1 takes a slightly increased load. The current in its series winding must increase, which strengthens its field and raises its electromotive force [thus causing it to take still more load.] On the other hand, as the system load is assumed to be fixed, generator No. 2 will at the same time drop some of its load, resulting in a weakening of its series field and a consequent further dropping of its load. In a very short time No. 1 will be driving No. 2 as a motor, and ultimately the breaker of at least one of the machines will open.

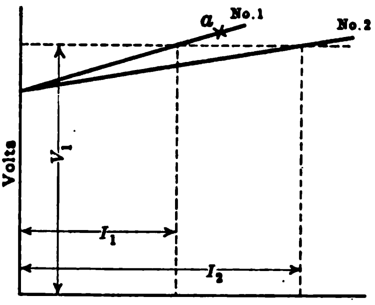


FIG. 336.—Characteristics of compound generators in parallel.

This condition is again illustrated by Fig. 336, which shows the individual characteristics of the two machines. Assume that the machines are operating at a voltage V_1, which corresponds

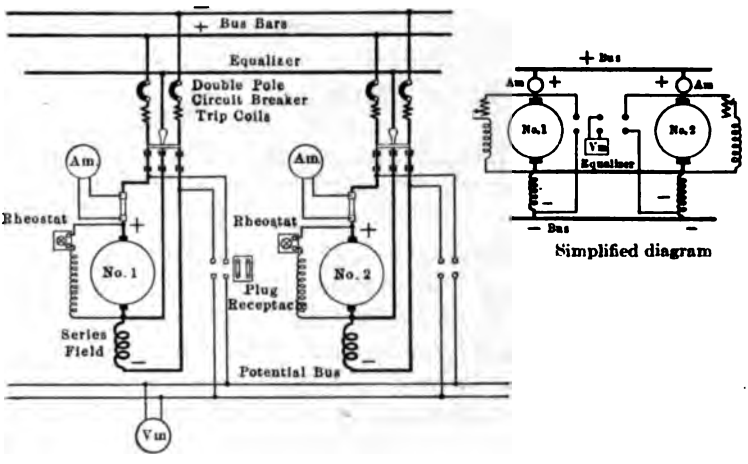


FIG. 337.—Typical connections for two compound generators operating in parallel.

to the respective currents I_1 and I_2. Assume that No. 1 takes a slightly increased load. Its voltage will then tend to rise to some point a. This increased voltage means that the machine

takes still more current and the effect will continue until ultimately the breaker opens.

These compound generators may be considered to be in unstable equilibrium. That is, any action tending to throw the machines out of equilibrium is accentuated by the resulting reactions.

The machines may be made stable by connecting the two series fields in parallel, Fig. 337. This connection, which in Fig. 337 ties the two negative brushes together, is a conductor of low resistance and is called the *equalizer*. Its operation is as follows: Assume that generator No. 1 starts to take more than its proper share of the load. This increased current will pass not only through the field of generator No. 1 but also, by means of the

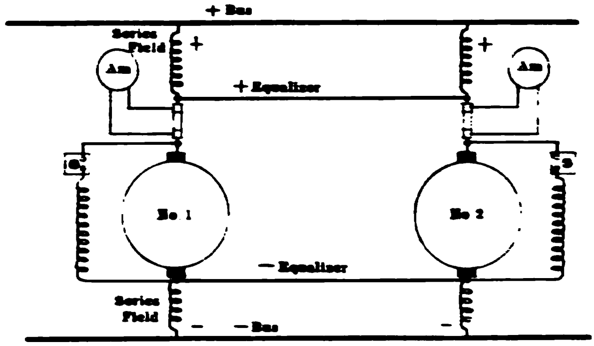


Fig. 338.—Compound generators requiring two equalizers.

equalizer, some of it will pass through the field of generator No. 2. Therefore, both machines are affected in a similar manner and No. 1 is unable to take the entire load.

To maintain the proportionate division of load from no load to full load, the following conditions must be satisfied:

(*a*) The percentage regulation of each armature must be the same.

(*b*) The series field resistances must be inversely proportional to the machine ratings.

It is not always possible to adjust compound generator characteristics by means of series field diverters so that they divide the load properly. Suppose, Fig. 337, that the series field of generator

No. 1 is shunted by a diverter. If the equalizer and bus-bar have negligible resistance, this diverter shunts the series field of generator No. 2 as well as that of No. 1. Therefore, the diverter merely drops the characteristic of the entire system but does not affect the *division* of load. The proper load adjustments may be made by means of a very low resistance in series with one of the series fields.

It should be noted that the desired division of load among either shunt or compound generators at any one load may be obtained by adjusting their field rheostats. However, it is usually desirable that this division remain constant at all loads, especially if an operator is not in continuous attendance. Therefore, it is desirable that generators operating in parallel have similar characteristics.

A compound generator with a single series field usually has a 3-pole switch, one blade of which connects the equalizer, as shown in Fig. 337. If a 3-wire generator (see page 394) having two series fields is to be connected, a 4-pole switch is necessary as there are two equalizers. (See Fig. 338.) The load ammeter in a compound generator should always be connected between the *armature* terminal and the bus-bars. If it is connected in the series field circuit, the ammeter may not indicate the generator current, due to the fact that some of the generator current may be passing through the equalizer.

Compound generators are put in service and taken out of service in the same manner as shunt generators, that is, the load is adjusted and shifted by means of the shunt field rheostat.

237. Circuit Breakers.—Generators, motors and electric circuits in general require protection from short circuits and overloads. The sudden load imposed by a short circuit may injure the generator or its prime mover. Wires may overheat under the short-circuit current, resulting in fire hazard. Two common devices are used for opening short circuits and overloads, the fuse and the circuit breaker. The fuse has a much lower first cost and occupies less space. On the other hand, it is worthless after being blown (unless it is of the refillable type) and considerable inconvenience often results from not having spare fuses at hand. The circuit breaker has a higher first cost and requires more space. On the other hand, it operates an indefinitely great

Fig. 339.—Two pole, 2000-ampere circuit breaker (Condit).

Fig. 340.—6000-ampere, electrically-operated circuit-breaker (Condit).

number of times without injury and is readily re-set. The action of a breaker is faster than that of a fuse. That is, it opens the circuit more quickly.

Practically all breakers operate on the same principle. The mechanism which presses the breaker contacts together is held by a trigger. This trigger is actuated by a solenoid plunger, the turns of the solenoid itself being in series with the circuit. When the current becomes excessive, the plunger is raised and so trips the trigger, allowing the breaker to spring open. Many breakers also have shunt solenoids, which allow them to be tripped from remote points.

The current in the breaker is usually carried by copper laminations which bridge the copper blocks, as shown in Fig. 339. On closing the switch, these laminations press on the blocks making a wiping contact. The carbon blocks parallel these copper contacts. They ordinarily carry a negligible portion of the current, but when the breaker opens they break contact later than the copper, and so interrupt the arc, which would otherwise burn the copper. The carbon contacts are cheap and easily renewed. Circuit breakers of large capacities are more or less complicated mechanisms, as is illustrated by the 6,000-amp. breaker shown in Fig. 340.

Circuit breakers should always be mounted at the top of the switchboard. If they are placed at the bottom, the arc which rises may cause personal injury or may damage the switchboard equipment.

CHAPTER XIV

TRANSMISSION AND DISTRIBUTION OF POWER

238. Power Distribution Systems.—Under modern conditions, most central stations generate power on a large scale as alternating current and transmit this power as alternating current. The reason for using alternating current in transmitting the power is that the voltage may be efficiently raised and lowered by means of transformers. Much less copper is required to transmit power at high voltages. The Thury system does transmit power as direct current at high voltage (see Par. 204), but is not used in this country.

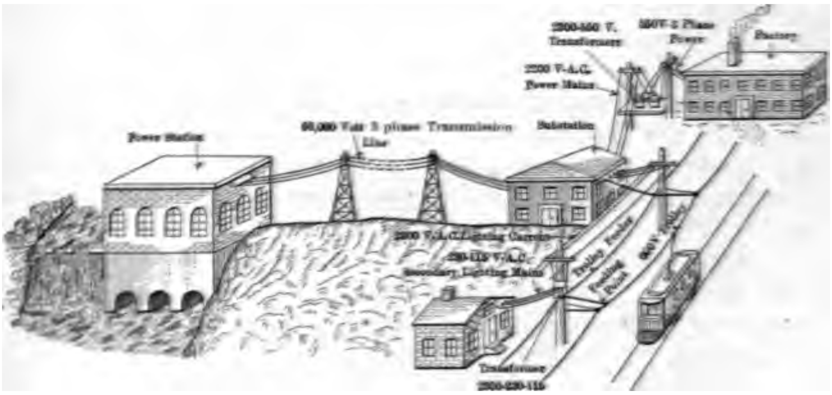


FIG. 341.—Typical power system.

Power is ordinarily utilized at comparatively low voltages (110, 220, 600 volts), but it cannot be economically transmitted to any considerable distance at these voltages. In fact, direct current for commercial use can be economically transmitted and distributed only in the most congested districts of large cities. Its advantages under these conditions are the absence of inductance and capacity effects, which are present with alternating current, and also the absence of eddy current losses in the cables. Another advantage of direct current is that a storage battery reserve can be readily utilized. Fig. 341 shows the general method of power distribution. Power is generated at the power

station, is transmitted as alternating current at high voltage to the substat ion (66,000 volts is shown; the transmission voltage is seldom less than 6,600 volts). At the substation it is either transformed to 2,300 volts alternating current by transformers or to 600 volts or 230 volts direct current by motor-generator sets or rotary converters. (Fig. 341 shows the substation supplying a trolley with 600 volts direct current; a 2,300-volt alternating current circuit supplies power for lighting, the voltage being transformed near the consumer's premises to a 230–115-volt 3-wire system; a 3-phase 2,300-volt alternating current power line supplies a factory, the voltage being transformed to 550 volts, 3-phase, by transformers. These systems are discussed more fully in Chap. XII, Vol. II.) The substation receives the power in large amounts and distributes it to the various consumers in smaller quantities. It bears the same relation to the power system as the middleman or retailer does to an industrial system.

239. Voltage and Weight of Conductor.—*The weight of conductor varies inversely as the square of the voltage,* when the power transmitted, the distance and the loss are fixed.

Let it be required to transmit the power P at the voltage V_1 and current I_1 over wires having a resistance R_1.

The current

$$I_1 = \frac{P}{V_1}$$

The power loss

$$P_1 = I_1^2R_1$$

Assume that the voltage is raised to V_2, the power, the loss and the distance remaining fixed.

The current

$$I_2 = \frac{P}{V_2}$$

The power loss

$$P_2 = I_2^2R_2 = P_1$$

Therefore,

$$I_1^2R_1 = I_2^2R_2$$

$$\frac{R_1}{R_2} = \left(\frac{I_2}{I_1}\right)^2 = \frac{(P/V_2)^2}{(P/V_1)^2} = \frac{V_1^2}{V_2^2}$$

That is, the conductor resistance varies *directly* as the square of the voltage. But the volume or the weight of a conductor of given length varies inversely as the resistance.

Let the weight of copper in the two cases be W_1 and W_2, respectively.

$$\frac{W_1}{W_2} = \frac{V_2^2}{V_1^2}$$

Therefore, the conductor weight varies inversely as the square of the voltage, when the power, the loss and the distance are fixed.

If the voltage of a system is doubled, the weight of the copper is quartered, other conditions being the same.

Example.—50 kw. are delivered at a distance of 500 ft. at 110 volts over a 400,000 C.M. feeder. (*a*) What is the power loss? (*b*) Repeat for 220 volts.

(*a*) The current

$$I_1 = \frac{50,000}{110} = 454 \text{ amp.}$$

If the cable had 454,000 C.M. (see Par. 69) the loss would be 454,000 $\times$ 1,000 $\times$ 10^{-5} watts = 4,540 watts. Actually the loss is $\left(\frac{454}{400}\right)^2 \times 1{,}000 \times 10^{-5} \times 400{,}000 = 5{,}150$ watts. *Ans.*

$$(b)\ I_2 = \frac{50,000}{220} = 227 \text{ amp.}$$

The loss is

$$\left(\frac{227}{400}\right)^2 4{,}000 = 1{,}290 \text{ watts. } \textit{Ans.}$$

The loss in (*b*) is one-fourth that in (*a*). Therefore, a 100,000 C.M. feeder, having just one-fourth the weight of the feeder in (*a*), would transmit the same power, the same distance, with the *same loss*.

240. Size of Conductors.—In transmitting or distributing power by direct current, four factors must be considered in determining the size of conductor.

(*a*) The wires must be able to carry the required current without overheating.

This is particularly important with inside wiring where fire risk exists. Tables of the permissible current-carrying capacity of wires are given in the Appendix, page 410.

(*b*) The voltage drop to the load must be kept within reasonable limits. This is particularly important when incandescent lamps constitute the load.

(*c*) The wires must be of sufficient mechanical strength. This is important when the wires are strung on poles. It is not advisable to use wires smaller than No. 8 A.W.G. for pole lines.

(*d*) The economics of the problem must be considered. Increasing the size of conductor means higher investment costs but less energy loss in transmission. That size of conductor should be chosen which makes the cost of the energy loss plus the interest on the investment a minimum. This may be modified in view of the considerations stated in (*a*), (*b*) and (*c*).

CONSTANT POTENTIAL DISTRIBUTION

241. Distribution Voltage.—About 110 volts has been found to be the most convenient voltage for incandescent lighting. It is not so high as to be dangerous to persons. Incandescent lamp filaments for voltages in excess of 110 volts become so long and of so small a cross-section that they are fragile. An even lower voltage than this would be desirable from the standpoint of the filament, but a lower voltage would be accompanied by an increase in the required weight of copper. Therefore, 110–115 volts has been standardized for lighting and for domestic use as being the most desirable when all factors are taken into consideration. Six hundred volts is commonly used for trolley distribution, because it is not so high as to give operating difficulties and it saves considerable copper as compared with systems of lower voltage. At the present time, 1,200, 2,400, and even 3,000 volts are used at the trolley in railway electrification, these higher voltages being for trunk line electrification, not for municipal traction.

242. Distributed Loads.—The load on a feeder or main may be concentrated at one or two points, as is generally the case with feeders, or may be distributed uniformly or non-uniformly along the conductors, as when lamp loads are located at various points along mains. (See Fig. 342.)

The conductors may be of uniform cross-section throughout their entire length, Fig. 342(*a*). This occurs where the mains are short and the voltage drop is small.

Where the mains are of considerable length, the minimum amount of copper for a given voltage drop is obtained when the mains are uniformly tapered, Fig. 342(*b*).

As it is impracticable to have a uniformly tapering conductor, a conductor of constant cross-section is run for a part of the distance, followed by another uniform conductor of lesser cross-section, and so on, as shown in Fig. 342(*c*). A good rule to remember is that the current *density* in each section should be the same. For example, the first section may consist of a 250,000 C.M. conductor, carrying 200 amp.; assume the second section carries 150 amp; it should be a $\frac{150}{200} \cdot 250{,}000 = 190{,}000$ C.M. conductor. Ordinarily 4/0 wire would be used for this second section.

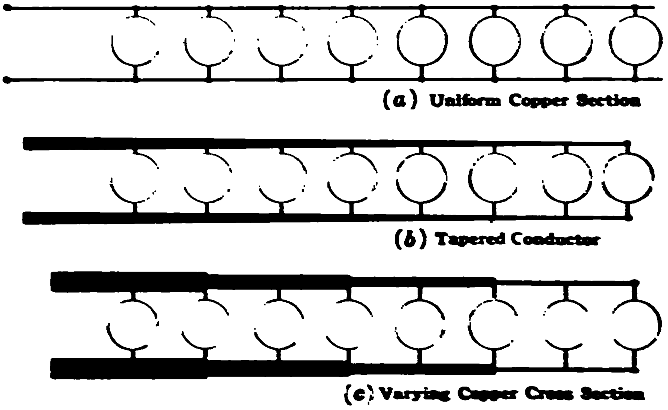


FIG. 342.—Copper cross-section of distributing system or of mains.

343. Systems of Feeding.—In order to keep a number of lamps at the same voltage without excessive copper, the *return loop* or *anti-parallel* system shown in Fig. 343(*a*) is often used. The two feeding wires are connected to opposite ends of the load. This system allows all the lamps to operate at nearly the same voltage and yet the voltage drop in the feeding wires may be large.

The objection to the return loop system is the extra length of wire required. This objection is often overcome by arranging the loads in the manner shown in Fig. 343(*b*), called the *open spiral* system. Where large groups of lamps are switched off and on at the same time, as in theaters and auditoriums, it is often possible to arrange the lamps in this way.

The open spiral may be closed at its ends, resulting in the *closed loop* system of Fig. 343(*c*).

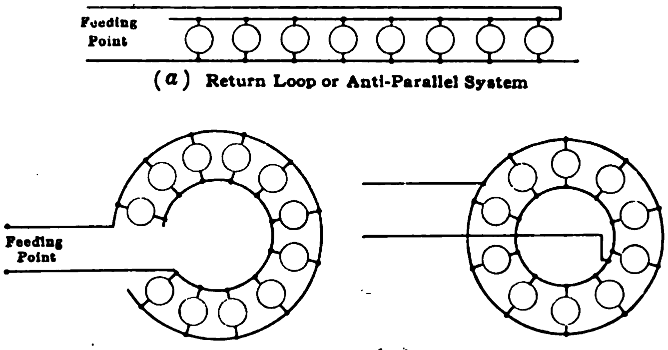


FIG. 343.—Systems of feeding.

244. Series-Parallel System.—Doubling the voltage of a system results in the weight of required copper being reduced to one-fourth its initial value. If 110-volt lamps be arranged so that two are always in series, as shown in Fig. 344, the system may be operated at 220 volts. The copper section will then be one-fourth that required for straight 110-volt distribution. The obvious disadvantages of the series parallel system are that

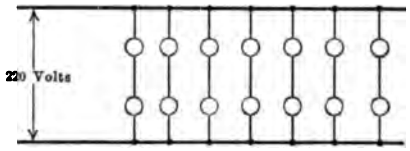


FIG. 344.—Series-parallel system.

lamps can only be switched in groups of two and if one lamp burns out, the lamp to which it is connected ceases to operate. Also, both of the lamps in series must be of the same rating.

THE EDISON 3-WIRE SYSTEM

245. Advantages.—The objections to the series-parallel system may be eliminated by running a third wire, called a ***neutral***, between the two outer wires. This neutral maintains all the lamps at approximately 110 volts. The advantage of a higher

25

saving in reducing the weight of copper is obtained by the use of this system. If there were no neutral wire, the 220-volt system would require one-fourth the copper of an equivalent 110-volt system. If it be assumed that the neutral of the Edison system is of the same cross-section as the two outer wires, the total copper for the Edison system is 3/8 or 37½ per cent. that for a 110-volt system of the same kilowatt capacity. Therefore the saving in copper is 62½ per cent. In practice, the neutral can be made smaller than the two outer wires so that the saving in copper is even greater than 62½ per cent.

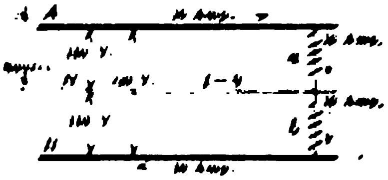


Fig. 345. Edison 3-wire system balanced loads.

The general plan of the system is shown in Fig. 345. Two wires *A* and *B* have 220 volts maintained between them, *A* being the positive and *B* the negative. A third wire *N* is maintained at a difference of potential of 110 volts from each of the other two wires. Therefore *N* must be negative with respect to *A* and positive with respect to *B*. That is, current tends to flow from *A* to *N*, and from *N* to *B*.

Fig. 345 shows the conditions which exist when the load on each side of the system is the same. Each of the loads *a* and *b*

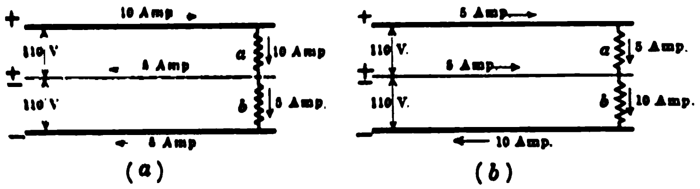


Fig. 346.—Unbalanced 3-wire systems.

takes 10 amp. The 10 amp. taken by load *a* passes through to load *b* and then back through wire *B* to the source. This is equivalent to a series-parallel system as both loads are equal and are in series. Under these conditions the current in the neutral wire is zero and the loads are said to be balanced.

Fig. 346(*a*) shows the conditions existing when the load *a* on the positive side of the system is 10 amp., and the load *b* on

the negative side is 5 amp. Under these conditions the extra 5 amp. taken by load *a* must *flow back* through the neutral to the generator or source. Therefore there are 5 amp. in the neutral returning to the generator. In Fig. 346(*b*) the load *b* is now 10 amp. and load *a* is 5 amp. Under these conditions the extra 5 amp. must *flow out* to the load through the neutral. It will be observed that the current in the neutral may flow in either direction, depending upon which load is the greater. Therefore, if an ammeter is used in a neutral it should be of the zero-center type. Moreover, it will be observed that the neutral carries the *difference* of the currents taken by the two loads. In practice the loads are usually so disposed that they are nearly balanced. Twenty-five per cent. unbalancing (that is, a neutral current which is 25 per cent. that in the outer wires) is usually allowed. Therefore, the current in the neutral is ordinarily much smaller than that in the outers and a much smaller conductor can be safely used. In practice the neutral is usually grounded.

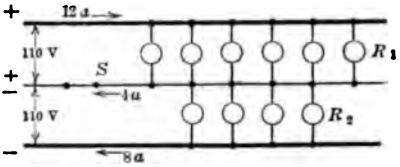


FIG. 347.—Effect upon the balancing of a 3-wire system of opening the neutral.

Effect of Opening the Neutral.—In practice, it is very desirable to keep the neutral of the 3-wire system closed under *all* conditions. The reason for this is illustrated by the following example.

Fig. 347 shows two lamp loads on a 3-wire system. The load on the positive side consists of 6 lamps each taking 2 amp., making a total of 12 amp. The load on the negative side consists of 4 lamps each taking 2 amp., making this load 8 amp. The voltage across each load is 110 volts so the resistance R_1 of the positive load is

$$R_1 = \frac{110}{12} = 9.17 \text{ ohms.}$$

The resistance of the negative load is

$$R_2 = \frac{110}{8} = 13.75 \text{ ohms.}$$

If the neutral now be opened at the point S, the two loads R_1 and R_2 are in series and therefore each must take the same current. The total resistance R is $R_1 + R_2 = 22.92$ ohms.

There is now 220 volts across these two loads in series, so that the current

$$I = \frac{220}{22.92} = 9.60 \text{ amp.}$$

The voltage V_1 across load R_1

$$V_1 = 9.60 \times 9.17 = 88.0 \text{ volts.}$$

The voltage V_2 across load R_2

$$V_2 = 9.60 \times 13.75 = 132 \text{ volts.}$$

This assumes that the resistance of the lamp filaments does not change. It will be observed, however, that the larger bank of lamps is operating at a much reduced voltage, resulting in a material decrease of candle-power, and that the smaller bank is operating considerably above rated voltage, which would result in the lamps burning out in a short time.

For the above reason the neutral of the 3-wire system is usually grounded and one rarely sees circuit breakers in the neutral wire of power plants.

246. Voltage Unbalancing.—The voltage on the two sides of a 3-wire system may become considerably unbalanced if the loads on the two sides of the system become unequal, as shown in Fig. 349.

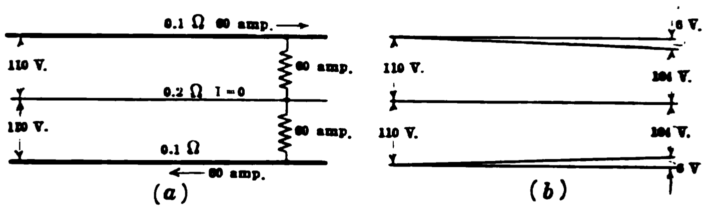


FIG. 348.—Voltage drop in a 3-wire system having balanced loads.

In Fig. 348(a) a load of 60 amp. exists on each side of the system. Each outer wire has a resistance of 0.1 ohm and the neutral has a resistance of 0.2 ohm. The generator voltage is 220 volts across the two outer wires.

As the two loads are equal, there is no current in the neutral wire. Therefore, the voltage drop per wire for the outers is

$$e = 60 \times 0.1 = 6.0 \text{ volts.}$$

The voltage across each load is 104 volts. There is no voltage drop along the neutral, as it carries no current. Fig. 348(*b*) shows a plot of the voltage distribution.

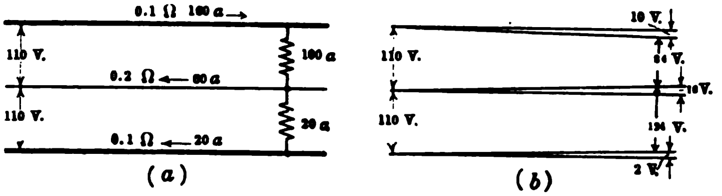


FIG. 349.—Voltage unbalancing in a 3-wire system having unbalanced loads

Assume that the loads are as shown in Fig. 349, 100 amp. on one side of the system and 20 amp. on the other side. This represents the same total amperes as in Fig. 348.

The drop in the positive wire

$$e_1 = 100 \times 0.1 = 10 \text{ volts.}$$

The drop in the neutral

$$e_2 = 80 \times 0.2 = 16 \text{ volts.}$$

Voltage across positive load

$$V_1 = 110 - 26 = 84 \text{ volts.} \quad \textit{Ans.}$$

The drop in the negative wire

$$e_2 = 20 \times 0.1 = 2 \text{ volts.}$$

Voltage across negative load

$$V_2 = 110 - 2 + 16 = 124 \text{ volts.} \quad \textit{Ans.}$$

There is now 40 volts difference between the voltages on the two sides of the system.

Under these conditions, the voltage across the load on the negative side is *greater* than the voltage on the negative side of the system at the power station. This rise in voltage from power station to load is obtained at the expense of the drop in the neutral. Fig. 349(*b*) shows these conditions graphically.

When motor loads are to be connected to a 3-wire system they are usually connected between the two outer wires rather than between an outer wire and neutral so that they will not produce any voltage unbalancing. In fact some power companies will not permit motor loads exceeding 1 hp. to be connected to neutral.

METHODS OF OBTAINING A 3-WIRE SYSTEM

There are various methods of obtaining a 3-wire system which are as follows:

247. Two-generator Method.—Two shunt generators may be connected in series as shown in Fig. 350. The positive terminal of one should be connected to the negative terminal of the other

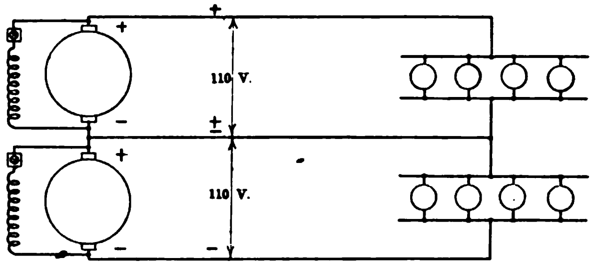


FIG. 350.—Two generators supplying a 3-wire system.

that is, the generators are in series between the outers. Both generators may be driven by the same prime mover. When connected in this manner, each machine supplies only the load on its own side of the line. The obvious objection to this method is that two separate machines are required.

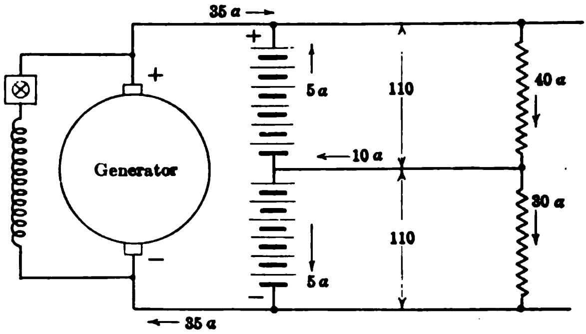


FIG. 351.—Storage battery giving neutral in a 3-wire system.

248. Storage Battery.—*A storage battery* may be floated across the line as shown in Fig. 351. The neutral wire is connected to the middle point of the battery. When the load is unbalanced, that half of the battery on the more heavily loaded side will discharge and the other half will be charged. Fig. 351 shows an

unbalancing of 10 amp. In this particular case the upper half of the battery supplies 5 amp., and the other 5 amp. in the neutral go to charge the lower half of the battery. The objections to this method of obtaining a neutral are the high maintenance cost of a storage battery and the difficulty of maintaining both halves of the battery at the same condition of charge.

249. Balancer Set.—A balancer set is a very common method of obtaining the neutral. This set consists of a motor and a generator mechanically coupled together. They are connected in series across the outer wires and the neutral is brought to their common terminal, as shown in Fig. 352.

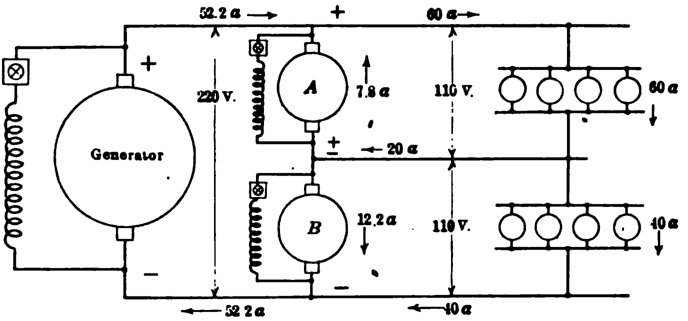


FIG. 352.—Balancer set giving neutral in a 3-wire system.

The action of this set may best be illustrated by the hydraulic analogy shown in Fig. 353. Water is supplied by the canal *A*. This water falls over a weir into canal *B* and may be made to do useful work in so doing. All this water is not needed between the canal *B* and the tail race *C* at the point of utilization *D*. Some of the water which is not needed at *D* passes to *C* through the water wheel shown in the figure. This water wheel is belted to a centrifugal pump operating between *B* and *A*. In virtue of the water passing through the water wheel some of the water in the canal *B* is pumped back to *A* by the pump, where it may be utilized again. The water wheel corresponds to the motor or machine *B*, Fig. 352, and the centrifugal pump to the generator or machine *A*.

If in Fig. 353 more water is required between canals *B* and *C* than can be supplied by the weir at *A*, the centrifugal pump may act as a water wheel and the water wheel as a pump. Some of

the extra water required in B will be supplied through the upper machine operating as a water wheel and discharging into B. In so doing the upper machine drives the lower machine as a pump. The lower machine then pumps water from C back to B. This condition corresponds to an excess of load on the negative side of the system of Fig. 352.

If in Fig. 352 there is an excess of load on the positive side of the system, as represented by 20 amp. in the neutral, 12.2 amp. of this 20 amp. flows through the motor and in dropping

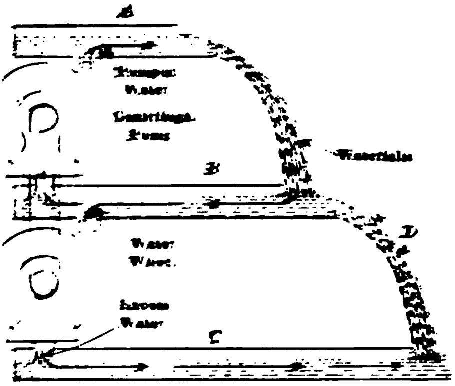


Fig. 353.—Water-wheel analogy of balancer set.

through 110 volts gives up its energy. The motor then causes the generator to pump 7.8 amp. back to the positive side of the line. This current distribution is determined in the following manner:

Each of the machines A and B is assumed to have 80 per cent. efficiency. Let I_1 be the generator current in machine A and I_2 be the motor current in machine B. The generator output will be $0.8 \times 0.8 = 0.64$ times the motor input. Assuming that the voltages are equal, actually they will be slightly unbalanced.

$$110 I_2 \times 0.64 = 110 I_1$$
$$I_1 + I_2 = 20$$

Solving

$$I_1 = 7.8 \text{ amp.}$$
$$I_2 = 12.2 \text{ amp.}$$

The machines will respond more readily to unbalanced loads if their fields are *crossed*, that is, if the motor field is across the

generator side of the line and the generator field is across the motor side of the line. In order that a generator may supply current, its terminal voltage must drop or its induced voltage must rise. In order that a motor may take load, either its terminal voltage must rise or its induced voltage must drop. The excess load on the positive side of the system (Fig. 352) tends to reduce the field of machine A and to increase that of machine B. These effects are the reverse of what is desired. If the generator field is across the motor side of the line, the increased voltage is now across the generator field and will raise the generator induced voltage. Therefore, its terminal voltage need not drop so much to take care of unbalanced currents. The same result may be

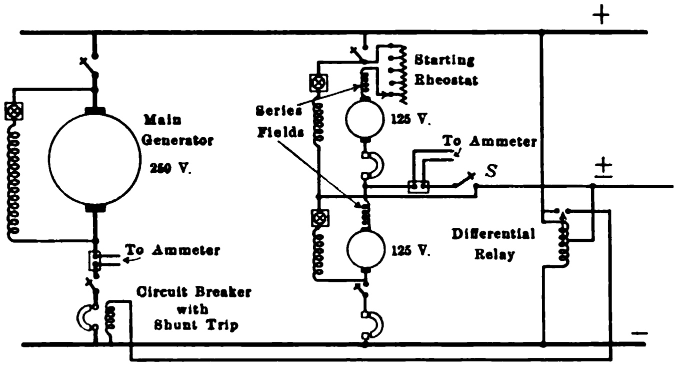


FIG. 354.—Connections of a 3-wire system using a balancer set.

obtained by compounding the two machines. The series fields should be so connected that the machine acting as a generator is cumulatively compounded, and that acting as a motor is differentially compounded.

Fig. 354 shows standard connections for a balancer set with series fields. The machines are started in series, with the neutral switches open and the shunt fields in series across the line. When the machines are up to speed the neutral switch S is closed. If the voltages on the two sides of the system become widely different, the currents in the two halves of the differential relay become unbalanced. This relay then closes the tripping coil circuit of the main generator breaker, resulting in the main generator circuit opening, even though *its* load is not excessive.

250. Three-wire Generator.—The three-wire generator or Dobrowolsky method is a very efficient method of obtaining a neutral. The details of the method can be understood better after alternating currents and the rotary converter have been studied.[1] The principle of the method is as follows: Alternating current is generated within a direct current armature as has already been shown. If slip rings be employed, alternating current can be obtained from the machine. A coil wound on an iron core has high inductance and offers a high impedance to this alternating current. The center of such a coil is at the center of gravity of the voltages generated within the armature. Further, this inductance coil offers very little resistance to the flow of direct current. Therefore, if the three-wire neutral be connected to the center of this coil, the voltage to either brush from the neutral will be the same. Moreover, any current flowing back through the neutral can readily flow back into the armature through this reactance. The connections of such a generator are shown in Fig. 355 (*a*). Sometimes, to obtain better balancing, two and even three reactances are employed. All have their neutrals tied together, as shown in Fig. 355 (*b*). Occasionally the reactances are placed within the armature. This arrangement requires but one slip ring, but increases the weight of the armature.

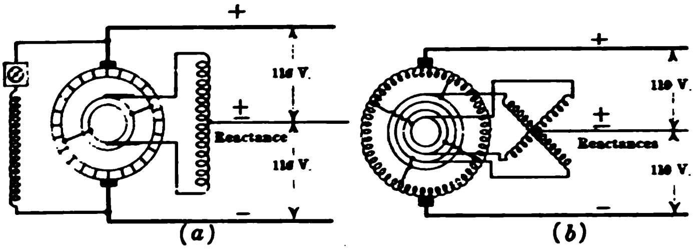


FIG. 355.—3-wire generator connections (Dobrowolsky method).

The Edison 3-wire system may be extended to 4, 5, 6, and 7-wire systems. (See Fig. 309, page 341.) The complications and number of wires prevent these multi-wire systems being extensively used.

[1] See Vol. II, Chap. XI.

251. Feeders and Mains.—In congested districts the mains form an underground network. This network is supplied at various points, called centers, by feeders connected to the direct current bus-bars at the power station. It requires a careful study of the various loads, amount of copper, etc., in order to determine the most advantageous feeding points or centers. Two or more substations may simultaneously feed the same centers. In order that the voltages at these centers may be determined and so maintained at the desired values, pilot or pressure wires run back to the station voltmeter. By means of a dial switch the operator is able to read the voltages at the various centers. Fig. 356 shows the cross-section of a concentric

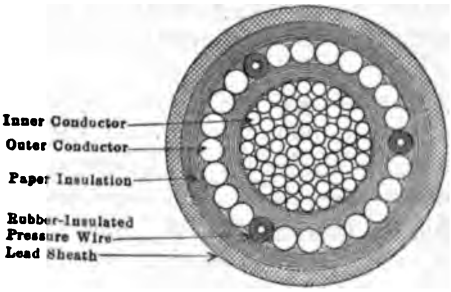


FIG. 356.—Cross-section of a 220-volt, 1,000,000 C.M. concentric cable.

1,000,000 C.M. cable. The outer and inner conductors are the outer wires of the Edison 3-wire system. The neutral is usually a separate wire of much smaller cross-section, or there may be one large neutral common to several feeders and mains. The three pilot or pressure wires are connected, one to each outer wire and one to the neutral at the feeding point. If the operator finds that the voltage is too low at the feeding point, he throws a feeder to a bus-bar of higher voltage. A large voltage drop can exist in such feeders, as no loads are taken off at intermediate points.

In practice the following are the percentage drops usually allowed: In feeders, 5 to 10 per cent.; in the distribution mains, 3 per cent.

The services are usually taken directly from the distribution mains and occasionally, extra large loads may be taken directly from junction boxes. Junction boxes are circular iron castings

containing a set of insulated bus-bars, to which either the distribution mains or the feeders are connected. Distribution mains are connected, through fuses, to suitable terminals already installed in the junction boxes. A junction box thus provides a convenient method of connecting the single feeding wires to the several distribution wires. The mains are always fused, but only disconnecting links are used for the feeders, it being deemed advisable to allow the feeders to burn themselves clear of any short-circuits.

252. Electric Railway Distribution.—Electric railway generators are generally compounded, the series field being on the negative side. The negative terminal is usually connected directly to ground or to the rail through a switch. The positive terminal feeds the trolley through an ammeter, a switch, and a circuit breaker.

On short lines, with light traffic, the trolley alone may suffice to carry the current to the car, as shown in Fig. 357 (*a*). Except in small installations, the trolley is of insufficient cross-section to supply the required power and at the same time to keep the voltage drop within the necessary limits. As the size of the trolley wire is limited by the trolley wheel, it cannot be conveniently increased. The same effect as increasing the size of the trolley may be obtained by running a feeder in parallel with the trolley and connecting the feeder to the trolley at short intervals, as shown in Fig. 357 (*b*). This is called the *ladder system* of feeding. The trolley and feeder together may be considered as forming a single conductor.

Where the density of traffic requires several feeders, the best results are obtained by connecting the feeders in the manner shown in Fig. 357 (*c*). Each feeder is protected by a circuit breaker.

The objections to the preceding methods of feeding are that trouble, due to a ground, for example, at any point on the trolley, involves the entire system. In cities where traffic is particularly dense, it is not permissible to take chances of having the entire system shut down due to a ground at one point only. Therefore, the trolley is sectionalized as shown in Fig. 357 (*d*). In this method the trolley is divided into insulated sections, each of which is supplied by a separate feeder. Trouble in one section

is not readily communicated to the other sections. This increased reliability is obtained at the expense of a less efficient use of the copper, as the feeders are unable to assist one another. In the preceding systems this mutual help is obtained.

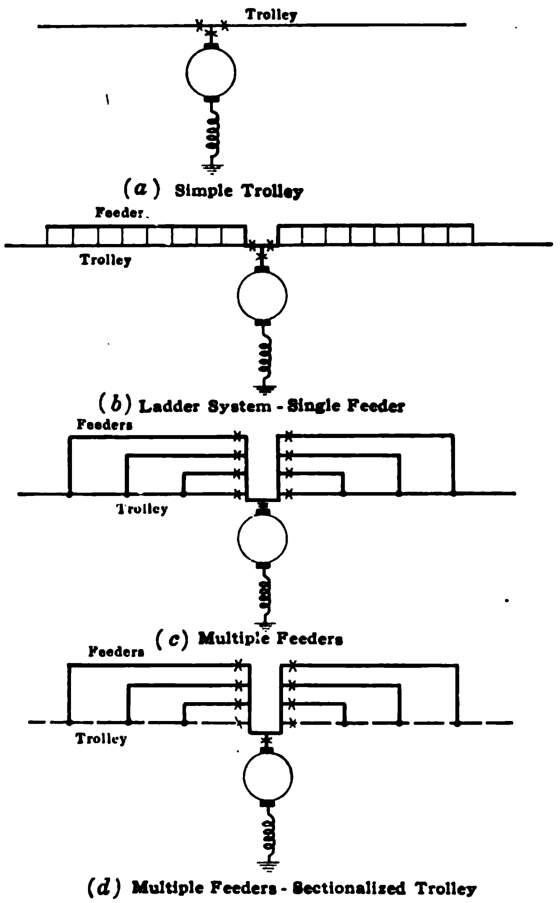


FIG. 357.—Methods of feeding a trolley system.

253. Electrolysis.—Most trolley systems use the track as the return conductor for the current taken by the car. The return currents not only pass through the tracks themselves, but seek the paths of least resistance by which they may return to the negative terminal of the station generator. Such currents in spread-

ing through the earth follow such low resistance conductors as water pipes, gas pipes, cable sheaths, etc., as shown in Fig. 358. The fact that the current *enters* and flows along these conductors in itself does no harm. However, it is obvious that such currents must ultimately leave these pipes as at (*a*), Fig. 358. In so doing they tend to carry the metal of the pipe into electrolytic solution, which ultimately results in the pipe being eaten away. To decrease the effects of electrolysis several expedients have been devised. The two most successful methods are the following: (*a*) Provide as good a return path through the track as is practicable. This is done by good bonding and by using insulated

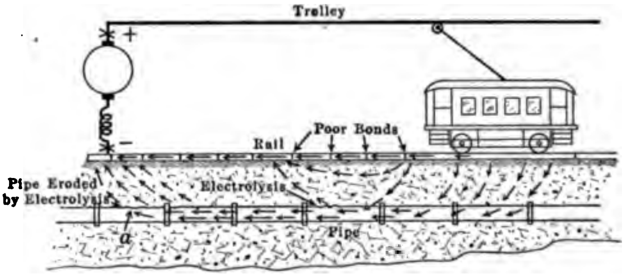


Fig. 358.—Electrolysis by earth currents.

negative feeders, that is, heavy copper feeders that are run back to the negative bus from various points along the track. Fig. 358 shows how poor rail bonds may cause the current to leave the track and enter the pipe. In some cities the total permissible drop in the ground return circuit must not exceed from 10 to 15 volts. (*b*) Discourage the current's entering the pipes by inserting occasional insulating joints in the pipes.

In testing for electrolysis the usual method is to measure the voltage existing between the track and the water pipes (as at a hydrant). The magnitude of this voltage indicates roughly the magnitude of the current which must be flowing from one to the other. The polarity shows which way the current is flowing. For instance, if the track is positive to the pipe, current must be flowing from the track to the pipe.

The electrolysis situation is still in an unsettled state both, as regards its mitigation and as to the ultimate responsibility for the damage resulting.

STORAGE BATTERY SYSTEMS

254. Central Station Batteries.—Fig. 359 shows a typical load curve of a central station. Between 11.00 P.M. and 5.00 A.M. the load is comparatively small, consisting of street lights and a few all-night commercial loads.

This portion of the load curve is called a "valley." The load increases rapidly from 5.00 to 7.00 A.M. due to commercial power loads, lights and perhaps to the beginning of street car service. The morning peak occurs about 8.00 A.M. The load drops off gradually until noon.

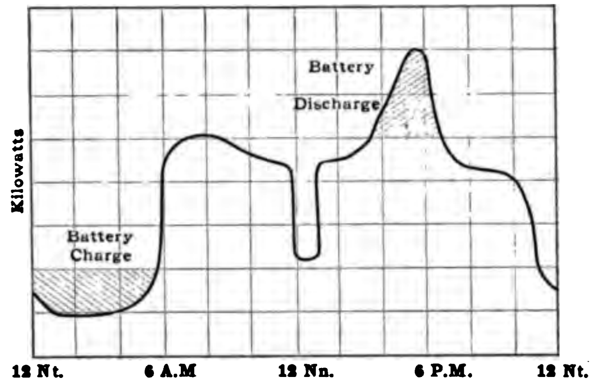


FIG. 359.—Battery smoothing out central station load curve.

The valley between 12.00 and 1.00 is due to the shutting down of the commercial loads because of the luncheon hour. The evening peak, which is usually the largest, occurs between 5.00 and 6.00 P.M. This peak may hold up for an hour, after which it drops to the evening load, which consists mostly of lighting. This load gradually diminishes to the all-night value.

Of course the power company must have sufficient station and distributing capacity to carry the peak. Even although this apparatus is in use only one hour a day, the investment charges are in effect 24 hours a day.

The ratio of the average load to the maximum load of a station is called the *load factor.*

Example.—A station delivers 192,000 kw.-hr. in a day and its peak load is 20,000 kw. What is the daily load factor?

$$\text{The average load} = \frac{192,000}{24} = 8,000 \text{ kw.}$$

$$\text{The load factor} = \frac{8,000}{20,000} = 40 \text{ per cent.} \quad \textit{Ans.}$$

Obviously a *high* load factor is very desirable. In fact power companies welcome any loads that will fill in the valleys of the curve and are usually prepared to offer attractive rates for such loads in order to improve their load factors and thus to utilize apparatus at times when it would otherwise be idle.

The load curve of a station may be smoothed out by the use of a storage battery. The battery may be charged at night and early morning and so fill in the valley of the load curve and then be discharged on the peak of the load curve, as shown in Fig. 359. This equalizes the load on the station and increases its load factor.

As a rule, batteries are not installed for the purpose of smoothing out the load curve. A storage battery operating under the best conditions is good for only a limited number of complete charges and discharges. Therefore, the battery maintenance is usually found to more than offset the economies effected by taking some of the load off the peak. Such batteries may be very useful in office buildings and other isolated plants, because it is often possible to shut down the entire lighting plant and run on the batteries at night, thus eliminating considerable labor charge.

Batteries are commonly installed as reserve in large central station systems. They are placed, therefore, near the center of the load. In case of a shut-down in the generating system or in the transmission system, the battery can help maintain service. For this reason pasted plate batteries are more often used, because of their high overload capacity. (See Par. 94.)

Storage batteries are also useful in taking care of unexpected loads. For example, a thunder storm may result in a sudden demand which could not be foreseen and so cannot be met immediately by the generating station, as it takes time to get up steam and put a generator on the line. A battery may be put

on the line immediately and so carry the sudden load increase until boilers and turbines can be brought into service. If the battery is already floating across the line it takes the load increase automatically.

255. Resistance Control.—In order to control the load taken by a generator connected to the bus-bars, it is necessary to change its induced voltage by adjusting the field current. It is not possible to adjust the voltage of a storage battery in this manner. One method of controlling the battery load is to have the battery voltage several volts higher than the bus-bar voltage and to insert resistance in series with the battery, as shown in Fig. 360. By adjusting this resistance, the load delivered by the battery may be controlled. The disadvantage of this method is the loss of power in the resistance and the voltage drop in the resistance, which depends upon the load.

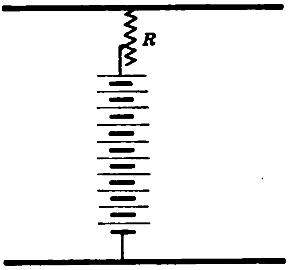


FIG. 360—Resistance control of battery discharge.

Even with constant load the resistance must be adjusted occasionally to compensate for the drop of battery voltage during discharge.

Example.—It is desired to discharge a storage battery, consisting of 115 cells each having an electromotive force of 2.1 volts and an internal resistance of 0.001 ohm, into 220 volt bus-bars so that the battery delivers 100 amperes. To what value must the series resistance be adjusted?

The total battery electromotive force

$$E = 115 \times 2.1 = 242 \text{ volts.}$$

The bus-bar voltage

$$V = 220 \text{ volts.}$$

The battery resistance

$$r = 115 \times 0.001 = 0.115 \text{ ohm.}$$

Let R = the added external resistance

$$100 = \frac{242 - 220}{0.115 + R}$$

$$100R = 22 - 11.5 = 10.5$$

$$R = 0.105 \text{ ohm.} \quad \textit{Ans.}$$

256. Counter Electromotive Force Cells.—If an electric current be sent through two plain lead plates immersed in dilute sulphuric acid, a simple storage battery is formed which immedi-

ately develops a counter electromotive force of about 2.0 volts. (See Par. 92.) Neglecting the small IR drop in such a cell, the counter electromotive force is practically independent of the current. This principle is utilized in controlling the current delivered by a battery.

Plain lead plates are immersed in dilute acid and are connected in series with the battery. If it is desired to decrease the discharge rate of the battery more of these cells are cut in. To do this an end cell switch, similar to that shown in Fig. 361, is used. The advantage of this method over the resistance control is that the opposing or control electromotive force is independent of the load.

257. End Cell Control.—A battery usually consists of a sufficient number of cells to give an electromotive force exceeding that of the bus-bars by an ample margin. To charge such a battery a booster may be used. (See Par. 102.) The electromotive force of the battery, and hence its load, may be controlled by cutting in or out the cells at the end of the battery.

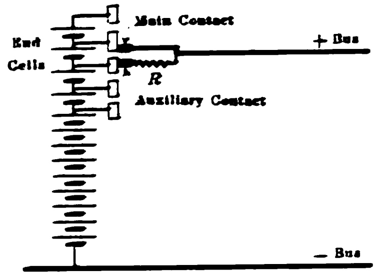


FIG. 361.—End cell control of storage battery.

It is essential to do this without opening the circuit. For this purpose a switch similar to that shown in Fig. 361 is used. The main contact is connected to the auxiliary contact by a resistance R. When sliding from one battery contact to the next the auxiliary contact maintains the circuit connections through the resistance R. Were there zero resistance between the main contact and its auxiliary contact the individual cells would be dead short-circuited during the transition period. The resistance R is usually so chosen as to allow the normal battery current to flow during the transition period. The end cell switches become rather massive in large battery installations and are often operated by a motor-driven worm. This also permits remote control.

The end cells, not being in continuous service, are discharged to a lesser degree than the others. Therefore they require individual attention on charging.

258. Floating Battery.—A battery is occasionally used to equalize sudden fluctuations of load such as commonly occur in railway systems. The battery voltage should be such that with an average load on the station it is just equal to the bus-bar voltage. The battery is then delivering no current and is merely "floating."

When a sudden load comes on the station, the bus-bar voltage drops. The battery then discharges and assists the generators. On the other hand, if the load drops to a low value, the bus-bar voltage rises and the battery charges.

As a rule the bus-bar voltage does not change enough to cause the battery to respond sufficiently to the load changes. In fact with over-compounded railway generators the reverse

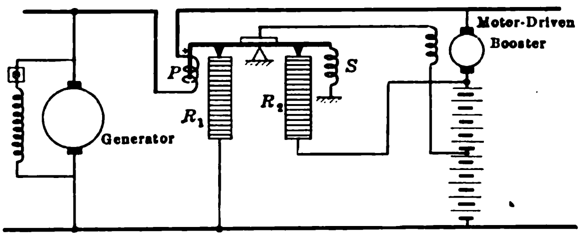


FIG. 362.—Regulating storage battery discharge with a booster set.

action might well occur. There are several methods of causing the battery to charge and discharge at the proper time. One typical method is shown in Fig. 362. The battery is connected to the bus in series with a motor-driven booster. Two carbon rheostats R_1 and R_2 are connected in series across the battery. The booster field is connected from their common point to the middle of the battery. If $R_1 = R_2$, the booster field is connected across two points of equal potential, the field current is zero and no voltage is induced in the armature. An increase of load, however, causes solenoid P to pull down on the lever. This compresses R_1 and releases the pressure on R_2. The resistances R_1 and R_2 now differ considerably so that the booster field is no longer across points of equal potential. A current now flows through the booster field causing the booster to generate an electromotive force of such polarity as to assist the battery to discharge. S is a spring.

In order to reduce the current flowing through R_1 and R_2 and the battery, the change of booster excitation is often accomplished through an intermediary exciter, whose field is connected in the same manner as the booster of Fig. 362.

Battery at End of Line.—Very poor voltage regulation may occur at the end of a trolley line, due to insufficient copper. Rather than to install more copper, it may be more economical to install a storage battery at the end of the line. This battery not only steadies the trolley voltage but tends to reduce violent fluctuation of the power station load as well.

As the voltage at the end of the line requiring a battery undergoes fluctuations of considerable magnitude, the battery is usually self regulating both as regards charge and discharge. With little load on the line, the voltage at the battery should be high enough to charge it. On the other hand, when a car is near the battery, the line voltage should drop to such a value as to allow the battery to discharge and assist the power station.

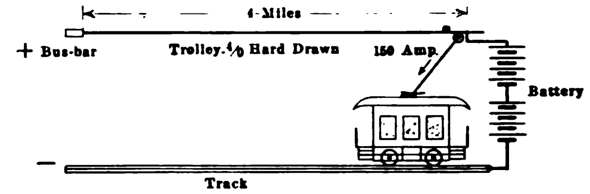


FIG. 363.—Battery floating at end of trolley line.

Example.—The bus-bar voltage (Fig. 363) at the station is maintained constant at 600 volts. A 4/0 trolley having a resistance of 0.26 ohm per mile extends out 4 miles from the station. The resistance of ground and track is 0.05 ohm per mile. At the end of the line a storage battery consisting of 240 cells is "floated." Each cell has an average electromotive force of 2.0 volts and a resistance of 0.002 ohm. At what rate will the battery charge when there is no load on the line? When the load at the battery is 150 amp., how much current does the battery supply and how much does the station supply?

The total resistance of the trolley and track

$$R = 4(0.26 + 0.05) = 1.24 \text{ ohms.}$$

The battery resistance

$$R_2 = 240 \times 0.002 = 0.48 \text{ ohm.}$$

The total battery electromotive force

$$E = 240 \times 2.0 = 480 \text{ volts.}$$

When there is no load between the station and the battery the current to the battery

$$I_1 = \frac{600 - 480}{1.24 + 0.48} = \frac{120}{1.72} = 70 \text{ amp. } \textit{Ans.}$$

To find the division of the 150-amp. load at the battery, first find the current at which the battery will just "float."

$$I^1 = \frac{600 - 480}{1.24} = \frac{120}{1.24} = 96.8 \text{ amp.}$$

That is, with a load of 96.8 amp. at the battery the line drop from the power station to the battery will be 120 volts, making 480 volts at the battery. Under these conditions the battery will neither charge nor discharge but will "float."

The remaining 53.2 amp. will be divided inversely as the trolley and battery resistance.

Let I_L be the line current and I_B the battery current.

$$\frac{I_B}{I_L} = \frac{1.24}{0.48}$$

$$I_B + I_L = 53.2$$

Solving

$$I_B = 38.4 \text{ amp.}$$

$$I_L = 14.8 \text{ amp.}$$

The station is already supplying 96.8 amp.

The total station current is then 96.8 + 14.8 = 111.6 amp. and the battery current is 38.4 amp. *Ans.*

This may be checked by calculating the voltage at the battery.

$$600 - (111.6 \times 1.24) = 461.6 \text{ volts}$$

$$480 - (38.4 \times 0.48) = 461.6 \text{ volts. (Check)}$$

259. Series Distribution.—In the parallel system of distribution the loads are all independent of one another. That is, a load applied at any one point does not affect any of the other loads, provided the voltage does not change. In the series system the loads are all in series with one another so that the same current passes through each. Therefore if the circuit of any one load be opened the current to all the other loads will be interrupted. As this is not permissible in practice, a load must be *short-circuited* when it is desired to remove it from service.

Power is usually supplied to a constant current system by one of two methods; the series generator, of which the Brush arc and Thomson-Houston machines are examples, and the constant current transformer operating in conjunction with the mercury arc rectifier. (See Chap. VII, Vol. II.) Both of these methods tend to maintain constant current under all conditions of load.

Therefore, if the circuit be opened and a very high resistance thus introduced, a constant current is maintained across a high resistance and a very high voltage results. For this reason the lamps used on a constant current system are protected by having a thin disc of paper between the lamp terminals (film cut-out). If the lamp burns out the high voltage across this paper punctures it and so prevents the circuit being opened.

The advantage of the series system is the small amount of copper required. This is due to the fact that the copper carries only the current of any single load. As the loads are in series the

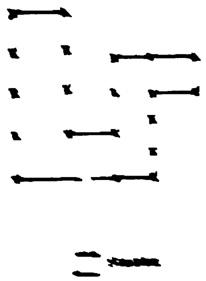


Fig. [illegible].—Open loop series circuit.

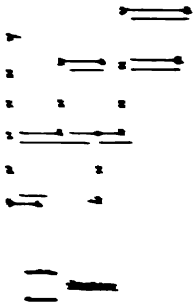


Fig. [illegible].—Parallel loop series circuit.

resulting voltage is high. Therefore this system is applicable only to outside work such as street lighting because it would ordinarily be dangerous to have such high voltages in buildings.

There are two general methods of connecting such series loads. In the open loop system, shown in Fig. [illegible], the circuit is connected to the loads without reference to the separation of the two conductors. This system is economical of copper.

In the parallel loop system the outgoing and return conductors are always kept near each other as shown in Fig. [illegible]. This system requires more copper than the other but facilitates testing for faults and [illegible] disturbances.

APPENDIX A

Relations of Units

Length

1 inch	=	2.54 cm.
1 foot	=	30.48 cm.
1 mile	=	1.609 kilometers

Area

1 circular mil	=	0.7854 sq. mil.
1 circular mil	=	0.000507 sq. mm.
1 sq. inch	=	6.452 sq. cm.
1 sq. meter	=	10.76 sq. ft.

Volume

1 cubic inch	=	16.39 cu. cm.
1 liter	=	1,000 cu. cm.
	=	0.2642 gallon
1 gallon	=	231 cu. in.

Weight

1 gram	=	981 dynes
1 ounce (av.)	=	28.35 grams
1 kilogram	=	2.205 lb.
1 ton	=	2,000 lb.
1 long ton	=	2,240 lb.
1 metric ton	=	1000 kg.
	=	2205 lb.

Work

1 joule (watt-second)	=	10,000,000 ergs
1 gram deg. Cent. (gram calorie)	=	4.183 joules
1 pound deg. Fahr. (B.t.u.)	=	252.1 gram deg. Cent. (gram calorie)
	=	777.5 ft.-lb.
1 kilogram-meter	=	9.81 joules
	=	7.233 ft.-lb.
1 foot-pound	=	1.356 joules
1 horse-power-second	=	178.3 gram deg. Cent. (gram calorie)
	=	0.7074 lb. deg. F. (B.t.u.)
	=	550 ft.-lb.

Pressure

1 atmosphere	=	14.70 lb. on sq. in.
	=	29.92 in. of mercury at 32° F.
	=	760.0 mm. of mercury at 32° F.
	=	33.94 ft. of water at 60° F.
1 lb. on sq. in.	=	702.9 kg. on sq. meter

APPENDIX B

Specific Gravities

Aluminum	2.67	Mercury	13.60
Copper	8.95	Nickel	7.83
Gold	19.26	Platinum	20.30
Iron, bar	7.48	Silver	10.55
Iron, wrought	7.79	Tin	7.29
Steel	7.85	Zinc	6.86
Lead	11.45		

1 cu. ft. of water weighs 62.5 lb.

APPENDIX C

Table of Turns per Sq. In. Solid Layer Winding*

(The Acme Wire Co.)

Size, A.W.G.	Single-cotton covered	Enamel and cotton	Single-silk covered	Enamel and silk	Enamel
10	87.5	84.5	.....	.....	92.5
11	109	105	.....	.....	117
12	136	130	.....	.....	147
13	169	161	.....	.....	184
14	210	199	.....	.....	231
15	260	248	.....	.....	292
16	321	304	.....	.....	366
17	396	374	.....	.....	458
18	488	456	.....	.....	572
19	598	556	.....	.....	715
20	772	722	865	807	907
21	947	890	1,075	1,010	1,150
22	1,155	1,075	1,330	1,230	1,425
23	1,410	1,303	1,650	1,510	1,780
24	1,720	1,575	2,045	1,860	2,220
25	2,080	1,910	2,520	2,290	2,800
26	2,500	2,310	3,090	2,830	3,540
27	3,020	2,770	3,810	3,460	4,440
28	3,630	3,300	4,690	4,220	5,570
29	4,270	3,910	5,650	5,100	6,950
30	5,100	4,630	6,950	6,200	8,730
31	5,920	5,330	8,410	7,300	10,650
32	6,950	6,300	10,000	8,900	13,500
33	8,120	7,300	12,080	10,650	16,900
34	9,430	8,410	14,500	12,600	21,000
35	10,850	9,610	17,300	14,900	26,000
36	12,350	10,850	20,400	17,300	31,900
37	.....	.....	23,700	20,400	40,000
38	.....	.....	27,800	23,700	49,300

*Standard Handbook, Sec. 5, Par. 98.

APPENDIX D

Table of Current-carrying Capacity in Amperes of Wires and Cables Under Various Conditions

Size, A.W.G. or cir. mils	National Electrical Code		Lead-covered cables		
			Single conductor		Three-conductor paper ins., 45 deg. Cent. rise
	Rubber ins.	Slow burning ins.	Rubber, 30 deg. Cent. rise	Paper or cambric, 40 deg. Cent. rise	
14	15	20	......	......	......
12	20	25	......	......	......
10	25	30	20	22	......
8	35	50	30	34	26
6	50	70	50	56	48
4	70	90	78	87	68
3	80	100	98	110	81
2	90	125	121	134	93
1	100	150	145	160	110
0	125	200	169	187	132
00	150	225	192	210	150
000	175	275	245	270	190
0000	225	325	285	315	225
250,000	235	350	320	360	255
300,000	275	400	370	415	300
400,000	325	500	460	515	370
500,000	400	600	550	605	......
750,000	525	800	750	830	......
1,000,000	650	1,000	900	1,030	......
1,500,000	850	1,360	1,200	1,450	......
2,000,000	1,050	1,670	1,400	1,590	......

QUESTIONS ON CHAPTER I

1. What metal is the most useful for magnetic purposes? Why? What other substances show magnetic properties?

2. Distinguish between a natural magnet and an artificial magnet. Under what conditions should soft iron be used for magnetic purposes? Hardened steel?

3. What is a magnetic field? What are lines of induction and do such lines actually exist? Distinguish between a north-seeking pole and a north pole. In what way does the magnetic circuit differ from the magnetic field?

4. What is the effect of breaking a bar magnet near the neutral zone? Explain how the newly created poles come into existence.

5. What is Weber's molecular theory of magnetism? How does it explain the phenomenon that occurs when a bar magnet is broken?

6. What is meant by consequent poles? Distinguish between consequent poles and poles obtained by breaking a bar magnet.

7. When a freely suspended south pole is brought into the presence of a north pole, what effect is noted? What effect is noted if the freely suspended south pole is brought into the presence of a south pole? What is the general law governing attraction and repulsion between like poles, and between unlike poles?

8. What is a unit pole? How is pole strength determined? What is the law governing the force of attraction and the force of repulsion between magnetic poles?

9. Distinguish between lines of force and lines of induction. Are both closed lines? In what way are lines of force and lines of induction similar? At what part of the magnetic circuit is the magnetic force quite distinct from the lines of induction?

10. What is unit field intensity? How are the lines of force related to field intensity? What relation exists in the magnetic field between field intensity and "lines of force per square centimeter?"

11. How many lines of force emanate from a unit pole? From a pole of strength m units? If B is the flux density within a steel rod of 1 sq. cm. cross-section, what is the pole strength at the end of the rod?

12. Of what does a compass needle consist? How is it used in practice to determine the correct polarity of motors and of generators? How is it possible to obtain accurate indications from a compass when it is used upon steel ships? How may a compass needle be used to map out an electrical field in the vicinity of a magnet?

13. How may the flux distribution in a certain region be determined by iron filings? Explain the relation of pole attraction and repulsion to the distribution of the magnetic lines existing near the poles.

14. What is magnetic induction? What is the relation between the inducing and the induced pole? How does magnetic induction explain the attraction of soft iron to magnetic poles? How may a compass become reversed? What is the use of a "keeper" in connection with a horseshoe magnet?

15. What general law governs the path taken by the lines of induction? How does this law explain the attraction of iron to the poles of a magnet?

16. What is the objection to the use of the bar magnet in practical work? What advantages have the ring and the horseshoe magnet over the bar magnet?

17. What is the principle underlying the compound or laminated magnet? Where are laminated magnets used in practical work?

18. How may sensitive instruments be shielded from stray magnetic fields that may exist in their vicinity?

19. How may a bar be magnetized by means of a bar magnet? By means of two bar magnets? In practice, how may magnets be magnetized by the use of electromagnets and also by means of electric current?

20. State why the compass needle does not point to the true north and the true south in most places on the earth's surface. What information is necessary in order to determine the true north from the indication of the compass needle? What is the dip of the needle?

PROBLEMS ON CHAPTER I

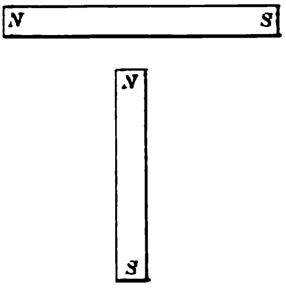


FIG. 1a.

1. Sketch the field around two bar magnets arranged as in Fig. 1a.

2. A uniform field is produced between two parallel polar surfaces of a magnet. A bar magnet is inserted in this field parallel to the lines of induction and with its north pole pointing to the north pole of the magnet. Sketch the resulting field.

3. In problem 2 show the ultimate effect upon the magnetic flux distribution of increasing the strength of the field due to the large magnet. What occurs to the bar magnet?

4. In problem 2 sketch the flux distribution when the bar magnet is perpendicular to the lines of induction.

5. Two poles of strength $m = 800$ and $m' = 1{,}000$ are 2 in. apart in air. What force (in lb.) is acting between them?

6. The two poles of a horseshoe magnet are 4 cm. apart. If each has a strength of 1,000 units, what force (lb.) is tending to pull them together?

7. A north pole of a bar magnet has a strength of 2,000 units. When it is 4 in. from one end of a long soft-iron bar, it induces a pole of 300 units on this end of the bar. With what force is the magnet acting upon the bar? Neglect the effect of the other poles. What pole is induced on the bar, and in what direction does the force act? Make a sketch.

8. A pole strength of 150 units acts with a force of 1.2 grams upon another pole 4 cm. away. What is the strength of the second pole?

9. A uniform magnetic field of 50,000 lines per sq. in. exists between two parallel polar surfaces. What force (grams) is acting upon an N-pole of 500 units placed in this field? Toward which pole will the N-pole tend to be drawn?

10. A magnetic field of 160,000 lines and having the shape of a truncated cone, exists between two parallel, plane surfaces having areas of 25 sq. cm. and 60 sq. cm. respectively. What force is exerted upon a unit N-pole if placed near the first surface? Near the second? Explain.

11. A N-pole has a strength of 100 units. How many lines of force emanate from this pole?

12. What is the flux density at a distance of 2 in. from this pole? What force would exist upon a unit S-pole placed at this distance from the pole?

13. A pole of 500 units exists at the end of a steel rod of circular cross-section and having a diameter of 0.8 cm. What is the approximate flux density in the rod?

14. A long steel rod has a square cross-section of 0.5 in. per side. The flux density at the center of the rod is 15,000 lines per sq. in. What is the strength of the poles at the end of the rod?

QUESTIONS ON CHAPTER II

1. What is the nature and general shape of the magnetic field about a conductor carrying an electric current? What relation exists between the direction of the current and the direction of the field produced about the conductor?

2. How may the above relations be shown experimentally? What simple rules enable one to remember the relation which exists between the current direction and the direction of the magnetic field?

3. The current in a conductor flows from left to right. In what direction will the north end of a compass needle point if held over the wire? If held beneath the wire?

4. If two parallel conductors carry current in the same direction, do these wires tend to separate or come together? Give two reasons for the answer. Repeat for two conductors carrying current in opposite directions.

5. A single loop of wire lying in the plane of the paper carries a current in a clockwise direction. What effect will be noticed if a compass is placed within this loop? Has this loop any properties in common with those of a bar magnet?

6. Show how several loops similar to the one mentioned in (5) may be combined to form a long solenoid.

7. Give three methods whereby the poles at the ends of a solenoid may be determined, provided the direction of the current through the solenoid turns be known.

8. What are commercial uses of the solenoid? Name seven such uses.

9. Explain fundamentally why the plunger is drawn into a solenoid when current flows in the solenoid winding.

10. Sketch the relation between the pull on the plunger and the position of the plunger in the solenoid.

11. What effect does "iron-cladding" have upon the pulling characteristic of the plunger? State one practical application of the simple solenoid; of the iron-clad solenoid. What effect does the stop have upon the solenoid characteristic? State a practical use of this type of solenoid.

12. Show the principle whereby a U-shaped solenoid attracts an armature. Explain the principle of operation of the telegraph relay; the ordinary electric door-bell.

13. Sketch a lifting magnet, showing its general construction. Where are such magnets used commercially, and in what way are they more economical than the older methods of handling material? Does the magnet itself do appreciable work when it is being used to handle iron and steel?

14. What is the disadvantage of the early types of magnetic circuits of dynamos, as represented by the Edison bi-polar type? How has the design of the magnetic circuits of the more modern generators overcome some of the disadvantages of the earlier ones. What should be the approximate ratio of the cross-section of the field cores of a multi-polar generator to the cross-section of the yoke? What general rule should be followed in the placing of the exciting ampere-turns upon a magnetic circuit? Does magnetic leakage between the poles of a generator represent a direct loss of power?

PROBLEMS ON CHAPTER II

15. A portion of a direct current feeder is shown in Fig. 15*a*. When a compass is held above the feeder the needle deflects as shown. In what direction does the current in the feeder flow, in or out of the duct?

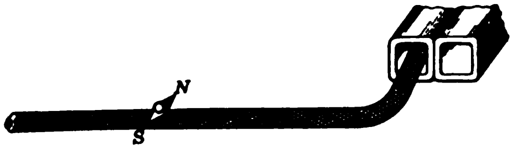


Fig. 15*a*.

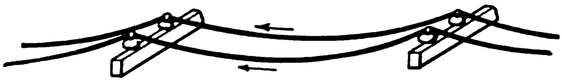


Fig. 16*a*.

16. Fig. 16*a* shows two positive feeders of a trolley system running upon a pole line and carrying current in the same direction. If the trolley wire drops upon the track, causing an enormous current to flow in the feeders for an instant, in what direction will these conductors tend to move and what is the direction of the force acting upon the insulators?

17. In Fig. 17*a* is shown the principle upon which one type of electric hammer operates. Two coils *C* and *C′* are connected in series and in the positions shown. *P*, a soft-iron plunger running in guides, actuates the hammering device. *A* coil *D*, encircling the plunger *P*, is excited continuously with direct current. If the terminals *a* and *b* of the coil *D* are of the

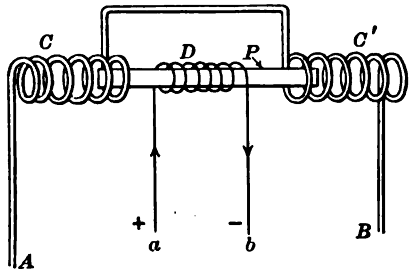


Fig. 17*a*.

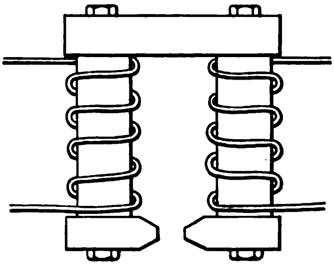


Fig. 18*a*.

polarity shown, indicate the polarity of the ends of the plunger *P*. If terminal *A* is + and terminal *B* is −, in what direction will the plunger *P* tend to move? If the polarity of terminals *A* and *B* is reversed, in what direction does the plunger tend to move?

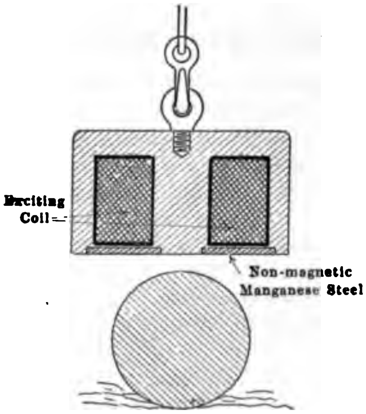


Fig. 20*a*.

18. Fig. 18*a* shows two coils on a simple horseshoe magnet. Connect these coils so that they aid one another. Sketch the magnetic field between the poles.

19. Assuming that one of the field coils of Fig. 38, page 27 is reversed, that is, the two coils "buck" one another, sketch the general appearance of the magnetic field. Will the total flux be increased or diminished by this method of connection?

20. Fig. 20*a* shows in cross-section a lifting magnet about to pick up a heavy iron sphere known as a "skull cracker" (used in breaking up scrap

iron). Sketch the magnetic lines and mark the poles existing under these conditions. The horizontal section of the magnet is circular. Assume that the current enters the paper in the right-hand section of the exciting coil.

21. Connect the coils *ab*, *cd*, *ef*, *gh*, in the multi-polar machine shown in Fig. 21*a*, so that the proper sequence of poles is obtained. Make the left-hand pole a north as shown. Sketch the paths of the magnetic lines.

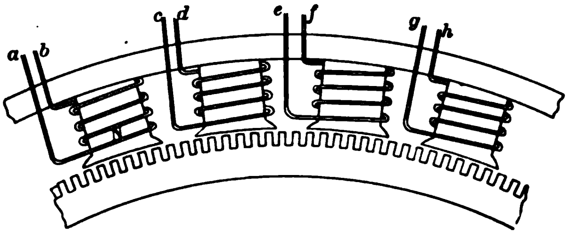


Fig. 21*a*.

Fig. 22*a*.

22. Fig. 22*a* shows the pole face of a generator and an armature tooth. Sketch the paths of the magnetic lines in passing from the pole face into the tooth and from the tooth to the rest of the armature iron. What is "fringing?"

QUESTIONS ON CHAPTER III

1. What is the mechanical analogue of resistance? What is the unit of resistance? How is it defined?

2. Distinguish between insulating materials and conductors. What is a "megohm?" A "microhm?"

3. May two conductors, each of the same material and of equal volume, have different resistances? Explain.

4. How does the resistance of a homogeneous material vary with its length and with its cross-section? What is specific resistance or resistivity?

5. If the volume of a substance is fixed, how does its resistance vary with its length? With its cross-section? If the volume is fixed and the length doubled, how is the resistance affected?

6. What is conductance and how does it vary with the length and cross-section? Distinguish between conductance and conductivity. What is the general meaning of "per cent. conductivity?"

7. What is the relation of the total resistance of a circuit to the resistances of its individual parts when these latter are connected in series?

8. What is the relation of the total conductance of a circuit to the conductances of its individual parts when these latter are connected in parallel? From this relation show how resistances connected in parallel may be combined into an equivalent resistance.

9. What is the meaning of the term "mil?" What is a square mil? A circular mil? What relation does one bear to the other? Where is the circular mil usually chosen as the unit of cross-section? What are its advan-

tages over such units as the square mil and the square inch? What relation does the number of circular mils in a circular cross-section bear to its diameter?

10. What is a cir.-mil-foot? What is its approximate resistance for copper? How may the resistance of a copper wire be determined if its length in feet and its cross-section in cir. mils be known?

11. How is the resistance of most of the unalloyed metals affected by temperature? What is the "temperature coefficient of resistance?" How is it used?

12. At what temperature would the resistance of copper be zero if the resistance decreased at the same rate that it decreases with ordinary drops of temperature? How may this principle be used to solve problems involving resistance and temperature?

13. What relation do the cross-sections of the wires in the A.W.G. bear to one another? How does this relation enable one to determine readily the resistance and weight of any given size of wire? What is the resistance of 1,000 ft. of No. 10 wire? What is the weight of 1,000 ft. of No. 2 wire?

14. What are the best conductors among the metals? Which is most commonly used and why? Compared with copper what are the advantages and the disadvantages of aluminum as a conductor? When are iron and steel used as conductors? Explain.

PROBLEMS ON CHAPTER III

23. Two conductors, *A* and *B*, of the same material, have the same length, but the cross-section of *A* is twice that of *B*. If the resistance of *A* is 30 ohms, what is that of *B*?

24. Two conductors, *C* and *D*, of the same material, have the same length, but the diameter of *C* is twice that of *D*. If the resistance of *C* is 30 ohms, what is that of *D*?

25. If the resistance of copper is 1.724 microhms per cm. cube at 20° C., what is the resistance of an inch cube at the same temperature?

26. A rectangular copper plate has a length of 18 in., a width of 6 in. and a thickness of 0.5 in. If the resistance of copper is 1.724 microhms per cm. cube, what is the resistance of the plate between the 6-in. edges? Between the 18-in. edges?

27. A phosphor-bronze strip $\frac{1}{8}$ in. $\times$ 1 in. and 4 ft. long has a resistance of 0.000597 ohm. What is its resistivity per cm. cube? Per in. cube?

28. No. 16 copper wire has a diameter of 51 mils and a resistance of 4.02 ohms per 1,000 ft. at 20° C. What is the resistance of 5 miles of 00 copper wire (diameter is 410 mils)?

29. A cylindrical conductor *A* has twice the diameter and twice the length of a cylindrical conductor *B*. If the resistance of *B* is 5 ohms, what is the resistance of *A*?

30. What is the resistance of a copper bus-bar 40 ft. long, made up of 4 bars of copper each 4 in. $\times$ $\frac{1}{2}$ in.? The resistance of copper is 1.724 microhms per cm. cube.

31. If copper weighs 0.32 lb. per cu. in. and costs $0.20 per lb., what is the cost of the bus-bar in problem 30?

32. (*a*) If aluminum bars ½ in. thick and of the same conductance were substituted for the copper in problem 31, what would be the ratio of radiating surfaces? Spacers are used between the bars. Neglect the ends as radiating surfaces.

(*b*) What should be the cost of aluminum per lb. in order that the aluminum bus-bars shall cost the same as the copper? Specific gravity of copper = 8.89, of aluminum = 2.70.

33. A 000 copper conductor 800 ft. long and having a diameter of 410 mils is drawn down so that its diameter is 258 mils. If the resistance of the 000, 800-ft. conductor was 0.05 ohm, what is the resistance of the entire length when its diameter has been reduced to 258 mils?

34. Determine the conductance of a copper rod, 1 in. diameter and 8 ft. long. Conductivity of copper = 580,000 mhos per cm. cube.

35. The resistance of a 4-ft. length of No. 8 wire is measured and found to be 0.00241 ohm at 20° C. What is its per cent. conductivity?

36. A copper bar ⅜ in. × 1 in. and 3.5 ft. long rolled from electrolytic copper is found to have a resistance of 0.0000755 ohm at 20° C. What is its per cent. conductivity?

37. The resistance of 500 ft. of No. 18 wire is measured at a temperature of 25° C. and found to be 3.35 ohms. What is its per cent. conductivity?

38. Three resistances of 4.2 ohms each, two resistances of 6.3 ohms each, and a resistance of 8.6 ohms are all connected in series. What is the total resistance of the combination?

39. Two resistances of 8 and 4 ohms are connected in parallel. What is the total resistance of the combination?

40. Three conductances of 6, 8, and 10 mhos respectively are connected in parallel. What is the resulting total conductance? What is the total resistance?

41. If the three conductances of problem 40 are connected in series, what is the resulting conductance? Resistance?

42. If all the individual resistances of problem 38 were connected in parallel, what would be the resulting resistance?

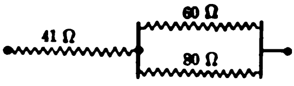


FIG. 43a.

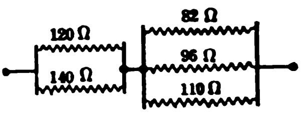


FIG. 44a.

43. A resistance of 41 ohms is connected in series with a group of two resistances of 60 and 80 ohms respectively, connected in parallel (Fig. 43*a*). What is the resulting resistance?

44. A group of two resistances, of 120 and 140 ohms in parallel, is connected in series with another group of 82, 96 and 110 ohms in parallel (Fig. 44*a*). What is the total resistance resulting from this combination?

45. A 0000 trolley wire (hard drawn) having a resistance of 0.0514 ohm per 1,000 ft. extends 5 miles from the power station. It is paralleled for 3 miles by a 250,000 C.M. cable, having a resistance of 0.0431 ohm per 1,000 ft., the feeder and the trolley being connected every half mile by taps (Fig. 45*a*). What is the total resistance of the overhead circuit from the power house to the end of the trolley line?

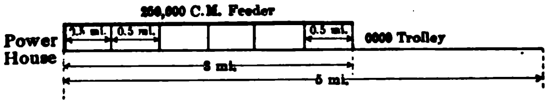


Fig. 45*a*.

46. The resistance of each rail of the trolley system of problem 45 is 0.080 ohm per 1,000 ft. including bonding. An insulated negative feeder consisting of a 0000 stranded cable of resistance 0.0509 ohm per 1,000 ft. runs from the power house and is bonded to both tracks $2\frac{1}{2}$ miles out. What is the total resistance of the return circuit, neglecting any conductance of the earth itself?

47. How many cir. mils in a rod of 1 in. diameter? 0.75 in.? 0.5 in.? 0.25 in.? $\frac{1}{8}$ in.?

48. What is the diameter of a wire having a cross-section of 168,000 C.M.? 66,400 C.M.? 62,500 C.M.? 8,100 C.M.? 400 C.M.?

49. Assuming that the resistivity (per cir.-mil-ft.) of copper is 10 ohms, determine the resistance of 2 miles of wire having a cross-section of 10,000 C.M.?

50. Determine the resistance of a telegraph loop between two stations 30 miles apart, if the wire is copper of $\frac{1}{8}$ in. diameter.

51. Hard drawn copper wire, such as is used for trolley wire, has a resistivity 2.7 per cent. greater than that of annealed copper. Determine the resistance of 5 miles of 000 trolley wire, if the resistivity (cir.-mil-ft.) of annealed copper is 10.37 ohms.

52. What is the resistance of 2 miles of 700,000 C.M. stranded copper cable?

53. What is the resistance at 0° C. of a reel of 0000 annealed copper wire. the wire weighing 400 lb., if the resistance per 1,000 ft. of 0000 copper is 0.050 ohm at 25° C.?

54. What is the resistance of the wire of problem 53 at 50° C.?

55. The resistance of a copper telegraph circuit was found to be 40 ohms when the external temperature was 0° C. What would be its resistance at a maximum summer temperature of 40° C.?

56. The resistance of a shunt field coil of a generator is 44 ohms at 22° C. What is its resistance at 0° C.? At 76° C.?

57. The resistance of an armature winding of a shunt motor is found to be 0.042 ohm at 25° C. What is its hot resistance when it attains a temperature of 70° C.?

58. A direct current feeder has a resistance of 0.007 ohm. What is its change in conductance between the lowest winter temperature of $-20°$ F. and the maximum summer temperature of 100° F.? What is the percentage change?

Without consulting the Wire Table solve the following problems:

59. Estimate the resistance of 1,000 ft. of No. 13 bare copper wire; of No. 16.

60. Estimate the resistance and weight of 1,000 ft. of No. 18 bare copper wire; of No. 24.

61. Estimate the resistance and weight of 2,000 ft. of No. 8 bare copper wire. Of 800 ft. of No. 1. Of 500 ft. of 0000.

62. Estimate the weight, resistance and cir. mils of 600 ft. of 0 bare copper wire. Of 600 ft. of 00.

QUESTIONS ON CHAPTER IV

1. What is the unit of electric current and how is it related to the unit of electric quantity? What is the nature of potential difference and of electromotive force? What are the mechanical analogies of electromotive force and why?

2. What is the nature of voltage drop in a line? Can it be compared to pressure drop in a pipe? Is it possible to supply power over a line and have the voltage at the load equal to the voltage at the sending end of the line? Explain. Is there a voltage loss in the return wire to the generator as well as in the outgoing wire? Can potential exist without a current flowing? Illustrate.

3. What is meant by "difference of potential"? Is it possible to have two or more emf.'s and yet have no difference of potential between certain points?

4. How should a voltmeter ordinarily be connected in a circuit? Is an ammeter connected in the same way as a voltmeter? Why should an ammeter never be connected across a line?

5. What fundamental relation does Ohm's Law express? In what three forms is the law expressed? Under what conditions is it most convenient to use each of these?

6. How are series-connected resistances combined to equal an equivalent resistance? How are parallel resistances combined? What relation does the division of current in a two-branch parallel circuit bear to the resistance of each branch? What relation exists among the currents when the circuit has three branches?

7. What is the unit of electrical power? How may it be expressed in terms of volts, amperes and ohms, taken two at a time? Differentiate carefully between power and energy. What is the unit of electrical energy and what relation does it bear to the unit of power? What is mechanical horsepower? What relation does it bear to the electrical units of power?

8. Discuss the various forms in which energy is stored or in which energy *may* appear. Describe the energy cycle involved in a steam-driven electrical

power plant. In what form does the energy appear ultimately? Approximately what is the over-all efficiency of a modern power system?

9. How is a B.t.u. defined? A gram-calorie? What is the relation between a gram-calorie and a watt-second?

10. What simple relation exists between the voltages at the sending and receiving ends of a power feeder and the efficiency of transmission?

11. Under what conditions is the voltage drop in each foot of wire independent of the total current? How is this principle utilized in solving electrical problems? Can this method be applied to obtaining the power loss? Explain.

PROBLEMS ON CHAPTER IV

63. A storage cell has a constant potential difference of 2.1 volts at its terminals. What current flows when 0.4 ohm is connected across its terminals?

64. A carbon filament incandescent lamp has a cold resistance of 330 ohms and a hot resistance of 240 ohms. What current does it take when it is first connected to 115-volt mains? At what current does it operate?

65. A 110-volt, 25-watt tungsten lamp has a cold resistance of 40 ohms and a hot resistance of 480 ohms. What current does it take when it is first switched to 110-volt mains and what current does it take when it has attained normal operating conditions?

66. A 220-volt generator has a field resistance of 160 ohms, including the rheostat. What current flows in the field?

67. A 550-volt generator has a field resistance of 350 ohms and the rheostat has a resistance of 45 ohms. What current does the field take? What should be the resistance of the rheostat in order to reduce the field current to 1.2 amp.?

68. A carbon rheostat has a resistance of 0.24 ohm and carries a current of 40 amp. What is the voltage across the rheostat?

69. What voltage must a generator develop to supply 25 amp. to an electric oven in which the heating coils have a resistance of 8.5 ohms and the connecting wires a total resistance of 0.25 ohm.

70. A telegraph relay is wound for 150 ohms and operates at 40 milliamperes. What should be the voltage of the circuit battery if it is to operate the relay over a line having a resistance of 30 ohms?

71. A series lighting system consists of 118 lamps, each having a resistance of 7.2 ohms and requiring 6.6 amp. If the line resistance is 100 ohms, what is the voltage of the generator supplying this system?

72. An incandescent lamp takes 0.25 amp. at 110 volts. What is its hot resistance?

73. When a copper bus-bar carries 1,580 amp., the voltage drop across a 6 ft.-length is found to be 1.26 millivolts. What is the resistance per ft. of the bus-bar?

74. The voltage drop across the series field of a compound generator delivering 250 amp. is 0.7 volt. What is the resistance of the series field?

75. A direct-current multiple arc lamp takes 6.0 amp. at 110 volts. If the drop across the arc is 70 volts, what is the resistance of the "ballast?"

76. Fig. 76*a* shows a lamp bank, having a total resistance of 7.5 ohms, being supplied from a 115-volt generator over connecting wires having a resistance of 0.15 ohm per wire. What current does the lamp bank receive?

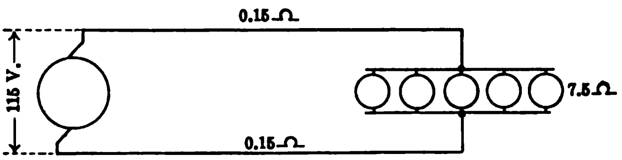


Fig. 76*a*.

77. An electromagnet has four spools of 1 ohm each, all connected in series. Two wires, each having a resistance of 0.05 ohm, connect the magnet to 115-volt mains. What current does the magnet take? If one coil becomes "grounded" so that half of its resistance is short-circuited, what will the magnet current be?

78. Determine the equivalent resistance of a circuit having four resistances of 16, 20, 30 and 40 ohms in parallel. If the current in the 16-ohm resistance is 2 amp., determine the current in each of the other resistances.

79. The series field winding of a generator has a resistance of 0.004 ohm and is shunted by a diverter having a resistance of 0.012 ohm. What is the voltage drop across the series field when the generator delivers 400 amp.?

80. In problem 79 how will the 400 amp. divide between the diverter and the series field?

81. A 000 hard-drawn trolley wire is 5 miles long and is paralleled by a 0000 annealed copper feeder. What is their combined resistance? What is the voltage drop in the feeder when a total current of 100 amp. is flowing in the two?

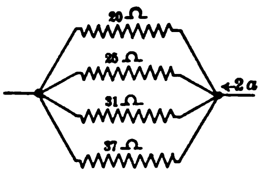


Fig. 82*a*.

82. Four selective relays connected in parallel are supplied by a common wire shown in Fig. 82*a*. If their resistances are 20, 25, 31 and 37 ohms respectively, what is the voltage across their terminals when 2 amp. are supplied by the common wire?

83. In problem 82 how will the 2 amp. divide among the four relays?

84. Two ammeters, one having a 50-amp. scale and the other a 100-amp. scale, are connected in parallel so as to measure a current greater than 100 amp. If the 50-amp. instrument has a resistance of 0.002 ohm and the 100-amp. instrument a resistance of 0.0012 ohm, what will each read when 130 amp. flows in the circuit?

85. To feed a trolley wire at a given point, two feeders, one 350,000 C.M. and the other 250,000 C.M., parallel the 0000 hard-drawn trolley wire. When the current demand upon the system is 600 amp., how does it divide among the feeders and trolley?

86. Fig. 86*a* shows a 120-volt generator supplying lamp loads over mains, the mains having a resistance of 0.3 ohm each. The loads are as follows: 6 gem lights, 305 ohms each; 10 tungsten lamps, 290 ohms each; and 4 tungstens, 150 ohms each. What current does the generator deliver?

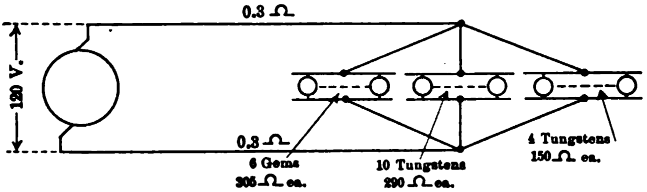


FIG. 86*a*.

87. Fig. 87*a* shows a 115-volt generator supplying lamp loads. Indicate the currents at each part of the system, and the voltage at the various lamp terminals. What is the voltage at *ab*?

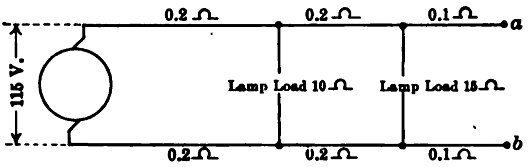


FIG. 87*a*.

88. A resistance of 50 ohms is connected in series with two parallel resistances of 75 and 100 ohms. These are in turn connected in series with a group of three parallel resistances of 120, 150 and 180 ohms. What is the total current of the system when it is connected across 100-volt mains? How much current does each resistance take and what is the voltage across each resistance?

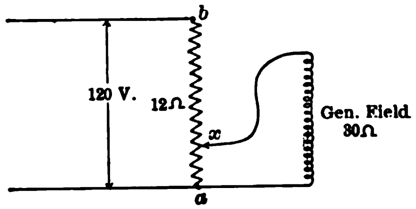


FIG. 89*a*.

89. Fig. 89*a* shows a drop wire, used for regulating the field current of a generator from zero to its maximum value. The total resistance of the drop wire *ab* is 12 ohms and that of the field is 30 ohms. If the line voltage is 120 volts, what current does the generator field take when the contact *x* is ¼ the distance from *a* to *b*? ½ the distance? ¾ the distance?

90. A gas-filled lamp takes 0.8 amp. from 110-volt mains. What is its rating in watts?

91. A generator delivers 1,670 amp. at 600 volts. What is its kilowatt rating?

92. The resistance of the series field of a compound generator is 0.002 ohm. What power is lost in this field when the generator delivers 1,200 amp.?

93. A 2,000-amp. shunt has a resistance of 0.000025 ohm. What power is lost in the shunt when carrying its rated current?

94. A tungsten lamp having a hot resistance of 202 ohms is connected across 110 volt mains. What is its watt rating?

95. Four street car heating units, each having a resistance of 55 ohms, are connected in series. If the trolley voltage is 600 volts, what power do the heaters take?

96. A shunt motor takes 75 amp. at 220 volts and delivers 20 hp. What is its efficiency?

97. A generator delivers 250 amp. at 230 volts. If it has an efficiency of 92 per cent., what horsepower engine is required to drive it?

98. An electroplating bath takes 80 amp. at 20 volts for 2.4 hours, and then 50 amp. at 60 volts for 1.5 hours. How much energy is consumed? At 4 cents per kw.-hr., what is the cost of the required energy under the above conditions?

99. How many joules are supplied to a 25-watt lamp burning 4 hours? If the supply voltage is 110, how many ampere-hours are delivered to the lamp?

100. Energy costs 8 cents per kw.-hr. Determine the cost of heating 2 quarts of water at a room temperature of 25° C. to the boiling point (100° C.). Assume that the efficiency of the heater is 80 per cent. (Water weighs 8.35 lb. per gal.)

101. A water-barrel rheostat contains 30 gal. of water. How long must 45 amp. at 220 volts flow before the temperature of the water is raised to 200° F. from a room temperature of 70° F.? (1 gal. water weighs 8.35 lb.). Assume no losses.

102. A factory, 1 mile from a power station, takes a maximum current of 120 amp. over a 250,000 C.M. feeder. If the station voltage is maintained constant at 600 volts, what is the greatest change of voltage that occurs at the factory from no load to the maximum load? What is the efficiency of transmission at the maximum load and at half this load?

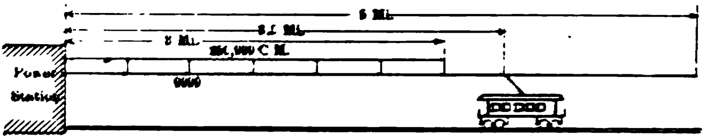


FIG. 103a

103. An electric railway is fed by a 5-mile trolley line of 0000 hard-drawn copper. A 250,000 C.M. feeder parallels this trolley for 3 miles, being tapped

in every half mile (Fig. 103*a*). The resistance of the ground return may be considered as 0.02 ohm per mile. If the station voltage is 600, what is the voltage at the car when it is 3½ miles from the station and is taking 110 amp. What is the voltage at the end of the line at this time? What is the efficiency of transmission?

104. Fig. 104*a* shows two loads, one of 500 amp. ¼ mile from the power station, and another of 200 amp. 1,000 ft. farther along. A 1,000,000 and a 500,000 C.M. cable are in parallel to the first load; a 750,000 C.M. runs from the first to the second load. The voltage at the 200-ampere load is 220 volts. What is the station voltage and the efficiency of transmission?

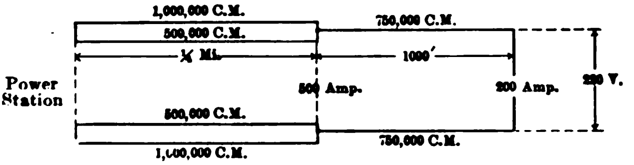


Fig. 104*a*.

Solve the following problems by the methods outlined in Pars. 68 and 69. Do *not* consult the wire tables.

105. A 1,000-ft. length of 200,000 C.M. cable supplies a certain power load. What is the total drop in the cable if the load is such that the cable operates at the *normal density?* What is the power loss under these conditions?

106. If, in problem 105, the cable operates at a density corresponding to 1,500 cir. mils per amp., what is the total voltage drop? What is the power loss under these conditions?

107. A 200-amp. load is to be supplied from the 600-volt bus-bars of a power station at a distance of 0.5 mile. The voltage drop cannot exceed 10 per cent. of the station voltage. What size feeder is necessary, and what is the efficiency of transmission?

108. A 40-hp. motor is to be supplied with power at a distance of 500 ft. from 230-volt bus-bars. The voltage drop cannot exceed 15 volts. The motor has an efficiency of 90 per cent. What size wire is necessary and what is the efficiency of transmission?

QUESTIONS ON CHAPTER V

1. What is the effect upon the terminal voltage of a battery of applying a load to its terminals? Explain. Why does the electromotive force of a cell differ from the terminal voltage? Under what conditions are they the same?

2. Is it possible to make a *direct* measurement of the internal voltage of a cell when it is delivering current? How may this internal voltage be calculated if the battery resistance be known?

3. To what is the internal resistance of a battery due?
Is this resistance a constant quantity?

4. If the electromotive force and the resistance of a battery be known, how may the current delivered to an external resistance be calculated? If

the battery becomes short-circuited what current does it deliver? What becomes of the energy that the cell develops under these conditions?

5. Under what conditions may a battery be made to receive electrical energy? What relation does the direction of current flow bear to its direction when the battery delivers energy? If a generator has a voltage equal to that of the battery, what effects are noted when the generator is connected to the battery, terminals of like polarity being connected together? What effect is noted when the generator voltage is raised above this value? What is meant by the battery "floating?"

6. Before current can be sent into a battery, what voltage must first be applied? Explain why the voltage in excess of that of the battery alone is effective in causing the flow of current. What is a very common illustration of a battery receiving energy?

7. If several cells are connected in series, what is the resultant electromotive force of the combination? What is the resultant resistance of the combination? How may the current be found if the external resistance be known?

8. Under what conditions do batteries operate most satisfactorily in parallel? What is the electromotive force of the combination under these conditions? What is the relation between the external current and the current in the individual cells? What is the relation between the total battery resistance and the resistances of the individual cells? If the resistances of the individual cells are not equal, how may the resistance of the entire battery be found? What relation does the current delivered by each cell bear to the resistance of the cell? What relation exists among the terminal voltages of individual cells connected in parallel?

9. What is a series-parallel grouping of cells? What is the voltage of the entire battery? How may the resistance of the battery be found if the resistance of the individual cells be known? How may the current in an external circuit be found if the external resistance, the electromotive forces and resistances of the individual cells and their arrangement be known?

10. In general, how should cells be grouped to obtain the best economy? How should cells be arranged to obtain the maximum power output?

11. What two fundamental principles are stated in Kirchoff's laws? If several currents meet at a junction, how should their direction of flow be taken into account?

12. How should a rise in potential be represented? A drop in potential? When passing from a $-$ to a $+$ terminal of a battery, what should be the sign of the potential change and why? When passing from $+$ to $-$? When passing through a resistance in the direction of the current does a rise or a drop in potential occur? What then should be the proper sign to use? When passing along the resistance in opposition to the current what sign should be used? Why?

13. If the assumed direction of a current in a network is in error, how is this fact indicated in the result?

PROBLEMS ON CHAPTER V

109. A Le Clanché cell has an open circuit voltage of 1.4 volts. When the cell delivers 5 amp. its terminal voltage drops to 1.3 volts. What is the internal resistance of the cell?

110. A starting battery, consisting of three storage cells connected in series, has an open circuit voltage of 6.4 volts. When delivering 90 amp., its potential drops to 5.0 volts. What is the internal resistance of the battery and of each cell?

111. A gravity cell has an open circuit voltage of 0.9 volt and an internal resistance of 0.3 ohm. When its terminal voltage is 0.83 volt, what current is it delivering?

112. When the storage battery of problem 110 is supplying only the lighting load of 14 amp., what is its terminal voltage?

113. An automobile starting battery, when being charged, has an electromotive force of 6.6 volts and an internal resistance of 0.03 ohm. What voltage must the charging generator supply to the battery in order to charge it at the 30-amp. rate?

114. A storage battery consists of 55 cells each having an electromotive force of 2.1 volts and a resistance of 0.002 ohm. What current will it take if connected across 120-volt bus-bars? What power is being delivered to the battery? How much is stored and how much is lost as heat?

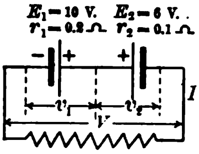


FIG. 115*a*.

115. In Fig. 115*a* are shown two cells connected in series and in series with a 3.7 ohm resistance. Determine the current I, the power p_1 and p_2 developed in each cell, the power P_1 and P_2 delivered by each cell, the power lost in each cell, the voltage v_1 and v_2 across each cell and the voltage V across the resistance. (If a cell is absorbing energy, the power developed is negative.)

116. Two Le Clanché cells, each having an electromotive force of 1.35 volts and an internal resistance of 0.15 ohm, are connected in series and supply current to a 10-ohm resistance wire. What current flows in the wire and what is the terminal voltage of the battery?

117. A station battery, consisting of 55 storage cells, all in series, each of which has an electromotive force of 2.2 volts and a resistance of 0.0005 ohm, supplies current to a load having a resistance of 5 ohms. What current does the battery deliver?

118. If the battery of problem 117 were accidentally short-circuited, what current would flow?

119. Each of five dry cells has an electromotive force of 1.4 volts; three have an internal resistance of 0.1 ohm and two have an internal resistance of 0.12 ohm. If these cells are all connected in series and to a circuit having a

resistance of 12 ohms, what current flows? What is the voltage across each cell?

120. Each of two starting batteries has an electromotive force of 6.5 volts; one has an internal resistance of 0.008 ohm and the other a resistance of 0.010 ohm. What is the equivalent resistance of the battery consisting of the two connected in parallel? What is the terminal voltage of the combined battery when it delivers 150 amp.? How does this current divide between the two individual batteries?

121. What is the resistance of a battery obtained by connecting all the cells of problem 119 in parallel? When a resistance of 0.8 ohm is connected across the terminals of this battery, what current flows? What is the terminal voltage of the battery and how much current does each cell deliver?

122. A battery consists of four storage cells all connected in parallel. The internal resistances of these cells are 0.006, 0.004, 0.003 and 0.0025 ohm respectively. If the electromotive force of each is 2.2 volts, what current does the battery deliver when its terminal voltage is 1.9 volts?

123. Twenty-four dry cells are arranged in rows of six in series and the four rows in parallel. The electromotive force of each cell is 1.4 volts and the resistance of each is 0.1 ohm. What is the total battery voltage and what is its total resistance? If an external resistance of 2.5 ohms is connected across its terminals, what current flows?

124. Arrange the cells of problem 123 so that the maximum amount of power may be supplied to a load resistance of 0.6 ohm. Under these conditions how much power is absorbed by the resistance and how much is lost in the battery?

125. A certain load is such that the potential difference at its terminals must not be less than 6 volts. Twelve storage cells, each having an electromotive force of 2.1 volts and a resistance of 0.02 ohm, are available. How should these be connected so that the maximum efficiency is obtained? When the load requires 10 amp., what is the battery terminal voltage? What is the load resistance? What is the battery efficiency?

126. Arrange the cells in problem 125 so that the maximum amount of current will be delivered to the load resistance. What is the efficiency of the battery under these conditions?

127. A telegraph battery consists of 12 gravity cells, each having an electromotive force of 0.9 volt and a resistance of 0.2 ohm. How should these cells be connected so as to operate most satisfactorily a 20-ohm relay over a 50-ohm circuit? What is the battery efficiency under these conditions?

128. Two batteries *A* and *B* (Fig. 128a), having electromotive forces of 4 and 3 volts and resistances of 1.2 and 1.0 ohm respectively, are connected in parallel, positive terminal to positive terminal. What current flows through a 2-ohm resistance connected across the battery terminals? What is the battery terminal voltage and how much current does each battery deliver?

129. Two batteries, having electromotive forces of 6 and 5 volts and resistances of 1.0 and 0.5 ohms respectively, are connected in parallel, positive terminal to positive terminal (Fig. 129a). These two supply current through a 1.0-ohm resistance to charge a 2-volt battery of a resistance of 0.3

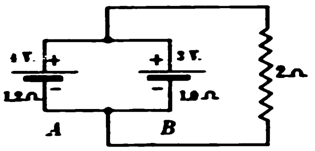


Fig. 128a.

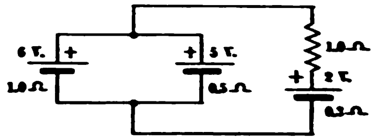


Fig. 129a.

ohm: the 2-volt battery is so connected that the current flows *in* at its positive terminal. What is the charging current of the 2-volt battery? What is its terminal voltage?

130. Across the terminals of a 12-volt, 0.2-ohm battery, a resistance wire of 10 ohms is connected. The negative terminal of a 6-volt battery, whose resistance is 0.15-ohm, is connected to the negative terminal of the 12-volt battery. The positive terminal of the 6-volt battery is connected to the resistance wire at a point $^3\!/_4$ its length from the negative terminals (see Fig. 130a). Determine the currents and the terminal voltages of each battery.

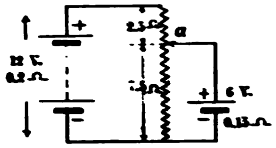


Fig. 130a.

131. (*a*) Determine the currents and the terminal voltages of each battery in problem 130 if the 6-volt battery is reversed.

(*b*) At what point must the contact *a*, Fig. 130*a*, be placed upon the resistance wire so that no current flows in the 6-volt battery circuit?

132. Two sub-stations *A* and *B* feed into the same distributing center. The voltage at the bus-bars of station *A* is maintained constant at 600 volts

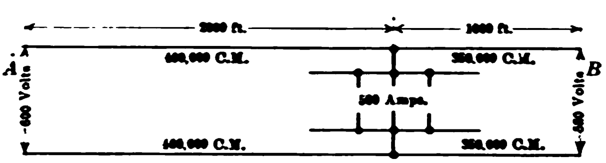


Fig. 132*a*.

and that at station *B* is maintained at 580 volts. Station *A* feeds a distance of 2,000 ft. through 400,000 C.M. cable and station *B* a distance of 1,000 ft. through 350,000 C.M. cable (see Fig. 132*a*). When the load at the distributing center is 500 amp., how much does each station supply? How

much power does each station supply, and how much is received at the distributing center?

133. Fig. 133*a* shows a distribution system. The voltage at the sub-station *A* is maintained constant at 240 volts. A radial feeder extends from *A* to each of the distributing centers *B*, *C* and *D*. The feeder to *B* is 2,300 ft. long and 2,000,000 C.M. equivalent; that to *C* is 1,800 ft. long and 2,500,000 C.M. equivalent; that to *D* is 2,000 ft. long and 2,000,000 C.M. equivalent (per wire in every case). A tie line 1,100 ft. long and of 500,000 C.M. connects *B* and *C* and another similar line connects *C* and *D*. At *B* is a load of 1000 amperes; at *C* a load of 500 amperes; and at *D* a load of 800 amperes. Find the voltage at each of the distributing centers *B*, *C*, and *D*.

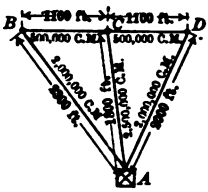


FIG. 133*a*.

QUESTIONS ON CHAPTER VI

1. What occurs if two copper strips be immersed in a dilute sulphuric acid solution and a voltmeter connected between them? If the two copper strips be replaced by two zinc strips? By two lead strips? Under what conditions may a voltage between the strips be obtained?

Would a voltage exist if the sulphuric acid were replaced by some other type of solution? Name three other such solutions?

2. What is meant by one metal being electrochemically positive to another? If metal *A* is electrochemically positive to metal *B*, what will be the direction of the current flow between them within the cell? What will be the direction of the current flow between them through the external circuit?

What is an electrode? What is the cathode? The anode?

3. In what form is the energy stored within the cell? What changes take place in the electrodes when the cell delivers current? Distinguish between a primary cell and a secondary cell.

4. What are the four requirements for a satisfactory primary cell?

5. What is the nature of the internal resistance of a cell? In what manner may this resistance be reduced? In what way does increasing the size of the elements of a cell increase its current capacity? Its electromotive force?

6. What voltage does a voltmeter indicate when it is connected to the terminals of a cell which is open-circuited? If the circuit is suddenly closed, to what is the initial voltage drop due? To what is the excess drop over this initial drop due? Explain the part that hydrogen plays in polarization. Describe two general methods of reducing polarization.

7. Describe the construction of the Daniell cell. What electrodes and what electrolytes are used? For what type of work is it designed? What is the electromotive force of this cell?

8. In what way does the gravity cell differ from the Daniell cell? Which electrode requires replacing? What occurs with the other electrode? What is the cell electromotive force and for what type of work is the gravity cell designed?

9. Describe the Edison-Lalande cell. What electrolyte and what electrodes are used? In what way does its electrolyte differ from the cells already described? What is the chief advantage of this type of cell? What is its electromotive force and what is its terminal voltage when delivering a current?

10. What materials are used for the positive and for the negative electrodes in the Le Clanché cell? What is the electrolyte? What is its electromotive force? When planning to use the cell commercially, what voltage per cell should be allowed? What materials are introduced in the cell to reduce polarization? How is the cell renewed? For what type of work is this cell best suited?

11. What is the function of a Weston cell in distinction to the uses made of other types of cells? In practice what two common electrical quantities are most easily reproduced and maintained? What must be the characteristics of a standard cell? How is the Weston cell constructed and how is its permanency insured? In what way does the saturated cell differ from the normal cell? Why cannot the voltage of the Weston cell be measured with an ordinary voltmeter?

12. In what way does a dry cell resemble a common type of wet cell? Is a dry cell really "dry?" Of what is the positive electrode composed? The negative? What is the electrolyte and how is it placed in the cell? What materials are placed between the carbon and the zinc and what are their functions?

13. What is the electromotive force of a dry cell when new? After it has stood idle for some time? What is the magnitude of the internal resistance when new and is it subject to change? How does the polarization effect compare with the internal resistance effect? How much current should a good cell deliver upon short circuit? What is the terminal voltage when a cell delivers current?

14. To what cause is the cell's becoming exhausted principally due? Can this cell be temporarily revived by any means? Name some of the commercial applications of dry cells.

15. In what way is a storage cell renewed when it becomes discharged? What condition concerning the materials of the cell is necessary for proper functioning of the cell? What two general types of storage cells are in commercial use?

16. Describe a very elementary experiment which illustrates the underlying principle of the lead cell. State the change that occurs in each of the lead strips; what voltage is observed to exist at different times in the experiment. What gases are evolved and from which plate does each emanate?

17. Even although both of its plates are of lead show that the existence of an emf. in a lead storage cell does not in any way violate the principle governing the emf. of electric cells in general. When the cell is approaching discharge what change in the materials would account for the approach of the voltage to zero? In what way is the 2.5 volts per cell utilized in the process of charging?

18. What change in the electrolyte during the charge and the discharge of a cell is shown by the chemical equation? Why is a cell composed of plain lead plates not useful in practice? Give two reasons. Describe briefly the Planté process and describe two plates that are formed by this process.

19. Describe the Faure or pasted process for making battery plates. What are the advantages and the disadvantages of pasted plates over the Planté plates? What commercial conditions demand a pasted plate and why? How does the life of a pasted plate compare with that of a Planté plate?

20. Describe briefly the construction of the "Iron-clad" Exide cell and its principal use in practice.

21. What are the two general classes into which storage batteries may be divided? What types of plate are best suited for regulating duty and for emergency duty in stationary batteries? Why?

22. What two types of containing tanks are used for stationary batteries? Under what conditions is each used and why? In what manner should the joints and seams in lead-lined tanks be made non-leakable? How are the plates suspended in the lead tank? What factors must be considered in designing and installing a lead-lined wooden tank?

23. What three types of separators are in general use? Name the advantages and the disadvantages of each type. For what type of battery is each kind commonly used? What one precaution must be taken in handling wood separators? Why?

24. What should be the specific gravity of a fully charged battery having Planté plates? Pasted plates?. What precaution should be taken in diluting sulphuric acid for storage battery use? What simple device is used for determining specific gravity? How is this device adapted for use with vehicle and portable batteries?

25. What change takes place in the electrolyte during the charging period? What is the effect of gassing on the specific gravity? What change takes place in the specific gravity after the charging has ceased? Explain. How does the specific gravity of the electrolyte change during discharge? What practical use is made of these changes of specific gravity?

26. When a battery is received, what special attention should be given to the wood separators? In what manner should the jars be installed? How should the plates be placed in position? Why is an initial charge necessary and what should be its duration?

27. What happens to the active material in a cell if it is allowed to stand idle over long periods? In what way may injury to the battery from this cause be avoided? If it is desired to withdraw a battery from service for an indefinite period, what procedure should be followed?

28. What are the requirements of a vehicle battery that make its design different from that of a stationary battery? What changes are made in the plates? Separators? Specific gravity of the electrolyte? How is a battery made up? In what way does a vehicle battery differ from a stationary

battery in the manner of shipment? What special attention should be paid to the electrolyte?

29. In what manner is the rating of a storage battery expressed? What is meant by the 8-hour rate? Can as many ampere-hours be extracted from a cell at the 3-hour rate as at the 8-hour rate? To what is this difference due? If a cell is apparently exhausted after discharging at the 3-hour rate, would it be possible later to extract any further current from it? What can be said of the overload capacity of a storage battery?

30. What two general methods of charging are commonly employed? In each method and with pasted plates what value of current should be employed when the charging commences? When does it become necessary to reduce this current? What are the objections to pronounced gassing in a cell? How does the charging rate with Planté plates differ from that with pasted plates?

31. Name a very common example of constant current method of charging. What care should be taken in the connecting up of the battery? Describe a simple test by which the determination of the correct terminal polarity may be ascertained.

32. What is the one great advantage of the constant potential rate of charging? About what voltage per cell is necessary in this method?

33. When a battery is just floating on a bus-bar and it is desired to charge it, in what manner may the necessary excess potential for charging be obtained? Does the generator employed supply the entire energy necessary for charging?

34. What change occurs in the electromotive force of a cell during the charging period? What corresponding changes occur in the terminal voltage? To what is the discrepancy between the cell electromotive force and the terminal voltage due? Can it be said that the voltage characteristic of a storage battery is such that its use upon lighting circuits is practicable?

35. What is lost by a lead storage battery during its period of service? With what should this loss be replaced except in rare instances? What circumstances justify the addition of acid to a cell? What care should be taken in the selection of water for use with storage batteries?

36. In what manner can the freezing of the electrolyte in a storage battery be absolutely prevented? How does a rise of temperature affect the rating of a storage battery?

37. Compare roughly the kilowatts per pound of plate for a given cell at different discharge rates. Repeat for kilowatts per pound of cell. Compare the above factors for three different types of cell, stating the type of service for which each type is best adapted.

38. Of what is the positive plate, the negative plate and the electrolyte composed, in an Edison cell? In the chemical reaction that takes place both on charge and on discharge, what part does the electrolyte play? How does its specific gravity change during charge and discharge?

39. Describe briefly the mechanical construction of the Edison cell, stating the method of holding the plates and connecting them with the binding posts. What kind of a tank is used for this cell? What is

the advantage of this type of construction? For what purpose is the valve necessary and what care does the valve require? How is the battery mounted?

40. In what way does the normal rating of an Edison cell differ from that of a lead cell? What is the voltage per cell? Is it possible to tell accurately the condition of charge by readings of either voltage or of specific gravity? How can complete charge be assured?

41. What should be used to replace evaporation of the electrolyte? Is any greater care required in the selection of water for the Edison battery than for the lead battery? Explain.

42. State the advantages of the Edison battery over other types of storage batteries. What are some of the commercial applications of the battery and what factors limit the applications of the battery? Compare the weights per kw. with similar weights for the lead cell.

43. In what terms is the efficiency of a storage battery expressed? Is the ampere-hour efficiency a true indicator of efficiency?

44. State the reason why the ratio of the kilowatt-hours of discharge at the 3-hour rate to those of charge at the 8-hour rate does not give the true efficiency. Give some of the factors which determine the efficiency of a battery.

45. What is the order of magnitude of the kilowatt-hour efficiency of a lead storage battery? The ampere-hour? Why do the two differ? In what manner does the cycle of operation of a storage battery affect the efficiency?

46. What is the approximate kilowatt-hour and the ampere-hour efficiency of an Edison battery?

47. State some of the factors which govern the selection of a storage battery for any particular purpose.

48. State a simple method of producing copper plating upon a carbon brush such as is used with generators. Which electrode is connected to the positive terminal of the supply and which is connected to the negative terminal? When copper is used in connection with a copper sulphate solution, is there any marked change in the electrolyte? Explain.

49. Can copper be plated from a solution in which neither terminal is copper? What voltages in the plating bath must the supply voltage overcome? How are these voltages reduced to a minimum? Is electroplating considered a high voltage or a low voltage process? In what way are plating baths connected, when possible?

50. Show how the gravity cell is an electroplating bath which supplies its own electroplating current.

51. Describe briefly the process of electrotyping.

PROBLEMS ON CHAPTER VI

134. A Daniell cell has an electromotive force of 1.07 volts and an internal resistance of 0.2 ohm. (*a*) What is the maximum current which it can deliver? The size of the cell is increased in such a manner that the plate area is doubled. (*b*) What is the new electromotive force? (*c*) What is the approximate maximum current that the cell can now deliver?

135. Two gravity cells have electrodes of the same materials and solutions of the same kind, concentration, etc., but one cell has each linear dimension twice that of the other, making its volume eight times greater. The two cells are connected with terminals of like polarity together. If the electromotive force of the smaller cell is 1.0 volt and its resistance 0.4 ohm, how much current flows between the two cells? Give reasons for the answer. If the plate area of the larger cell is four times that of the smaller, what will be its short-circuit current, approximately?

136. A Le Clanché cell has an electromotive force of 1.43 volts on open circuit. A load of 2 amp. is suddenly applied and the terminal voltage drops to 1.25 volts almost instantly. After a lapse of some time it drops to 1.06 volts. What is the actual internal resistance of the cell and what is the "electromotive force of polarization"? What is the total apparent cell resistance?

137. A telegraph relay has a resistance of 150 ohms and the loop resistance of the sending circuit is 1,600 ohms. The relay requires 50 milliamperes for satisfactory operation. How many gravity cells, each having an electromotive force of 1.09 volts and an internal resistance of 0.4 ohm, are required at the sending end? How should they be connected?

138. A railway signal circuit consists of a 4-ohm relay, a track and connecting resistance of 3 ohms. The relay requires 75 milliamperes to attract its armature satisfactorily. Two gravity cells, each having an electromotive force of 1.05 volts and a resistance of 0.3 ohm, are used to operate this signal relay. What extra resistance in series with the battery is required?

139. A certain signal motor requires 4 amp. at 10 volts at its terminals for satisfactory operation. The leads from the battery to the motor have a total resistance of 1.4 ohms. How many Edison-Lalande cells would be necessary to operate this system? (See Par. 87.)

140. Three Le Clanché cells, connected in series, are used to operate a door opener which has a resistance of 1 ohm. The resistance of the connecting wires is about 0.5 ohm. What is the approximate current taken by the door opener? (See Par. 88.)

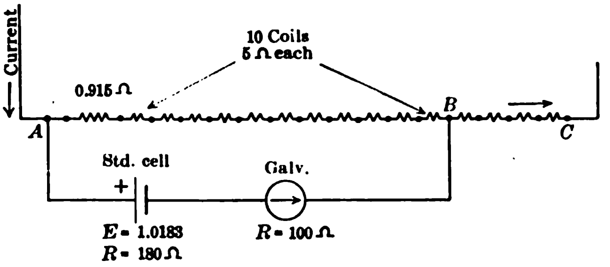


FIG. 141*a*.

141. A Weston cell having an electromotive force of 1.0183 volts is connected to a potentiometer wire as shown in Fig. 141*a*, in order to calibrate the wire *AC*. Between *A* and *B* is a resistance of 0.915 ohm and 10 coils

having a resistance of 5 ohms each. The cell has a resistance of 180 ohms and the galvanometer a resistance of 100 ohms. When the current in AC is 0.021 amp., how much current passes through the galvanometer?

For what value of current in the wire AC will the current through the cell and galvanometer be zero? Under these conditions what will be the voltage across each of the 5-ohm coils?

142. A voltmeter having a resistance of 1,000 ohms is used in an attempt to measure the electromotive force of the Weston cell in problem 141. What will the voltmeter read? Is this a practicable method of using the Weston cell as a standard?

143. The ignition system on an automobile requires 6 volts for satisfactory operation. How many dry cells should be recommended for this purpose?

144. A dry cell shows an open-circuit emf. of 1.2 volts and a short-circuit current of 4 amp. What is its internal resistance? What does this test show as regards the condition of this cell?

145. A certain flashlight has a 2 candle-power lamp whose efficiency is 1.2 watts per candle. If this lamp is operated by a single dry cell, in first class condition, approximately what current does it take and what is its voltage?

146. A storage cell is being charged at the normal rate as indicated by an ammeter. A voltmeter across its terminals indicates 2.2 volts. At what part of the charging period is the cell operating at this time? (See Fig. 104.)

147. A storage cell has an 8-hour rate of 40 amp. This rate is maintained constant for the 8 hours of charge. During this period the voltage rises according to the curve shown in Fig. 104. How many ampere-hours are delivered to the cell? How many watt-hours? (Note: Mark several equally spaced points on the voltage curve and take their average.)

148. If the cell of problem 147 discharges at the 8-hour rate and its voltage follows the 8-hour discharge curve of Fig. 104, how many watt-hours are discharged?

149. It is desired to dilute a quart of concentrated sulphuric acid, sp. gr. = 1.84, to make acid having a specific gravity of 1.240. How much water is needed and what is the total volume of acid when the solution is mixed? State the procedure that should be followed in mixing the liquids.

150. A gallon of water weighs 8.3 lb. How much will 5 gallons of battery acid (sp. gr. = 1.210) weigh?

151. What is the percentage by weight of acid (sp. gr. = 1.84) in the acid solution of problem 150?

152. The hydrometer in a pilot cell of a stationary battery indicates a specific gravity of 1.190. How many more hours should the battery be left charging (Fig. 98)?

153. A hydrometer test of the electrolyte in a vehicle cell shows the specific gravity to be 1.185. If this cell is one of an electric vehicle battery engaged in propelling a vehicle, how near complete discharge is the battery?

154. A battery having a normal rating of 40 amp. at the 8-hour rate is just received new. How many ampere-hours should be sent through it before it is ready for active service?

155. The average charging voltage per cell in problem 154 is 2.3 volts. There are 40 cells in series and a 0.5-ohm resistance in series with the battery. At 5 cents per kw.-hr., what is the energy cost of getting the battery ready for service?

156. If a vehicle battery had a specific gravity of 1.200, what would be an estimate of its condition of charge?

157. A battery is charged at the 80-amp. rate for 6 hours. How many ampere-hours has it absorbed? If the ampere-hour efficiency is 95 per cent., for how many hours can it discharge 60 amp.?

158. If the battery of problem 157 is of the pasted plate type, what current will it discharge at the 3-hour rate? How many ampere-hours does it discharge at this rate? (See Par. 101.)

159. What current and how many ampere-hours will the battery of problem 157 deliver at the 1-hour rate?

160. A battery has a rating of 320 ampere-hours. At what value of current should the charging be started if the plates are of the pasted type? Of the Planté type? (See Par. 101.)

161. It is desired to charge a starting battery from 110-volt d.c. mains. The battery consists of three cells each having a terminal voltage of 2.5 volts when being charged at the normal rate of 12 amp. How much resistance must be inserted in series with this battery? What percentage of the power supplied is reaching the battery?

162. If two batteries each similar to that of problem 161 are being charged in series at the same rate, what series resistance is necessary? What percentage of the power supplied reaches the batteries?

163. A storage battery of 115 cells is floating on 230-volt bus-bars. It is desired that the battery begin to discharge when the bus-bar voltage is exactly 230 volts. On charge it is necessary to have 2.4 volts per cell. What capacity of booster is required if the normal charging current is 60 amp.? How much power is delivered to the battery by the booster? How much is supplied directly by the bus-bars?

164. If the booster generator of problem 163 has an efficiency of 78 per cent. and the shunt motor which drives it has an efficiency of 80 per cent., what power does the booster set take from the bus-bars?

165. A storage battery of 50 cells has a total internal resistance of 0.5 ohm and is charged from 115-volt d.c. mains. At the beginning of charge its electromotive force is 1.8 volts per cell. (*a*) What current does it take? After charging 4 hours the electromotive force rises to 2.0 volts per cell. (*b*) What current does it take at this time? (*c*) What must be the electromotive force per cell when the battery stops taking current? What method of charging is used and is it a desirable method?

166. The specific gravity in a vehicle battery is found to be 1.240. Is there any possibility of its freezing in the climate of the United States? Give reasons.

167. It is desired to install a 110-cell stationary battery having a capacity of 1.22 kw. per cell at the 4-hour rate. (*a*) What will be the approximate weight of the plates of this battery? (*b*) Of the total battery? (See Par. 105.)

168. What will be the weight of a 24-cell vehicle battery composed of "iron-clad" cells, this battery to have a total output of 1.28 kw. at the 8-hour rate?

169. Approximately how many Edison cells would be required for a 24-volt lighting project?

170. It is desired to install a generator to charge a 60-cell Edison battery. The normal charging rate is 20 amp. What size generator is necessary (kilowatts, amperes, volts)? (See Fig. 108.)

171. What will be the weight of a 50-cell Edison battery designed to deliver 15 kw.-hr. at the 8-hour rate? (See Par. 108.)

172. What will be the weight per kilowatt of this battery?

173. A lead cell is charged at a 40-amp. rate for 10 hours with an average potential difference across its terminals of 2.3 volts. It discharges 45 amp. for $8\frac{1}{2}$ hours at an average terminal voltage of 1.95 volts. What is its ampere-hour efficiency? What is its watt-hour efficiency?

174. A storage battery in its discharged condition is charged at the 100-amp. rate at an average voltage of 250 volts for 9 hours. It delivers 105 amp. at an average terminal voltage of 220 volts for 8 hours before it is again in the discharged condition. What is its kilowatt-hour efficiency?

175. An Edison battery of 12 cells is charged for a period of 6 hours at the 25-amp. rate and the average terminal voltage per cell is 1.65 volts. The battery discharges 5 hours at the 28-amp. rate with an average terminal voltage of 1.2 volts. What is its ampere-hour and what is its watt-hour efficiency?

176. One ampere-hour will deposit 0.843 gram of copper upon the cathode in an electroplating bath. If the voltage across a plating bath is 12 volts and the current is 12 amp. and the current is allowed to flow for 6 hours, how many kilograms of copper are deposited and how many kilowatt-hours are utilized in the process?

QUESTIONS ON CHAPTER VII

1. If a coil carrying a current be placed in a magnetic field, what effect is noticed? Give two explanations of this effect. Of what importance is this principle?

2. How is the principle of the moving coil adapted to measuring small currents in the D'Arsonval galvanometer? How is the coil suspended? How is the current led in and out of the coil? Why is a soft-iron core placed between the poles?

3. What two common methods are used to read the galvanometer deflection? What is meant by the "damping" of a galvanometer? How may this damping be accomplished?

4. How may a galvanometer be protected from excessive currents? Sketch the connections of two types of shunt. What are the advantages of the Ayrton shunt?

5. What was the underlying principle of the early types of electrical instruments? What two factors caused these instruments to be inaccurate?

6. Show that the movement of a Weston d.c. instrument is an evolution of the D'Arsonval galvanometer. How is the moving coil pivoted? How is the current led to the coil? What means are used to oppose the motion of the coil? Is the coil damped? Explain. What is meant by a "radial field" and what effect does it have on the calibration of the instrument scale? Why are the top and the bottom springs coiled in opposite directions? Is it possible to utilize the movement of a Weston instrument as a galvanometer?

7. Of what order of magnitude is the current that will give full-scale deflection in a Weston instrument? Is it possible to use the instrument for measuring current in excess of this value? Explain.

8. Describe briefly the construction of a shunt. Why are four posts or terminals necessary? Show that when a Weston instrument is used in connection with a shunt, it is acting as a voltmeter.

9. What law does the current follow in dividing between the shunt and the instrument? Why should the resistance of the shunt and the resistance of the instrument remain constant? What errors may be caused by the heating of the shunt or of the instrument?

10. In what way may an instrument be made to have several scales? In general, when is an internal shunt used? An external shunt?

11. Does the movement of a voltmeter differ materially from that of an ammeter? In what important respect does the voltmeter differ from the ammeter? How is the current in the coil of a voltmeter limited when the voltmeter is connected across the line?

12. Is it possible for a voltmeter to have more than one scale? Explain. What is meant by a multiplier or extension coil?

13. In what manner may the heating effect of an electric current be utilized to measure the value of the current? State some of the advantages and the disadvantages of hot wire instruments.

14. Show the connections that are used in measuring resistance with a voltmeter and an ammeter. What precaution should be taken in connecting the voltmeter? What special type of voltmeter contact should be used in measuring very low resistances?

15. Show the connections that can be used in measuring resistance by a voltmeter alone. What is the order of magnitude of resistances that can be measured by this method? What special type of voltmeter is often desirable for this work and why? To what type of resistances is this method especially applicable?

16. Sketch an arrangement of four resistances, a battery and a galvanometer, whereby one of the resistances may be measured. How is the condition of "balance" in the bridge detected? Prove the law of proportionality that exists when this condition of balance has been reached.

17. What two types of bridge are in general use? Compare them from the standpoint of ease of manipulation; plug-contact resistance; convenience.

18. Give briefly the procedure which should be followed in obtaining a balance with a plug bridge.

19. In what way does the slide wire bridge resemble the Wheatstone bridge? Compare it with the Wheatstone bridge from the standpoint of simplicity and accuracy.

20. Give the connections whereby the slide wire bridge may be put to practical use in locating an earth fault in a cable. What is the name of this method? Explain why the galvanometer and battery do not occupy the same positions in the slide wire bridge of Fig. 133 as they do in Fig. 132.

21. Sketch the connections used in the Varley loop. Upon what arm is the balance obtained? What additional factor must be known before the position of the fault can be determined? Was it necessary to know this factor in the Murray loop test? Which is the simpler method? What possible sources of error exist?

22. Why is it desirable in practice to know the insulation resistance of cables? Why is the voltmeter method not always practicable? What is the general principle of the method described in Par. 124?

23. What method is used to obtain readable deflections of the galvanometer under all conditions of circuit resistance? Why is it desirable to keep the 0.1 megohm in circuit continually and does it introduce any appreciable error?

24. What other factor besides the resistance of the insulation affects the value of the current flowing in the circuit? What time of electrification has been adopted as standard in commercial measurements of insulation resistance? What precautions should be observed in the installation of cable testing apparatus?

25. Upon what standard do potentiometer measurements primarily rest? Against what is the standard cell balanced? What care as regards polarity must be observed if a balance is to be obtained? Why is a "nul" method the only one which will give satisfactory results when a standard cell is used?

26. Show how a wire may be calibrated and marked in volts, after the standard cell balance has been obtained. Is it possible to measure other electromotive forces with this standardized wire? What method is employed in such measurements?

27. Does the Leeds & Northrup potentiometer whose connections are shown in Fig. 138 differ materially from the simple device sketched in Fig. 137? What minor changes are necessary? Where are the one-tenth volt divisions located and how are they utilized when obtaining a balance? How are the smaller decimal divisions obtained? What resistances are used in each of these units? What is the working current of this potentiometer?

28. What provision is made for the variations in the voltages among standard cells? What protection is afforded the galvanometer during the preliminary adjustments?

29. What is the maximum voltage measurable with this potentiometer alone? By what means can voltages in excess of this be accurately measured? Is the device used for increasing the voltage range of the potentiometer in any way complicated? What is meant by a "drop wire" and how may it be used to vary the voltage when the supply is at constant voltage?

30. Is this potentiometer, as a voltage-measuring device, adapted to measuring currents? What is the principle underlying the measurement of current? What is a standard resistance? Why does it have four posts? In what units of resistance are standard resistances generally manufactured? Why is it desirable that their temperature remain normal and what means are adopted to accomplish this?

31. What instruments are generally used in measuring the power in a direct current circuit? Do these instruments take any power themselves? What should be the relative positions of the voltmeter and the ammeter when the power delivered to a high resistance is being measured? When that delivered to a low resistance is being measured?

32. Describe a wattmeter. In what way do the fixed and moving coils differ in construction? In their manner of connection to the circuit? Why are the instrument deflections proportional to power? What care is necessary when using this type of instrument with direct currents?

33. What does a watthour meter measure? Upon what familiar electrical device is it based? From what source are its field coils supplied? Its armature? To what is the torque acting upon the armature proportional?

34. Why is a retarding device necessary and what must be the law of retardation? Upon what principle does this device operate?

35. At what values of meter load does friction produce the greatest error? Explain. How is this friction error practically eliminated?

36. What methods are used to reduce friction in a watthour meter? What are some of the causes of a meter running slow? How is the recording dial of a meter actuated?

37. Why is it usually very important that a watthour meter register accurately? What load and measuring devices are necessary in testing a meter?

38. What is the fundamental relation between the revolutions of the disc and the energy registered by the meter? What measurements are made in checking the meter?

39. What two adjustments are made to change the meter speed? What is the effect of moving the magnets nearer the center of the disc? Nearer the periphery? At what loads is this adjustment made?

40. What adjustment is made to correct the meter registration at light loads? Why is this adjustment made at light rather than at heavy loads?

41. In what general respect does a three-wire meter differ from a two-wire meter?

42. Describe in a general way the construction of a meter which makes the meter practically astatic and therefore enables it to be used near bus-bars

carrying heavy currents. What two elements in a meter are most likely to be affected by stray fields? How are these elements safeguarded from these effects?

PROBLEMS ON CHAPTER VII

177. A galvanometer has a resistance of 351 ohms. What should be the resistance of a shunt for use with this galvanometer if it be desired that $\frac{1}{10}$ the total current of the line pass through the galvanometer? If it be desired that $\frac{1}{100}$ of the line current pass through the galvanometer?

178. The resistance of a certain galvanometer is 495 ohms. Design a shunt which will allow $\frac{1}{10}$, $\frac{1}{100}$, and $\frac{1}{1000}$ the line current to pass through the galvanometer.

179. An Ayrton shunt (Fig. 179*a*) has a resistance from *A* to *B* of 10,000 ohms. It is used to shunt a galvanometer having a resistance of 2,000 ohms. When the shunt is set at the 0.001 point (the resistance *AC* = 10. ohms), determine the current through the galvanometer when 1 milliampere flows in the line.

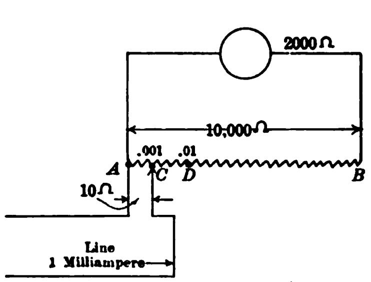


FIG. 179*a*.

180. If the line contact be moved to the 0.01 point at *D* (Fig. 179*a*), the resistance *AD* being 100 ohms, determine the current through the galvanometer when the line current is 1 milliampere.

181. Repeat problem 180 when the line contact is moved to *B*. Compare this result with those of problems 179 and 180. If the shunt were removed what current would now pass through the galvanometer? How much does the shunt reduce the ultimate sensitivity of the galvanometer?

182. A 50-scale millivoltmeter has a resistance of 2 ohms. It is desired that it measure a current of 75 amp. at full-scale deflection. What should be the resistance of the shunt under these conditions? How much current flows through the instrument and can it be neglected as compared with the current in the shunt?

183. Find the resistances of shunts necessary for measuring currents of 150 amp. and 500 amp., full-scale deflection, with the instrument of problem 182.

184. An instrument has a resistance of 25 ohms. It is used to measure a current of 60 amp. The shunt has a resistance of 0.00075 ohm. How much current passes through the instrument? Through the shunt? What is the rating of the instrument in millivolts?

185. It is desired to measure a current of 50 amp. An internal shunt, 5-scale ammeter alone is available. This instrument has a resistance of 0.01 ohm. What should be the resistance of a shunt to be used with this instrument?

186. A 15-scale voltmeter has a resistance of 160 ohms. What should be the resistance of a multiplier to increase the range of this instrument to 150 volts?

187. A 150–15-scale voltmeter has a total resistance of 17,500 ohms. What is the resistance between its 15-volt scale binding posts? What multiplier resistance will give this instrument a range of 600 volts?

188. It is desired to measure the potential difference between a trolley and ground. A 40,000-ohm and a 10,000-ohm resistance are connected in series between the trolley and the ground. Across the 10,000-ohm resistance a 50-scale voltmeter having a resistance of 5,100 ohms is connected. When this instrument reads 45 volts, what is the trolley voltage?

189. What multiplier resistance would have been necessary in problem 188 to have obtained the same multiplying power for the voltmeter?

190. A 100-watt lamp when connected across d.c. mains is observed to take 0.9 amp. at 115 volts. What is its hot resistance?

191. When the armature of a 220-volt, 10-hp. motor is stationary, a current of 40 amp. gives a voltage drop across its terminals of 8 volts. What is the resistance of the armature?

192. The current in problem 191 was taken from d.c. mains whose potential difference was known to be 115 volts. What resistance was connected in series with the armature?

193. The resistance of a sample of copper bus-bars is measured by the method shown in Fig. 126. When the ammeter reads 140 amp., the millivoltmeter reads 3.5 millivolts. The bus-bar is 0.5 in. by 2 in. in cross-section and the distance between voltmeter contacts is 3 ft. (*a*) What is the resistance of the sample? (*b*) What is its resistance per cm. cube? (*c*) What is its per cent. conductivity? (See Par. 39, Chap. III.) Res. of standard copper = 1.724 microhm-centimeters.

194. It is desired to obtain the resistance of a 70-lb. rail. A current of 350 amp. is sent through the rail and a millivoltmeter is connected between two contact points on the rail spaced 6 ft. apart. The millivoltmeter reads 5.1 millivolts. What is the resistance per ft. of the rail?

195. A 300-scale voltmeter having a resistance of 35,000 ohms is connected across d.c. mains and indicates 225 volts. It is then connected in series with an unknown resistance across these same mains. It now indicates 48 volts. What is the value of the unknown resistance?

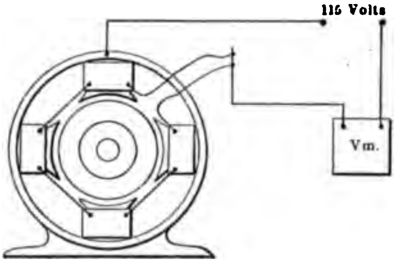


Fig. 196*a*.

196. A special 150-scale 100,000-ohm voltmeter, when connected across d.c. mains, reads 115 volts. The iron frame of a generator is connected to one wire of these mains and the copper of the field coil is connected to the other wire through the voltmeter, as shown in Fig. 196*a*. Under these conditions the voltmeter reads 8 volts.

What is the resistance in megohms of the insulation of the field circuit to the frame of the machine?

197. In a Wheatstone bridge measurement the unknown resistance is connected between one end of the arm M and the arm P. (See Fig. 128.) When a balance is obtained M = 10 ohms; N = 1,000 ohms; P = 1.426 ohms. What is the value of the unknown resistance?

198. A resistance whose value is known to be between 10 and 20 ohms is connected to a Wheatstone bridge as shown in Fig. 128. What are the best values of M and N to use? If the unknown resistance is 16.72 ohms, what will P read when a balance is obtained?

199. An unknown resistance is measured by means of a 100-cm. slide wire bridge. A known resistance of 100 ohms is inserted at the 100-cm. end of the bridge. (See Fig. 132.) A balance is obtained when the slider reads 32.4 cm. What is the value of the unknown resistance?

200. If a 10-ohm resistance be used as the known resistance in problem 199, what will be the reading on the slide wire when a balance is obtained?

201. A cable 1,200 ft. long, wound on a reel, is known to have a fault in its insulation. It is immersed in a tank of water and the Murray loop test is used to locate the fault. The slide wire bridge, 100 cm. long, reads 18.4 cm. when the balance is obtained. What is the distance from one end of the cable to the fault?

202. An installed two-conductor cable of 4/0 copper is 3,200 ft. long. Due to a burn-out both conductors are short-circuited and grounded at the same point. To locate the fault a single 00 conductor of another cable which parallels the faulty one is looped to one conductor of the faulty cable at the

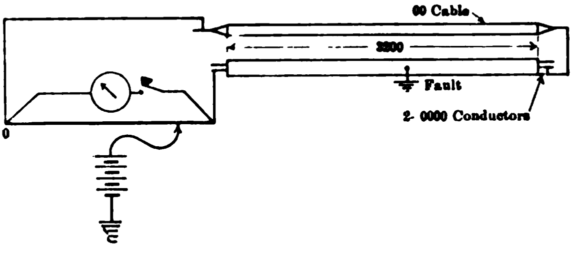


Fig. 202a.

far end, as shown in Fig. 202a. The perfect conductor is connected to the low reading end of the slide wire and the faulty conductor to the 100-cm. end. A balance is obtained at 89.4 cm. How far out on the faulty conductor is the burn-out located?

203. In a Varley loop test for a fault in a 1/0 conductor, 3,500 ft. long, this conductor is looped back through a perfect 00 conductor. The ratio arms are M = 10 and N = 1,000 ohms. (See Fig. 134.) A balance is obtained when P = 585 ohms. The bridge measurement of the entire loop shows its resistance to be 0.70 ohm. How far out is the fault?

204. One conductor in a cable containing two No. 14 wires a and b, each 8,000 ft. long, is known to be grounded. The two are looped at the

far end and the Varley loop test made. P is connected in series with conductor a. The two arms M and N (Fig. 134) are each set at 100 ohms. A balance cannot be obtained with P, as the galvanometer is found to deflect the same way with $P = 0$ and $P = \infty$. P is then shifted over in series with b, the other conductor, and a balance obtained when $P = 12.6$ ohms. In which conductor is the fault and how far is it from the home end of the cable?

205. In an insulation test of a cable the connections are made as in Fig. 135. When the cable is short-circuited and the Ayrton shunt is set at 0.0001, the galvanometer deflects 12.8 cm. The short circuit is then removed, putting the cable in circuit, and the galvanometer deflects 19.8 cm. with the shunt set at 1.0 after the cable has been charged for 1 minute. What is the insulation resistance of the cable?

206. The cable in problem 205 is 1,100 ft. long. What is its insulation resistance in megohms per mile?

207. It is desired to measure the terminal voltage of a storage battery by means of a standard cell. The ratio and rheostat arms of a Wheatstone bridge (Fig. 207a) are connected across the terminals of the storage battery and a standard cell having an electromotive force of 1.0176 volts is connected across a 1,000-ohm coil. The galvanometer in the standard cell circuit stands at zero when 1,050 additional ohms are unplugged in P. What is the terminal voltage of the storage battery?

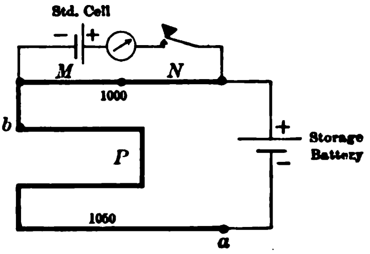


Fig. 207a.

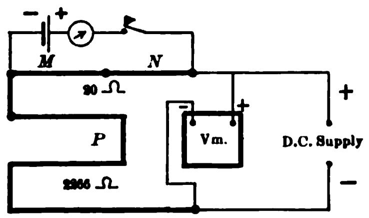


Fig. 209a.

208. The storage battery of problem 207 is of such comparatively large capacity that its electromotive force and terminal voltage are sensibly the same when delivering the small current required by the resistance of 2,050 ohms. To measure the electromotive force of another cell which is not capable of delivering any appreciable current, its negative terminal is connected to the point a and its positive terminal to the point b through a key and galvanometer (Fig. 207a). P is then adjusted until this galvanometer reads zero. P is then read and found to be 890 ohms. What is the electromotive force of this cell?

209. It is desired to calibrate a voltmeter at the 115-volt point. No potentiometer is available. The voltmeter is connected in parallel with the arms of a bridge box (Fig. 209a) and 115 volts is impressed upon this circuit. A standard cell, which is known to have an electromotive force of 1.0180 volts, is connected across the two ratio arms in series with a key and

galvanometer, the proper polarity being observed and 20 ohms are unplugged in these two arms. The galvanometer reads zero with the key depressed when 2,266 ohms are unplugged in *P*. What correction should be applied to the voltmeter at this point?

210. The power to a 25-watt tungsten lamp is being measured with a voltmeter and an ammeter. The voltmeter, which has a resistance of 12,000 ohms, is connected directly across the lamp terminals. When the ammeter reads 0.23 amp. the voltmeter reads 117 volts. What is the true power taken by the lamp? What per cent. error is introduced if the instrument power be neglected?

211. In measuring the power taken by a low resistance rheostat, an ammeter having a resistance of 0.0008 ohm and a voltmeter having a resistance of 120 ohms are used. When the ammeter reads 70 amp. the voltmeter, which is connected directly across the resistance, reads 2.1 volts. What is the true value of the resistance? What per cent. error is introduced by the voltmeter current? What error is introduced by connecting the voltmeter outside the ammeter?

212. In a test of a direct current watthour meter the average voltmeter reading is 118 volts and the average ammeter reading is 21.4 amp. Thirty revolutions are counted and the time is found to be 42.6 seconds. If the meter constant is 1.0, what is the per cent. accuracy of the meter at this load? What adjustment should be made to bring it nearer the correct registration?

213. The meter load (problem 212) is dropped to 1.0 amp. but the voltage is still 118 volts. It takes 62.6 seconds for the disc to make two revolutions. What is the per cent. accuracy of the meter at this point? What adjustment should be made in order to bring it nearer the correct registration?

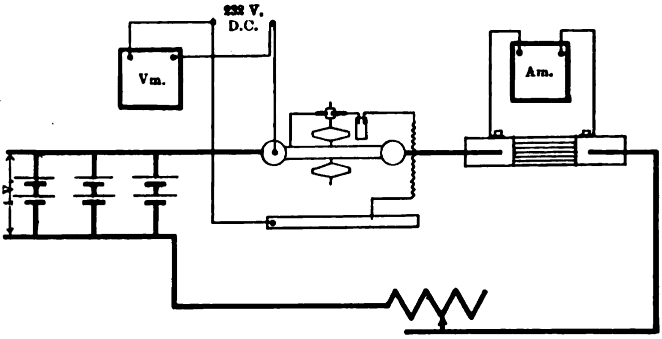


FIG. 214*a*.

214. In order to make a laboratory test of a 2,000-amp., 220-volt, astatic watthour meter, of the type shown in Fig. 147, its current coils are supplied with current from a 4-volt storage battery and its armature circuit, which has a resistance of 2,200 ohms, is connected across the 220-volt mains, as shown in Fig. 214*a*. A calibrated voltmeter is connected in parallel

with the armature circuit and an external shunt ammeter is connected in series with the current terminals of the meter. The meter constant is 150. The corrected voltmeter reading is 232 volts and the ammeter reads 1,980 amp. The meter makes 40 revolutions in 45.8 seconds. What is the per cent. accuracy of the meter at this load? How much power is required for this test? How much power would be required if the meter current were supplied at 232 volts?

QUESTIONS ON CHAPTER VIII

1. In what way does the magnetic circuit resemble the electric circuit? In what two ways do they differ from each other? Why cannot magnetic flux be readily confined to definite paths? In a general way how does the precision obtainable in magnetic calculations compare with that obtainable in electrical calculations?

2. Are ampere-turns dependent on current alone? On turns alone? What is the numerical relation between magnetomotive force and ampere-turns? To what quantity in the electric circuit does magnetomotive force correspond?

What is reluctance and to what does it correspond in the electric circuit? What is the basic unit of reluctance? How is permeance related to reluctance and to conductance?

What is meant by permeability?

To what quantity in the electric circuit does magnetic flux correspond?

3. How is the reluctance of a magnetic path related to its length? To its cross-section? To its permeability? How are reluctances in series combined? In parallel?

4. Why is it usually necessary to represent the relation between magnetizing force and flux, in iron and steel, by curves? What general shape does the lower part of such curves have? The upper part? What is meant by saturation? How may a permeability curve be obtained from a *B-H* curve? How does the variation of permeability compare with such variation of electric resistance as is due to heating?

5. State the simple law governing the relation between flux, mmf. and reluctance. To what law in the electric circuit does this law correspond?

6. Why is a method of trial and error sometimes necessary in solving magnetic problems?

7. Upon what three factors is the magnetomotive force acting upon a circuit dependent? How may the 0.4π be eliminated from computations in centimeter units? In inch units? How are magnetization curves plotted in order to reduce computations to the simplest basis?

8. If the excitation acting upon a sample of iron be increased from zero to some definite value and then decreased again to zero, does the magnetic flux return along the same path? If the excitation be decreased to zero, does the magnetic flux return to zero? How may the magnetic flux be brought back to zero? What is a cycle? A hysteresis loop? Remanence? Coercive force? What does hysteresis represent in terms of energy?

9. How is the hysteresis loss related to the loop area? How may the loss be calculated under practical conditions? How is the loss related to the maximum flux-density? What is the Steinmetz Law?

10. How is the geometrical position of the lines of induction related to the current in a circuit? Does this relation suggest the term "linkages?" How may these linkages be calculated? What relation does inductance bear to the total linkages?

11. Is it possible to produce an electromotive force in a circuit which is insulated from everything else? How, in a general way, is this electromotive force produced?

If an induced current is allowed to flow in a coil, what reaction will exist between this current and the inducing agent? If the inducing agent as, for example, a bar magnet, be withdrawn from a coil, will the induced electromotive force have the same direction as when the bar magnet was inserted in the coil? What reaction will be produced between the induced current and the inducing agent?

12. Upon what two factors does induced electromotive force depend? Is it possible to determine the value of this electromotive force in volts if these factors are known?

What is Lenz's Law?

13. If the flux linking a coil be made to change by altering the value of the current in the coil itself, show that an electromotive force is induced. What is the relation of this electromotive force to the direction of the current flowing in the coil? How does this relation affect the rapidity with which the current builds up to its Ohm's Law value?

14. What is the "time constant" of a circuit and by what two quantities may it be expressed? In a general way, what does it indicate as regards the circuit? Does the time lag of current in a circuit have any practical importance?

15. If an inductive circuit carrying a current be short-circuited, why does not the current die out immediately? To what is this tendency of the current to persist due?

What is the nature of inductance as regards its effect upon circuit changes? To what mechanical property does it correspond?

How does the effect of inductance manifest itself when the current of a circuit is interrupted? How can it be shown that this condition is not produced by the current alone? To what is this arc due? Under what conditions in practice may it become a menace? How may this menace be partially or wholly removed?

What personal dangers exist in opening inductive circuits?

16. Upon what three factors does the electromotive force of self induction depend?

Does the establishment of a magnetic flux require an expenditure of energy? Is energy expended in maintaining this flux after it is once established? What becomes of the power required by electromagnet field coils? Cite instances where electomagnetic energy manifests itself.

Is it possible to calculate this energy? Upon what two factors does it

depend? How may the energy of generator fields be very materially reduced, before opening the circuit?

17. How does the gas-lighting spark coil utilize the electromotive force of self induction in its operation? How is it connected in the circuit? Show that the spark coil can be considered as a reservoir in which magnetic energy is stored and later liberated. Explain why the coil produces a hot spark.

18. Is it possible for a magnetic flux produced by one coil to induce an electromotive force in another coil from which the first is insulated? Does this in any way correspond to the production of electromotive force by the insertion of a bar magnet in the second coil? What is the relation of the direction of the induced voltage in the secondary when the primary circuit is closed to its direction when the primary circuit is opened? Upon what three factors does this electromotive force depend?

19. Is it possible for *all* the flux produced by one coil to link another? What is the definition of the "coefficient of coupling" of two circuits?

How is mutual inductance defined? How may it be utilized to determine the induced voltage?

How may the mutual inductance of two circuits be materially increased?

20. Explain how the action of the induction coil depends upon mutual induction? How is the primary current interrupted and why is it necessary that this current be interrupted?

21. Upon what two factors does the pull between magnetized surfaces depend? How does this pull vary with the flux density?

PROBLEMS ON CHAPTER VIII

215. A certain electromagnet has two exciting coils, each of which has 2,200 turns. (*a*) When these two coils are connected in series, what is the total number of ampere-turns acting on the magnet if 3 amp. are supplied from the line? (*b*) If the coils are connected in parallel and the total current supplied is 3 amp., what is the number of ampere-turns?

216. If one of the coils in problem 215 has a resistance of 80 ohms and the other a resistance of 60 ohms, what is the line voltage in (*a*)? What is the line voltage in (*b*) and what are the ampere-turns per coil and the total ampere-turns?

217. A certain exciting coil has 1,400 turns and has a resistance of 160 ohms. What are the ampere-turns when this coil is connected across 120-volt mains? Another coil in every way similar to this one is placed on the same magnetic circuit and connected in series with it across the same 120-volt mains. What ampere-turns now act on the magnetic circuit?

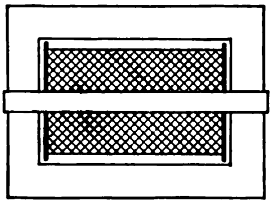


Fig. 219*a*.

218. What is the magnetomotive force in gilberts in problem 215 (*a*) and (*b*)? In problem 217?

219. In a certain iron-clad solenoid, Fig. 219*a*, the reluctance of the yoke is negligible compared with that of the plunger. When the plunger is inserted the lines of induction passing through the central core are observed

to increase from 350 to 52,000. **What is the permeability of the plunger at this flux density?**

220. A steel field core of a dynamo is 4 in. in diameter and carries a magnetic flux of 1,280,000 lines. What is the flux density in lines per sq. in. and per sq. cm.?

221. A magnet plunger, of circular cross-section and 1.5 in. diameter, carries a flux of 200,000 lines. What is the flux density in lines per sq. in. and per sq. cm.?

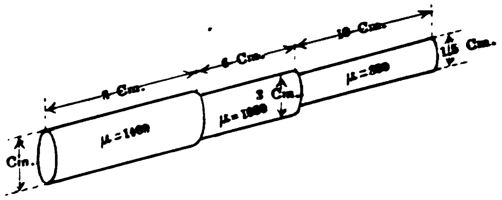


FIG. 225a.

222. The field core of a dynamo is 3 in. long and 4 in. in diameter. At 80,000 lines per sq. in. it has a permeability of 700. What is the reluctance between opposite ends of this field core at this flux density?

223. The two iron pole pieces of an electromagnet are 6 in. in diameter and are spaced ½ in. apart, forming the air gap. What is the reluctance of this gap? Neglect fringing.

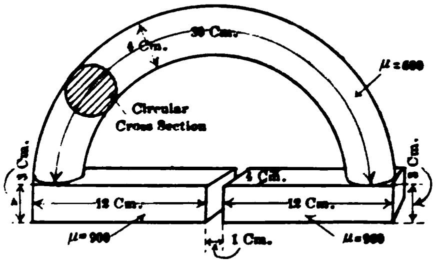


FIG. 226a.

224. If the iron pole pieces of problem 223 are cylindrical and have axial lengths of 1 in., what is their reluctance if their permeability is 1,200?

225. Fig. 225a shows three portions of a magnetic circuit. Compute the reluctance of each portion and the total reluctance of the combination. Each portion is circular in cross-section.

226. Compute the reluctance of the magnetic circuit shown in Fig. 226a.

227. Fig. 227a shows a magnetic circuit composed of two branches, which are similar and which are in parallel. Compute the reluctance of each half and the total reluctance of the circuit. The iron has a permeability of 600 throughout.

228. Two coils, each having 1,600 turns, are placed on the magnetic circuit shown in Fig. 226*a*, and are connected in series in such a way that they act in conjunction. What is the magnetomotive force acting on the circuit when 1 amp. flows in each coil? What is the resulting flux? What is the flux density in the gap, in lines per sq. cm.?

229. If a field coil of 1,000 turns is placed upon the central core of the magnetic circuit shown in Fig. 227*a*, and 5 amp. are sent through the coil, what is the resulting magnetomotive force? What are the total flux and the gap density in lines per sq. in.?

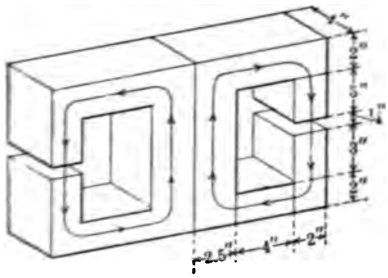


FIG. 227*a*.

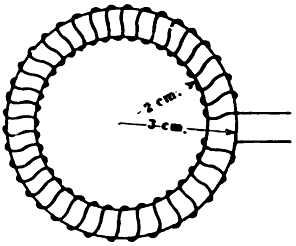


FIG. 230*a*.

230. An anchor ring is shown in Fig. 230*a*. Determine its reluctance when the permeability of the iron is 800. What is the magnetomotive force if there are 200 turns wound on this ring and 1.5 amp. are sent through the winding? What are the flux and the flux density?

231. A gap 1 mm. long is cut in the anchor ring (Fig. 230*a*). The corresponding reduction of flux density in the iron increases its permeability to 1,000. Determine the magnetomotive force, the reluctance, the flux and the flux density.

232. Assume that the ring, Fig. 230*a*, is made of cast steel whose magnetization and permeability curves are shown in Figs. 151 and 152 respectively. Determine the flux and the flux density in the steel and air gap when 1 amp. flows in the winding of 200 turns. (Use the trial and error method of Par. 138.)

233. Repeat problem 231, assuming the ring is made of cast iron and that 1 amp. flows in the winding.

234. Determine the ampere-turns necessary to send a total flux of 6,000 lines through the ring and the air gap of problem 233.

235. Assuming that the magnet of Fig. 227*a* is made of cast steel whose permeability curve is shown in Fig. 152, determine the number of ampere-turns on the central core necessary to send a flux of 400,000 lines through each gap. Neglect fringing and leakage.

236. Repeat problem 235 using the cast steel magnetization curve of Fig. 154.

237. The magnet shown in Fig. 237*a* has a yoke of cast steel and pole pieces of cast iron. Using the magnetization curves of Fig. 154, determine the ampere-turns necessary to send 120,000 lines through the air gap. Neglect fringing.

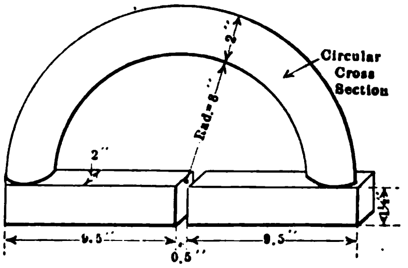


FIG. 237*a*.

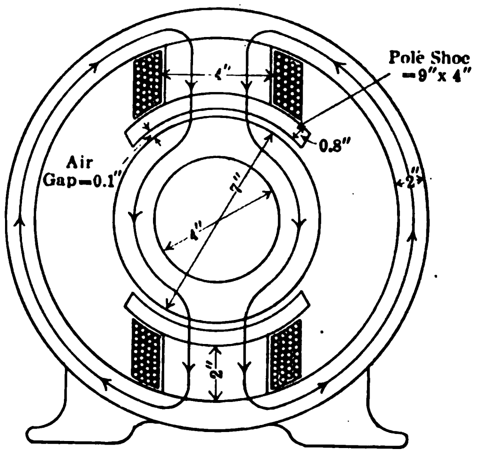


FIG. 238*a*.

238. Fig. 238*a* shows the magnetic circuit of a 2-pole dynamo. The field cores are of cast steel and are 4 in. square. The armature is of O.H. sheet steel and has a net axial length of 3.6 in. over the iron; the yoke is of cast iron and has a cross-section of 2 × 6 in. Using the magnetization curves of Fig. 154, determine the necessary field ampere-turns for an average flux density of 30,000 lines per sq. in. in the air gap.

239. Repeat problem 238 assuming the air gap to be 0.075 in. and that only 80 per cent. of the flux in the yoke and field cores enters the armature. $\left(\text{Leakage factor } = \frac{100}{80} = 1.25.\right)$

240. Determine the hysteresis loss in ergs per cu. cm. per cycle for cast iron operating at densities between 30,000 lines per sq. in. positive and negative. (Use data of Par. 143.)

241. A transformer yoke of silicon steel has a volume of 600 cu. in. What is the hysteresis loss in ergs per cu. in. per cycle if the maximum flux density is 40,000 lines per sq. in.?

242. In a certain electromagnet having 800 turns a current of 5 amp. produces 1,200,000 lines of induction. What are the total linkages? What are the linkages per ampere? What is the inductance of the circuit?

243. When a current of 12 amp. flows in a certain exciting coil of 2,000 turns, 2,000,000 lines of induction link the coil. What are the linkages? What is the inductance in henrys?

244. Assuming that the permeability of the magnetic circuit of problem 243 remains constant, determine its inductance when the current is doubled. Determine the inductance when the turns are doubled.

245. In a closed magnetic circuit of cast steel the net ampere-turns per in. are 20. The cross-section of the magnetic path is 12 sq. in. and its net length is 30 in. If 1 amp. flows in the exciting coil, determine the inductance of the circuit using the curve of Fig. 154.

246. Repeat problem 245 for an exciting current of 2 amp., or double the value of that in problem 245. To what is the change of inductance due?

247. When the exciting current of an electromagnet is flowing, there are 1,800,000 lines of induction linking the circuit. The exciting coil has 2,400 turns. If the exciting current is interrupted, requiring 0.5 second to completely rupture the arc, what is the average induced voltage across the ends of the exciting coil?

248. Re-compute problem 247, assuming that the circuit is interrupted in 0.2 second.

249. A certain electromagnet has an inductance of 2.8 henrys and a resistance of 5 ohms and is connected across 110-volt mains. What is the time constant of the magnet? How long will it take the current to reach 63 per cent. of its ultimate value? What will be the value of the current at this instant? Illustrate the rise of the current by a sketch, marking the values involved in the problem.

250. If the resistance of the electromagnet of problem 249 be doubled, what does the time constant of the circuit become? How long does it take the current to reach 63 per cent. of its ultimate value? What is the ultimate value? Illustrate by a sketch and compare with problem 249. In which problem is a given value of current first reached?

251. Six amperes flow in the exciting circuit of problem 247. Compute the induced electromotive force when the circuit is opened in 0.5 second, using equation (75).

252 If the exciting current of problem 245 is reversed in 0.1 second, determine the voltage induced across the ends of the exciting coil.

253. A certain generator field circuit has an inductance of 2 henrys and carries 100 amp. The induced voltage across the field terminals must not exceed 1,000 volts. What is the minimum time which can be allowed for opening the field? How much energy is liberated in opening this field? What is the average power during the opening period?

254. Repeat problem 253 with the total field resistance doubled by means of the field rheostat.

[illegible] [illegible] A [illegible] Fig. [illegible] [illegible] [illegible] [illegible] per cent. [illegible] [illegible] [illegible] [illegible] [illegible] 12 [illegible] 30 [illegible] When [illegible] [illegible] 220,000 [illegible] How many [illegible] B? What [illegible] [illegible] current [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] B? [illegible]

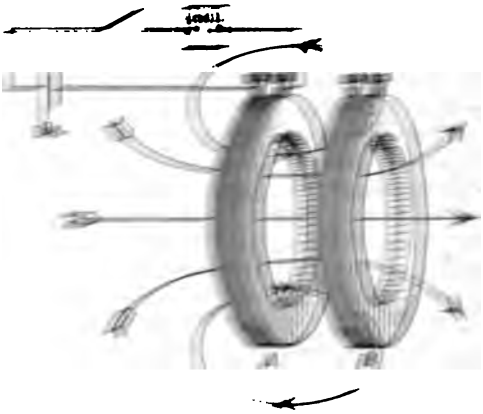


Fig. [illegible]

[illegible] The same [illegible] that was produced in A by [illegible] amp. is produced in B by [illegible] amp. What voltage is induced in A upon interrupting the [illegible] amp. of B in [illegible] second? What voltage is induced in B?

[illegible] Determine the mutual inductance in henrys of coils A and B in problems 255 and 256. What is the self inductance of A? Of B?

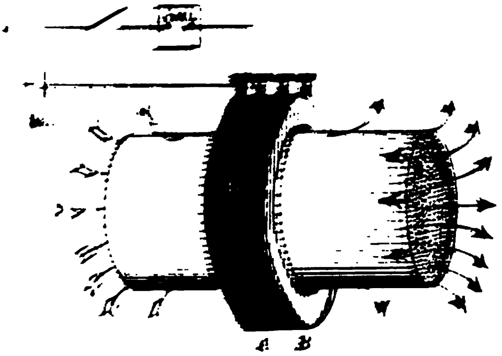


Fig. [illegible]

[illegible] Coils A and B of problem [illegible] are now linked magnetically by an iron core as shown in Fig. [illegible] so that practically all the flux of one links with the other. [illegible] amp. in A now produces 300,000 lines in the joint

magnetic circuit. How many amperes in B will produce this same flux? What is the self inductance of A? Of B?

259. If the 0.1 amp. in A of problem 258 is interrupted in 0.05 second, what electromotive force is induced in A? In B? What is the mutual inductance of the circuits? The self inductance of A? Of B?

At what rate must the current in B be interrupted to induce 10 volts in A? If the current in B is 0.05 amp., in what time should the circuit be opened?

260. The flat pole pieces of an electromagnet are in contact with each other and a total flux of 2,000,000 lines passes from one to the other. If each cross-section is 4 in. $\times$ 5 in., what force in pounds is necessary to pull these pole pieces apart?

QUESTIONS ON CHAPTER IX

1. If two ellipsoids near each other are connected to the terminals of an electrostatic machine, upon what portions of the ellipsoids will the density of charge be greatest? Would any considerable change be observed in these charges if the wires to the machine were disconnected? How can it be shown that charges are "bound."

What force exists between two charges of unlike sign? What is its direction? For charges of like sign?

2. If a positive charge is brought into the neighborhood of an insulated and uncharged ellipsoid or sphere, what phenomenon occurs? What is the relation of the induced charge to the inducing charge? Distinguish between free and bound charges. How may it be shown experimentally that a difference exists?

3. How does a small positive electrostatic charge act when placed near two conducting bodies between which a difference of potential exists? Can the distribution of electrical stress in the air between such bodies be represented by lines in a manner similar to that used in showing magnetic distribution? Where do electrostatic lines originate and terminate? Compare them with lines of induction and lines of force in this respect.

4. What is the effect in a dielectric medium of increasing the density of electrostatic lines beyond a certain value? Is this same effect noted in the magnetic circuit and in the electric circuit?

5. If a needle or other sharp projection be raised to a high potential, what effect is noted at this projection? What is the condition of the air in this region? What is the effect of a further increase of potential?

Distinguish between an insulator and a dielectric. Name two substances that are good insulators but poor dielectrics; good dielectrics but poor insulators. In what terms is dielectric strength expressed?

6. What is the effect of applying a voltage to an electric condenser? What is the order of magnitude of the time required by a current to charge such a condenser? Why does the current cease to flow? To what hydraulic phenomenon can this be compared?

7. How can it be shown that electricity is actually stored in a condenser? How does the quantity which can be stored in a condenser vary with the voltage? What simple relation does this give between charge, capacitance and voltage?

8. What is the usual effect of inserting some dielectric medium other than air between condenser plates? What is "specific inductive capacity" and to what magnetic property is it analogous? What is the specific dielectric constant of glass? Of mica? Of rubber?

9. How is the equivalent capacitance of condensers connected in parallel determined? To what electric circuit condition is this analogous?

10. How is the equivalent capacitance of condensers connected in series determined? What is the relation among the electric charges on each of a number of condensers connected in series? To what equation in the electric circuit is the equation relating to the equivalent capacitance of condensers in series similar?

11. How can the voltage across each of a number of condensers in series be calculated if the line voltage and the individual capacitances be known? Are these voltage relations dependent at all upon the insulating properties of the dielectrics? In the case of leaky condensers, upon what does the ultimate voltage distribution depend?

12. How may it be shown that electric energy can be stored in a condenser? Upon what factors does this energy depend?

13. Upon what factors does the capacitance of a parallel plate condenser depend? What is the effect upon the capacitance of changing the area of the plates? Of decreasing the distance between them? Of substituting hard rubber or glass for air?

14. What two methods are commonly employed in the measurement of capacitance? Upon what fact does the ballistic galvanometer method depend? What relation exists between the quantity passing through the galvanometer and its maximum throw?

Should the measurement be made upon "charge" or upon "discharge?" Explain. How is the galvanometer calibrated?

15. Describe the bridge method of capacitance measurement. Compare it with the Wheatstone bridge method of resistance measurement. How does the bridge formula for capacitance differ from the formula employed when resistance is measured? What is the source of power and what simple detector is used in the capacitance bridge?

16. How may a disconnection in a cable be located? Upon what principle does this method of measurement depend? Is this method applicable if the fault is grounded?

PROBLEMS ON CHAPTER IX

261. A condenser has a capacitance of 12 m.f. What is its charge when the potential between its plates is (a) 220 volts? (b) 440 volts? (c) How much current flowing at a uniform rate is necessary in order that the condenser may be charged in 0.2 second in each case?

262. It is desired to store 70 microcoulombs in a condenser at a potential of 750 volts. What should be the capacitance of the condenser?

263. What is the potential across a 40-m.f. condenser in which the charge is 0.002 coulomb? How long must a current of 1 milliampere flow in order to charge this condenser to the above potential?

264. A certain condenser consisting of two parallel plates, with air as dielectric, has a capacitance of 0.00012 m.f. A slab of glass is placed between the plates occupying the entire space. The capacitance is now found to be 0.00072 m.f. What is the specific inductive capacity of the glass?

265. The condenser of problem 264 is charged to a potential of 300 volts between plates and the supply then disconnected. Glass is then inserted between the plates completely filling the space. This insertion of the glass in no way changes the value of the electric quantity on the plates. What is the condenser voltage after the insertion of the glass?

266. A plate condenser, with air as dielectric, has a capacitance of 0.0012 m.f. and 300 volts is impressed across its terminals. The condenser is then immersed in a bath of transformer oil having a dielectric constant of 2.5, the voltage supply remaining connected. What is the charge on this condenser before and after immersion in the oil?

267. Four condensers having capacitances 12, 16, 20 and 30 m.f. respectively are connected in parallel across 220-volt mains. What is the charge on each and what must be the capacitance of a single condenser to replace the four?

268. Three condensers connected in parallel across 400-volt mains have charges of 600, 800 and 1,000 microcoulombs. What is the capacitance of each and what single capacitance would replace the three?

269. The four condensers of problem 267 are connected in series across these same mains. What is the voltage across each of them and what single condenser would replace the four? What is the charge on each condenser?

270. Four condensers are connected in series. The voltages of the condensers are 50, 70, 80 and 100 volts respectively. This combination of condensers can be replaced by a single condenser having a capacitance of 6 m.f. What is the capacitance of each condenser?

271. A condenser has a capacitance of 20 m.f. What is the stored energy in the condenser when the voltage across it is 100 volts? 200 volts? In what ratio is the energy increased if the voltage is doubled?

272. Three condensers having capacitances of 20, 40 and 60 m.f. respectively are connected in series across a 600-volt supply. (*a*) What is the voltage across each? (*b*) What is the energy of each? (*c*) What is the energy of the system?

273. Determine the stored energy of the system when the three condensers of problem 272 are connected in parallel across the same voltage.

274. An air condenser consists of three plates. The two outer ones are connected together as one terminal and the other terminal is formed by the intermediate plate between the two outers. The dimensions of each plate are 12 in. × 12 in. and the plates are spaced $\frac{1}{16}$ in. apart. What is the capacitance of this condenser?

275. If the space between the plates of the condenser of problem 274 is filled with paraffin, having a dielectric constant of 2.1, what does the capacitance become?

276. A high voltage condenser is to be made of alternate layers of glass and tin foil, the glass having a dielectric constant of 8. The glass is 3/64 in. thick and the tin foil is 2 mils thick and its dimensions are 3 in. × 4 in. How many plates and sheets of tin foil are necessary to make a condenser having a capacitance of 0.02 m.f.? If the glass plates are 5 in. × 6 in., what is the size of the completed condenser?

277. In a bridge measurement of condenser capacitance the bridge is connected as shown in Fig. 183 (*b*). When a balance is obtained $R_1 = 100$, $R_2 = 1{,}242$, $C_2 = 0.4$ m.f. What is the value of C_x, the unknown capacitance?

278. In a test for a cable fault the apparatus is connected as shown in Fig. 184. In the capacitance measurement of the part *x* the galvanometer has a ballistic throw of 4.2 cm. In the measurement of the capacitance of the perfect cable plus the looped end of the faulty cable the deflection is found to be 16.4 cm. If the length of each conductor is 1,800 ft., how far from the point of test is the cable broken?

QUESTIONS ON CHAPTER X

1. In what way is the flux linking the coil of a generator armature varied? How does this induce voltage? How does this voltage vary with the speed? The flux? The number of turns in the coil?

2. If instead of regarding this voltage as due to the change of flux linking a coil, it is considered as being due to the individual conductors cutting flux, is the ultimate result in any way affected? If the voltage is considered as being due to the cutting of lines by individual conductors how does this voltage vary with the length of conductor? The flux density? The velocity of the conductor?

3. What definite relation exists among the direction of the induced emf., the direction in which the conductor moves and the direction of the flux? What simple rule enables one to determine these relations?

4. What is the value of the emf. induced in a rotating coil, (*a*) when the coil is in the plane perpendicular to the flux? (*b*) When its plane lies parallel to the flux? Does the voltage ever reverse its sign? Explain.

5. How may the alternating current produced in a coil be changed to direct current? What is the effect of adding coils to the rotating member? To what are the "ripples" in a voltage wave due?

6. In what way is the open coil type of armature different from the closed coil type? Which type is the gramme ring armature (Fig. 192)? Show that the resultant electromotive force is different in the two types, even though the number of coils and turns be the same.

7. Name two serious objections to the ring winding. How are these objections overcome in the drum winding? What two methods are used to fasten conductors on armatures? Which is the better method and why?

8. What is meant by "coil pitch" and what is its relation to pole pitch? What relative slot positions do the two sides of a coil occupy? Why?

What is meant by a "winding element?" May it consist of more than one conductor? Explain.

9. What is "front pitch?" "Back pitch?" Average pitch? What is the relation between the number of winding elements and the number of coils? The number of commutator segments?

10. In a simplex lap winding how many commutator segments does the winding advance each time that a coil is added? What three fundamental conditions must be fulfilled by a winding? What is a winding table and what is its practical value?

11. Why is it sometimes desirable to place more than two winding elements in a slot? In what type of generator is this necessary? Is the conductor numbering and are the winding relations in any way affected? What one condition should be imposed upon this type of winding and why?

12. What is meant by "paths through an armature?" How is the current output of a machine affected by increasing the number of paths? How is the voltage affected? The power output? How many paths are there in all simplex lap windings?

13. What is meant by a duplex winding? Show that such a winding may be composed of two simplex windings each lying in alternate slots. How many closures may such a winding have and what is its degree of re-entrancy in each case?

14. If a duplex winding does not close after one passage around the armature, is the number of segments even or odd? When does such a winding close? How many times does it close and therefore what is its degree of re-entrancy?

15. In a winding whose multiplicity is m, how many winding elements separate a given element from the next returning element? How many armature paths are there in a 6-pole machine having a simplex lap winding? A duplex lap winding? A triplex lap winding?

16. To what causes are unequal voltages in different paths of an armature winding due? Do equalizing connections do away with these inequalities? What is the purpose of equalizing connections? What care should be taken regarding the number of slots per pole when equalizing connections are used? Why?

17. What is the fundamental difference between a lap and a wave winding? Does the direction of induced emf. in opposite sides of a coil differ in the two types? Explain.

18. After a wave winding has passed under every pole, in passing around an armature, what relation should it bear to its starting position if the winding is simplex? What would a closure after one passage around the armature mean?

19. Show that the definitions of front pitch and back pitch in a wave winding do not differ from the similar definitions in a lap winding? Can the front pitch be even? Odd? Can the back pitch be even? Odd? Can the two be equal? Can the average pitch be even? Odd? When is a winding progressive? Retrogressive? Explain.

20. Is it always possible to fit a wave winding to an armature having a fixed number of slots if all the slots are utilized? Explain? What makeshift may be used to accomplish the desired result?

21. If the number of pairs of poles is even, is the number of commutator segments even or odd? Answer if the number of pairs of poles is odd?

22. What is the minimum number of brush sets that can be used in a wave winding? What is the maximum number that it is possible to use? When would two sets be used and why? Why is the maximum number usually desirable?

23. How many paths are there in a simplex wave winding? In what way is the number of such paths affected by the number of poles? How many paths in a duplex wave winding? A triplex wave winding?

24. When is it desirable to use a wave winding and why? A lap winding? Give specific reasons.

25. In addition to forming a part of the magnetic circuit, what other function does the yoke of a generator perform? Of what two materials is it made and why? Describe a process whereby the yoke is made without casting.

26. Of what materials are the field cores made? The pole shoes? What are the two general shapes of the core sections? Where is each used?

27. Is the armature a solid casting? If not, how is it built up? By what two methods are the stampings produced? How are they held in position when placed upon the armature? What is the purpose of the ventilating ducts?

28. Sketch two general types of slot. Where is each used? What two methods are used to prevent the conductors from being affected by centrifugal forces?

29. Of what is the commutator made? What insulation is used between segments? How are the segments clamped together? How are the coil connections made?

30. What is the purpose of the brushes? Of what material are brushes usually made? What pressure is used to hold the brush on the commutator? What is the purpose of the plating on the brush? What is the purpose of the pig-tail?

PROBLEMS ON CHAPTER X

279. A coil 20 cm. square, having 50 turns, rotates at a speed of 600 r.p.m. in a uniform magnetic field having a density of 200 lines per sq. cm. (*a*) What is the average voltage induced in the coil?

(*b*) If the flux and the speed are both doubled, what average voltage is obtained?

280. A wire 40 cm. long moves at a speed of 2,000 cm. per second through a magnetic field having an intensity of 6,000 lines per sq. cm. How many volts are induced between the ends of this conductor?

281. A uniform magnetic field is just sufficient in cross-section to pass perpendicularly through a coil 40 in. $\times$ 12 in. The coil has 80 turns. If

the coil slides out from this field in 0.001 second and at a uniform rate, what voltage is induced due to the change in flux linking the coil? What voltage is generated by the cutting of the flux by the individual conductors? (Work with the coil sliding in the two directions, one parallel to the 12-in. side and one parallel to the 40-in. side.)

282. An armature has 40 slots. Design a 2-layer, 4-pole, simplex lap winding, in which the back pitch is 21 and the front pitch is 19. Make a winding table.

283. Repeat problem 282 making the front pitch 21 and the back pitch 19. Which winding is progressive and which is retrogressive?

284. Design a 2-layer, simplex lap winding for a 6-pole, 40-slot machine, choosing the proper pitches.

285. An 8-pole armature has 128 slots and 6 winding elements per slot. Determine a correct value of back and front pitch for a simplex lap winding. Sketch a few slots with their winding elements and connections. How many commutator segments are necessary?

286. Repeat problem 285 for a winding with 8 elements per slot.

287. A 6-pole, simplex, lap-wound armature delivers a total current of 228 amp. at 220 volts. How many amperes per path through the armature? How many volts per path? What is the kilowatt rating of the machine?

288. If the machine of problem 287 had a duplex lap winding, what would be the amperes per path? Per brush?

289. Repeat problems 287 and 288 for an 8-pole, 200-kw., 220-volt generator.

290. Make a winding table for a 60-slot, 4-pole armature, the winding to be a duplex, doubly re-entrant winding. There are 2 winding elements per slot.

291. Repeat problem 290 using 61 slots and making the winding singly re-entrant.

292. A 4-pole armature has 33 slots and 2 elements per slot. Design a simplex wave winding for this armature, having a back pitch of 17 and a front pitch of 17. Make a winding table. (Check the pitch, using equation 100.)

293. Repeat problem 292 making $y_b = 19$, and $y_f = 15$.

294. Attempt to place a similar winding upon a 34-slot armature. Then omit one slot, using a dummy coil, and repeat.

295. An 8-pole, 550-volt, 50-kw. generator has a simplex wave-wound armature. How many amperes per path?

296. Repeat problem 295 using a duplex wave winding.

QUESTIONS ON CHAPTER XI

1. A certain armature has a fixed number of conductors on its surface. What are the separate effects on the induced voltage of (1) doubling the speed of the armature; (2) doubling the flux entering the armature; (3) reconnecting the armature so that the number of paths through the armature is doubled?

2. In a given generator, upon what two factors does the induced voltage depend? If the speed of the generator be maintained constant, upon what factor does the induced voltage depend?

3. Show that a similarity should exist between two curves plotted as follows:

1. The field ampere-turns of a generator as abscissa and the flux leaving one of its north poles as ordinate.

2. The field current of the same generator as abscissa and the induced armature voltage at constant speed as ordinate.

4. In the curve relating ampere-turns of the field and the flux of one north pole, why does not the flux start at zero value? Why is the first part of the curve a straight line? At the higher values of field current why does the induced voltage increase less and less rapidly for any given increase in field current?

5. Is there any difference between the saturation curve obtained with *increasing* values of field current and that obtained with *decreasing* values? Explain any difference.

6. Sketch the connections used in determining a saturation curve. (1) Using a simple field rheostat. (2) Using a drop wire with the field. Give two reasons why the generator should be separately excited.

7. Show that Ohm's Law can be expressed graphically. What two quantities are plotted when expressing Ohm's Law in this manner?

8. Sketch the connections of a shunt generator. Is the field of comparatively low resistance or of high resistance? Explain.

9. Explain in detail how a shunt generator "builds up." What limits the voltage to which a machine can build up?

10. What is the critical field resistance? Give three causes, each of which may prevent the generator building up. What tests and remedies should be used for each?

11. What is the general direction of the flux produced by the current in the armature conductors? What effect does this have upon the resultant flux in a machine? How does it affect the position of the neutral plane? What effect does the change in position of the neutral plane have upon the brush position?

12. What is the relation of the direction of the armature field to the brush axis? When the brushes are moved forward in a generator what is the resulting direction of the armature field? Into what two components can this be resolved? What is the effect of each component?

13. Which conductors on an armature produce a demagnetizing effect? Which produce a cross-magnetizing effect?

14. Sketch the conductors on the armature, together with the poles, for a loaded multi-polar machine, indicating the current directions in the various conductors. Sketch a curve showing the values of armature magnetomotive force along the armature surface. Show the flux produced by this magnetomotive force when acting alone.

15. Show the effect of the above flux on the distribution of the total flux along the armature surface. How is the neutral zone affected? What change must be made in the brush position?

16. Name four methods by which armature reaction is either eliminated or reduced. State the principle of each method.

17. Sketch an ideal commutation curve assuming uniform current distribution over the brush.

18. What is the effect of having voltages induced in a coil during the time that it is being short-circuited by the brush? What limits the current in such a coil? How do these currents affect the uniform distribution of current over the brush?

19. Sketch commutation curves for the following conditions: (*a*) Brush advanced too far; (*b*) brush too far back; (*c*) brush too wide.

20. Why does an armature coil have self inductance? What is the effect of this self inductance during the commutation period? What effect does the voltage of self induction have upon the relation of the brush position to the neutral zone?

21. What is the order of magnitude of the voltages induced in a coil undergoing short circuit? If the voltages are low what makes them so objectionable?

22. What is the advantage of copper brushes over carbon? Why are carbon brushes used almost universally?

23. What evidence points to the fact that the taking of current from the commutator by the brushes is not pure conduction? To what is "high mica" due? How may it be reduced or even eliminated? Name two methods.

24. In general, what is the effect of arcing on the commutator? Why should any appearance of arcing be a reason for eliminating the cause of the arcing as soon as possible? Why is it not desirable to use emery paper or cloth in grinding brushes or smoothing the commutator?

25. What changes occur in the flux at the geometrical neutral of a generator as load is applied? What is the effect of these changes upon the brush position? Why do the brushes have to be moved ahead of the load neutral plane?

26. Show that instead of moving the brushes forward in order to obtain the proper commutating flux, the same result may be obtained by the use of a commutating pole.

27. Why is the commutating pole connected in series with the armature? Why has it an unusually long air gap?

28. What is the relation of the polarities of the main poles and of the commutating poles to the direction of rotation, in a generator? In practice how are the commutating poles adjusted to the proper strength?

29. Sketch the connections used in obtaining the shunt characteristic. Sketch the characteristic. Why does the machine finally "break down?" Why does the return curve from short cicuit not follow the curve obtained with increasing values of armature current?

30. Give three reasons why the voltage of a shunt generator drops as load is applied. Why are these three reactions cumulative? What prevents a generator from "unbuilding" as load is applied?

31. What effect does running a generator at higher than rated speed have upon its characteristic, provided that the field current is so adjusted that the no-load volts are the same in each case?

32. What is meant by generator regulation? Does a large value of the per cent. regulation indicate that a generator is a desirable one for supplying lamp loads? Explain.

33. What is meant by the "total characteristic" of a generator? What is its relation to the shunt characteristic? How may the total power developed within an armature be determined?

34. How may the objectionable drooping characteristic of the shunt generator be improved? How are these additional turns connected and in what way do they differ from the shunt field turns?

35. Show the difference between "long shunt" and "short shunt" connection. What is the effect of the connection upon the characteristic? Sketch the characteristics of an over-compounded, a flat-compounded and an under-compounded generator. Where is each used and why?

36. How is the degree of compounding in a generator adjusted? When do generators have two separate series fields?

37. What is the effect of speed upon the degree of compounding, if the no-load voltage is the same in each case? Compare this with the effect of speed upon the shunt characteristic and explain.

38. Show how the number of series turns for a desired degree of compounding may be determined. What is the armature characteristic and how may it be utilized?

39. How does the series generator differ fundamentally from the shunt generator in construction? In the type of load that it supplies?

40. Describe the external characteristic of the series generator and show its relation to the saturation curve.

41. In what way does the machine "build up?" What is meant by the critical external resistance? Why is it desirable to operate upon the right-hand side of the external characteristic?

42. Name a very common use of the series generator. Name two common types of machines. Why are special commutators necessary?

43. What is the "Thury system" of power transmission? Where is it used?

44. How may series generators be used to control the voltage at the end of a feeder? Upon what portion of the characteristic does such a generator operate? Sketch the connections. What precautions must be taken in the installation and operation of such a booster?

45. How may the speed of a prime mover affect the generator characteristic? Is such a drop in speed chargeable to the generator? How may it be taken into consideration?

46. State one essential difference between a unipolar generator and the ordinary type. What design is necessary to prevent the armature being short-circuited on itself? What is the advantage of this type of machine over the ordinary type and for what type of work is it best adapted? What are its disadvantages?

47. What is the basic principle of the Tirrill regulator? What is the function of the main control magnet? Of the relay magnet? Why cannot this regulator be applied directly to machines of large capacity? How may it be applied to these machines?

PROBLEMS ON CHAPTER XI

297. The pole faces of a shunt generator are 8 in. square and the average flux density under the pole is 45,000 lines per sq. in. The machine has 4 poles and there are 300 surface conductors on the armature. The machine is wave wound making two parallel paths through the armature. What is the induced voltage when the generator rotates at 800 r.p.m.? 1,000 r.p.m.?

298. If the current per path in problem 297 is 20 amp., what is the rating of the machine in kilowatts?

299. Repeat problems 297 and 298 for a simplex lap winding?

300. In an 8-pole, 220-volt generator, the pole faces are 12 in. square. The flux density under the poles at no load is 47,500 lines per sq. in. There are 16 slots per pole on the machine. The speed of the machine is 750 r.p.m. If the armature is lap wound, how many conductors per slot are necessary to give the rated voltage at no load?

301. The following data are for the saturation curve of a 20-kw., 220-volt generator, running at 600 r.p.m.,

Field current	0	0.5	1.0	1.5	2.0	2.5	3.0
Volts	10	62.5	125	178	212.5	235	245

Plot this saturation curve and then replot it for 550 r.p.m.

302. The generator of problem 301 is a 4-pole, lap-wound machine and has 440 conductors on the armature surface. There are 400 shunt field turns per pole. Plot a curve between flux per pole and ampere-turns per pole.

303. Determine the approximate number of ampere-turns required for the gap and for the iron at 220 volts, when the generator of problem 301 is operating at 550 r.p.m.

304. Determine the critical field resistance for both speeds in problem 301. Determine the field resistance necessary for the generator to build up to 220 volts at each speed.

305. When the generator of problem 301 is operating at 600 r.p.m. and the field resistance is adjusted so that the machine builds up to 220 volts, what current flows through the field due to the residual magnetism? What induced voltage results from this field current? What field current results from this last voltage?

306. A generator fails to build up. When the shunt field is connected across the armature, a voltmeter across the armature shows 4 volts. When this field circuit is opened the voltmeter reads 7 volts. What is the probable reason that the machine does not build up and what remedy is suggested?

307. The no-load flux of a bi-polar generator is 3,000,000 lines. When the generator is carrying its rated load and the brushes are in the no-load neutral plane, the armature itself produces a flux of 1,000,000 lines. Neglecting the effect of saturation, what is the *resultant* flux?

308. Repeat problem 307 when the brushes are advanced 30°.

309. There are 240 conductors on the surface of the armature of a bi-polar generator. The generator delivers 50 amp., making 25 amp. flowing in each conductor. If the brushes are advanced 15°, how many demagnetizing and cross magnetizing ampere-conductors are there? How many demagnetizing and cross magnetizing ampere-turns are there?

310. The brushes of a 4-pole generator are advanced 10 space degrees. The armature is lap wound and has 496 surface conductors. How many demagnetizing and cross magnetizing ampere-*turns* are there on the armature when the generator delivers 120 amp.?

311. Repeat problem 310 for a generator having the same number of poles and armature conductors and delivering the same current, but with a wave-wound armature. What is the ratio of the kilowatt capacities of the two machines?

312. A commutating pole circuit has a resistance of 0.08 ohm. The rated full-load current of the generator is 80 amp. The most satisfactory condition of commutation is obtained when 60 amp. flow in the commutating pole circuit. What must be the resistance of a shunt to be connected across this commutating pole circuit?

313. The terminal voltage of a shunt generator is 550 volts when the armature delivers 100 amp. If the armature resistance is 0.3 ohm, what voltage is being induced in the armature?

314. A 75-kw., 220-volt shunt generator has 228 volts induced in its armature when it is delivering its rated load at 220 volts. At the same time 12 amp are taken by the shunt field. What is the armature resistance?

315. The no-load voltage of the generator of problem 314 is 234 volts. What is its per cent. regulation? Why is the induced voltage at rated load not equal to the no-load volts?

316. A shunt genrator has a no-load voltage of 119 volts. It is specified that it shall regulate to within 6 per cent. What should be the terminal voltage when it delivers its rated load?

317. What is the total power being developed in the armature of the generator in problem 314? How much of this power is lost in the armature and how much is lost in the field? How much is available for delivery to the external circuit?

318. The terminal voltage of a generator is 500 volts when delivering 50 amp. The armature resistance is 0.8 ohm and the shunt field resistance is 250 ohms. What power is being generated in the armature? What is the *electrical* efficiency of the armature?

319. A compound generator has a no-load voltage of 230 volts. It supplies a 200-kw. load, situated 800 ft. distant, over a 1,000,000 C.M. cable. It is desired to maintain the voltage at the load constant at 230 volts from

no load to full load of 200 kw. What must be the no-load and the full-load voltage rating of the generator?

320. Repeat problem 319 for the condition in which it is desired that the voltage at the load rise from 230 to 240 volts from no load to full load.

321. The generator of problem 319 is connected long shunt. Its armature resistance is 0.007 ohm, the shunt field resistance is 15 ohms, and the series field resistance is 0.0015 ohm. What is the voltage induced in the armature with the 200-kw. load of problem 319? How much power is lost in the armature, in the shunt field, in the series field and in the cable? Of the power generated, how much reaches the load?

322. Repeat problem 321 for the 200-kw. load of problem 320. Due to the removal of the series field diverter the resistance of the series field is now 0.003 ohm.

323. It is desired to add series turns to a shunt generator, so that its rated load voltage is the same as the no-load voltage. There are 260 shunt turns per pole. It is found necessary to increase the shunt field current from 8 to 11 amp. in order to keep the rated load voltage equal to the no-load voltage. The rated current of the machine is 300 amp. How many series turns per pole should be added?

324. It is desired that the voltage of a 250-kw., 550-volt generator increase from 550 volts at no load to 600 volts at rated load. With the series field out of circuit and the shunt field excited from an external source it is found that this increase of voltage may be obtained by increasing the shunt field current from 6 amp. to 9 amp. There are 640 shunt turns per pole and 6 series turns per pole. The total series field resistance is 0.03 ohm. What must be the resistance of a shunt or diverter to be connected across the series field?

325. The terminal voltage of a series generator is 2,840 volts. The armature resistance is 20 ohms and the field resistance 25 ohms. What is the voltage induced in the armature when the machine delivers 6.6 amp.?

326. A 100-kw. load is situated 2,000 ft. distant from the 230-volt bus-bars of a station. The load is supplied over a 500,000 C.M. feeder. It is desired that when the load is 100 kw. the load voltage shall not be less than 225 volts. What must be the current and voltage rating of a series booster designed to maintain this voltage at the above value?

327. The booster in problem 326 has an efficiency of 80 per cent. It is driven by a shunt motor connected across the bus-bars, the motor efficiency being 80 per cent. What is the efficiency of transmission over the feeder?

QUESTIONS ON CHAPTER XII

1. In what way does a motor differ from a generator in the work which it performs? In general construction?

2. What effect is noted when a conductor carrying a current is placed in a magnetic field? How can this action be explained by two elementary laws of magnetism? What is the effect of reversing the current in the conductor?

3. To what three factors is this force proportional? If the flux is doubled how is the force affected? If the current is doubled?

4. State a convenient rule by which the relation among the direction of the current, the direction of the field and the direction of the force can be determined. What other simple method enables one to determine this relation?

5. What is torque? In what units is it expressed? In the British system? In the metric system?

6. Show that a coil carrying current when placed in a magnetic field develops a torque. In what position of the coil is the torque a maximum? When is it zero? What change in the connection to the armature should be made when the torque reaches its zero value?

7. Why are a large number of conductors upon the armature desirable? To what three factors is the torque of an armature proportional? In any one machine, to what two factors is the torque proportional?

8. How can it be shown that resistance alone does not determine the amount of current taken by a motor armature? Why must a motor of necessity be generating a voltage when it is rotating? What is the relation of this voltage to the direction of the current? To the direction of the applied voltage?

9. Is the counter electromotive force greater or less than the applied voltage? Why? By what quantity do the two voltages differ from each other?

10. Fundamentally, upon what two quantities does the speed of a motor depend?

11. In what direction is the flux of a motor distorted by armature reaction? In what direction should the brushes be moved as the load is applied to a motor? What general effect on the field flux does this movement of the brushes have? What is the effect upon the speed?

12. What is the relation between the main poles, the interpoles and the direction of rotation of a motor? How does this relation compare with the similar one for a generator?

13. When load is applied to a motor what is its first tendency? In the case of the shunt motor, how does this tendency affect the back electromotive force? The current flowing into the armature?

14. What two characteristics are very important in considering the suitability of a motor for commercial work?

15. How does the torque of the shunt motor vary with the load? Why? How does the speed vary with the load? Demonstrate. Ordinarily is its change of speed with load excessive? What effect does armature reaction have upon the speed? What is meant by "speed regulation?" Does the per cent. speed regulation have any significance as regards a motor's performance? To what general type of work is a shunt motor adapted and why?

16. How does the flux in a series motor vary with the load? How does this affect the variation of torque with load?

17. To what extent is the speed of a series motor affected by the application of load? By the removal of load? What precautions should be taken when the series motor is being installed for industrial purposes?

18. To what general types of load is a series motor adapted and why? For what reasons is it especially adapted to street car work?

19. What factors are plotted in giving the characteristics of a street car motor? Why?

20. In what way do the windings of a compound motor differ from those of a shunt motor? A series motor? In what two ways with respect to the shunt winding may the series winding be connected?

21. What is the speed characteristic of the cumulative compound motor? The torque characteristic? What advantage has it over the series motor? For what general type of work is it best adapted?

22. What is the nature of the speed and torque characteristics of the differentially compounded motor? Is this type of motor in general use? Explain. What precaution is necessary in starting this type of motor?

23. How may a motor be reversed? What is the effect of reversing the line terminals?

24. Why is a starting rheostat necessary for direct current motors? In what circuit is the starting resistance connected? Why should it not be connected in the line?

25. What two additions to the starting resistance of Fig. 299 are incorporated in a 3-point starting box? Why? Sketch the connections of a 3-point box. Show that the starting resistance which is in series with the shunt field when the arm is in the running position has little effect upon the field current.

26. Under what conditions of motor operation is a 3-point box undesirable? Why? Show that this objection is overcome by the use of a 4-point box. Sketch the connections of a 4-point box. What is the principle advantage of having the hold-up magnet in series with the shunt field?

27. Sketch the connections of a starting box containing the field resistance. Why is it necessary to short-circuit this resistance on starting? How is this accomplished?

28. How should a shunt motor be stopped? Give reasons. What is the effect of stopping the motor by throwing back the starting arm?

29. Sketch the connections of series motor starters. What is the advantage of the no-load release over the no-voltage release?

30. When are controllers used and why? What two functions may a controller perform outside actual starting duty?

31. What are two advantages of automatic starters in medium sizes of motors? In the larger sizes of motors? What limits the rate of cutting out resistance in the type shown in Fig. 304? How is this starter operated?

32. Upon what principle do the plungers and solenoids of the E.C. & M. controller operate? Why do the plungers remain down when the current is large? Why do they rise and close the contacts when the current decreases?

33. What is the principle of the magnetic blow-out? When is it used?

34. Of what material are resistance units for the smaller types of starting boxes made? The larger types?

35. What two factors only can be varied in securing speed control of a motor? In the armature resistance control, which of these factors is varied? What are the advantages of this method of control? Name two serious disadvantages.

36. What is the principle of the multi-voltage system? How are coarse adjustments of speed obtained? Fine adjustments? What is the objection to this system?

37. What factor in the speed equation is varied in the Ward-Leonard system of speed control? How many machines are necessary in this system? What is its chief advantage and where has it been used extensively? Name two disadvantages.

38. What factor in the speed equation is varied in the field control method? Name two distinct advantages of this method. What limits the range of speed obtainable? What type of motor is especially adapted to this type of speed control?

39. What principle is involved in the speed control of the Stow motor? Why can a wide range of speed, with good commutation, be obtained with this motor?

40. Upon what principle does the Lincoln motor operate? What are its advantages?

41. What is meant by series-parallel control of railway motors? Why is such control desirable? Sketch the half speed and the maximum speed connections in a 2-motor car. In a 4-motor car.

42. Give three reasons why it is objectionable to place the main controller on the platform in the larger sizes of electric car equipment. How are these objections overcome? Give two other reasons why automatic control is desirable.

43. What is the general principle underlying the multiple-unit control? What is the train line?

44. Name briefly the sequence of closing of the contactors in starting a train.

45. What is meant by "dynamic braking?" Where is it used? Can a motor armature be brought to a standstill by this method of braking? Explain. What is regenerative braking and when is it used?

46. Give two occasions where it is desirable to know the efficiency of a motor. What type of brake is often used for loading motors? Does this type lend itself to ready calculation of torque and power output of the motor? Explain. What is meant by the dead weight of the brake arm and how can it be determined and correction be made?

47. Describe a simple type of rope brake. How many balances are necessary in this type? What is a common method of cooling prony brakes?

48. In what way does a speed counter differ from a tachometer? Upon what principle is the magneto-voltmeter method of measuring speed based?

PROBLEMS ON CHAPTER XII

328. A bundle of 16 wires lies perpendicular to a magnetic field whose density is 800 lines per sq. cm. That part of the bundle of wires which lies

in this field is 25 cm. long. What force in kilograms is acting on the entire bundle when a current of 12 amp. flows in each wire, the direction of current being the same in each?

329. A gear having 130 teeth drives another having 60 teeth. The distance from the center of the first gear to the point of contact of the teeth is 6.5 in., the pitch circle having a diameter of 13 in. The pressure between the teeth at the point of contact is 400 lb. What is the torque in pound-feet developed by each of the gears?

330. A pulley having a diameter of 14 in. drives a 50-in. pulley with a 6-in. belt. The respective tensions in the tight and loose sides of the belt are 1,500 and 300 lb. respectively. What net torque in pound-feet is developed by each pulley?

331. A coil consisting of 16 turns of wire lies parallel to a magnetic field having a strength of 30,000 lines per sq. in. (See Fig. 286*a*.) The distance across this coil parallel to the field is 12 in. and 14 in. of active conductor lie in the magnetic field. What torque in kilogram-meters is developed by the coil when the current per conductor is 5 amp.? Sketch the coil and the magnetic field, indicating the directions of the forces acting.

332. Repeat problem 331 for a similar coil in which the current in each conductor is 8 amp. and the strength of field is 40,000 lines per sq. in. Obtain the result in pound-feet.

333. When the flux density in the air gap of a shunt motor is 45,000 lines per sq. in. and the armature current is 60 amp., the motor develops 80 lb.-ft. torque. What is the torque developed when the motor takes 30 amp., the flux remaining constant? 50 amp.?

334. When the load is entirely removed from the armature of problem 333, the motor armature requires 8 amp. to keep it running. What torque is required to overcome the motor losses? What is the torque available at the pulley in each case of problem 333, assuming that the no-load torque remains constant?

335. When the current (60 amp.) of problem 333 is halved, the flux is also halved. What torque is developed?

336. The armature of a shunt motor has a resistance of 0.04 ohm. When this motor is connected across 110-volt mains, it develops a counter electromotive force of 105 volts. What current does the armature take? What current would it take if it were connected across the same mains while stationary?

337. What counter electromotive force does this motor armature develop when it is taking 80 amp. from the mains? If this same machine were running as a generator what would be its internal electromotive force when the armature is delivering 80 amp. at 110 volts?

338. The armature of a 4-pole shunt motor has 420 surface conductors and is wave wound. What back electromotive force does it develop when rotating at 1,400 r.p.m.? The flux is 2,500,000 lines per pole. Its armature resistance is 0.2 ohm. What is its terminal voltage when the motor takes 50 amp. from the line?

339. What current does the armature of the motor in problem 338 take from the line when its speed is 1,360 r.p.m. if the terminal voltage and flux remain constant?

340. A shunt motor has an armature resistance of 0.1 ohm. When connected across 220-volt mains it takes 5 amp. and runs at 1,100 r.p.m. At what speed will it run when its armature current is 40 amp.? Neglect armature reaction.

341. A compound winding having a resistance of 0.05 ohm is added to the motor of problem 340. This increases the motor flux 20 per cent. between zero current and 40 amp. Assuming that the increase of flux is proportional to the current, find the speed at 40 amp. When running at 5 amp. the speed is adjusted to 1,100 r.p.m. Neglect armature reaction.

342. By what percentage should the flux of problem 340 be decreased in order that the speed at 40 amp. be the same as it is at 5 amp? Neglect armature reaction.

343. A 550-volt series motor has a series field resistance of 0.05 ohm and an armature resistance of 0.2 ohm. When taking 90 amp. from the line its speed is 480 r.p.m. What is its speed when it takes 40 amp. from the line? Assume that the saturation curve is a straight line and neglect armature reaction.

344. A 220-volt shunt motor has a field resistance of 100 ohms and its armature has a resistance of 0.15 ohm. The total line current is 50 amp. What is the back electromotive force of the armature?

345. The motor of problem 344 develops 35-lb.-ft. internal torque when taking 50 amp. from the line. What internal torque does it develop when taking 15 amp. from the line? Neglect armature reaction.

346. When the motor of problem 344 is running without load it takes 6.0 amp. from the line and runs at 1,000 r.p.m. What is its speed when taking 50 amp. and when taking 15 amp. from the line? What is its speed regulation in each case? What is the torque at the pulley in problem 345?

347. A motor runs at 800 r.p.m. when running light. It has a speed regulation of 3.5 per cent. What is its speed at its rated load?

348. When a series motor takes 40 amp. from the line it develops 120 lb.-ft. torque. What torque does it develop at 60 amp.? At 90 amp.? Assume that the saturation curve of the iron is a straight line.

349. The motor of problem 348 has an armature resistance of 0.2 ohm and a series field resistance of 0.04 ohm. If it runs from 600-volt mains and runs at 700 r.p.m. when taking 40 amp., what is its speed at 60 and at 90 amp.?

350. What is the motor speed of problem 349 when it takes 10 amp. from the line?

351. A shunt motor is rated at 44 amp. at 230 volts. Its field current is 1.0 amp. and its armature resistance is 0.2 ohm. It is desired that the motor start with 125 per cent. rated current. What should be the value of the starting resistance?

352. The motor of problem 351 reaches 25 per cent. of its rated speed before the second contact of the starting resistance is reached. When this

contact is reached it is desired that the armature current be 43 amp. What is the resistance between the first two contacts?

353. The motor of problem 351 reaches half speed when the starting arm reaches the third contact. Find the resistance between the second and third contacts. The current should again be 43 amp. when the arm touches the third contact.

354. A 220-volt shunt motor has an armature resistance of 0.15 ohm. When the armature takes 4 amp. from the line it runs at 1,000 r.p.m. It is desired to obtain 600 r.p.m. at 44 amp. by inserting resistance in the armature circuit. What external resistance is necessary? With this external resistance in circuit, at what speed will the motor run when the armature takes 22 amp.?

355. Repeat problem 354 for 300 r.p.m.

356. What percentage of the line power is delivered to the armature at 44 amp. in problems 354 and 355?

357. A motor when connected across the 110-volt mains of Fig. 309 runs at 600 r.p.m. What speeds can be obtained by the use of this system if the shunt field is kept constant? Neglect the I_aR_a drop in the motor armature.

358. In a Ward-Leonard system of speed control the efficiencies of the machines are as follows: M_1 (Fig. 310) 85 per cent.; G, 83 per cent.; M_2, 80 per cent. The line voltage is 220 volts. When M_2 delivers 7 hp. how much current is being supplied by the line? What is the over-all efficiency of the system?

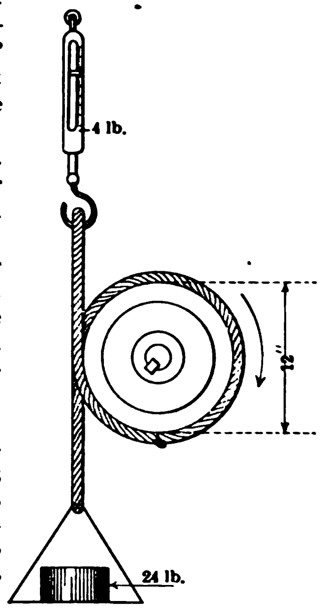


Fig. 362*a*.

359. In a brake similar to that shown in Fig. 317, the length L is 2 ft. The balance reading is 32 lb.; the dead weight of the arm is 2.8 lb.; the speed of the armature is 1,120 r.p.m. (*a*) What horsepower does this motor develop? (*b*) The motor input is 49 amp. at 220 volts. What is its efficiency at this load?

360. Repeat problem 359 for a balance reading of 23 lb. and a speed of 1,130 r.p.m. The motor input is now 36.2 amp. at 220 volts. The dead weight of the arm remains unchanged.

361. In a brake similar to that shown in Fig. 319, the diameter of the drum is 10 in. The speed is 1,400 r.p.m. One balance reads 19.8 lb. and the other reads 4.3 lb. (*a*) What torque does the motor develop at this load? (*b*) What is the horsepower output? (*c*) If the input is 18 amp. at 110 volts, what is its efficiency at this load?

362. Calculate the horsepower output developed by the brake shown in Fig. 362*a*. It is running at 1,500 r.p.m.

QUESTIONS ON CHAPTER XIII

1. What happens to the energy which is lost within electrical apparatus? Is it useful or otherwise? Explain in detail its effect on the apparatus.

2. Into what three groups can the losses in either a motor or a generator be classified? Name the losses under the first group, indicating how they are determined. Are they readily determinable?

3. What constitutes the losses of the second group? How are the losses supplied, electrically or mechanically? Upon what do they depend? How is the eddy current loss made small? What is meant by pole face loss and to what is it due? How is it reduced?

4. Why are all the losses except the copper loss grouped as one? Upon what do they all depend? If it is desired to duplicate stray power losses under two different conditions of load, what two factors must be maintained constant?

5. Show that if the losses in a machine are known at any particular load its efficiency can be calculated. Why is the formula for a generator different from that of a motor?

6. How may the efficiency of a generator be measured directly? What practical conditions make such measurements difficult? What effect do errors in the measurements have upon the precision of the results? What other objections are there to direct measurements of efficiency?

7. How is a machine ordinarily operated in order to measure its stray power? What measurements are made? To what is the stray power then equal?

8. In stray power measurements, how is the flux adjusted to the proper value? How is the speed adjusted? Does the flux adjustment have any effect upon the speed and if so how are any readjustments made?

9. For what purpose is a set of stray power curves desirable? Why cannot the stray power over the entire operating range of the machine be shown with one curve? What errors are introduced by using the field current as a measure of the flux and how is one of these errors partially neutralized?

10. For determining losses what is the advantage of the opposition method over the stray power method? Upon what principle does this method depend?

11. What assumption is it necessary to make in the opposition method? Does this assumption introduce appreciable error? In this method how are the two machines started and then adjusted? What instruments are used and what measurements are necessary? State the disadvantages of this method.

12. What determines the rating of a steam engine? A steam turbine? A gas engine? An electric machine? State reasons in each case?

13. State the effects of excessive temperatures upon the insulation of electric machinery. What insulating materials can withstand the highest temperatures?

14. What is the "hot spot" temperature and what difficulties accompany its measurement? Name one method by which an approximation of the *hot spot* temperature may be reached.

15. What other well-known principle is utilized to determine the temperature rise? What is meant by "ambient" temperature?

16. For what length of time should a temperature test be run? How may the temperature rise be accelerated? In what way may a machine's approach to constant temperature be determined? Why does the temperature of a machine rise more rapidly at the beginning of a test than at the end? What relations exist between the heat supplied and the heat dissipated when a constant temperature is reached?

17. Why must care be taken not to include the brush and contact resistance when measuring the armature resistance for temperature determinations? Where must the voltmeter leads be held?

18. What difficulties arise when the resistances of multi-polar armatures are measured for temperature determinations? How may these difficulties be eliminated? What precautions should be taken when the field temperature is being determined by resistance measurements? Why must the temperature measured in this way be still further increased?

19. Give five reasons why it is either necessary or desirable to operate shunt generators in parallel. What, in their characteristic, makes them especially adapted to parallel operation?

20. If one generator starts to take more than its share of the load, what is the resulting effect upon its voltage? Does this effect oppose the generator's taking additional load? Explain. What is meant by "stable equilibrium."

21. State in detail the steps necessary to put a machine in service. If the generator is put in service with its voltage equal to that of the bus, why does it not take load? What must be done in order that it may take its share of the load?

22. Describe the steps necessary to remove a machine from service. Why is it undesirable to pull the generator switch when the machine is delivering load? What is necessary as regards the generator characteristics in order that the machines may properly divide the load over their entire range of operation?

23. Show that over-compounded generators in parallel are in unstable equilibrium. What simple connection can make their operation stable?

24. What two conditions are necessary for two compound generators to divide the load proportionately over their entire range of operation?

25. Why does not a diverter change the division of load between two compound generators in parallel? What adjustment can be made to change the load division?

26. How many equalizers may be necessary in certain types of compound generators? How many poles must such a generator switch have?

27. Compare circuit breakers and fuses, stating the advantages and disadvantages of each. Which has the higher first cost? Which ordinarily has the lower maintenance cost? Which requires the more space? Which operates the faster? Under what conditions should each be used?

28. Upon what principle do circuit breakers in general operate? How is a "wiping contact" secured? What is the purpose of the carbon contacts? Why should breakers always be mounted at the top of a switchboard?

PROBLEMS ON CHAPTER XIII

363. The eddy current loss in a generator is 300 watts when it is running at 800 r.p.m. and with a flux of 1,000,000 lines per pole. (*a*) What is the eddy current loss when the speed is raised to 1,000 r.p.m., the flux remaining unchanged? (*b*) What is the loss at 800 r.p.m. with a flux of 1,200,000 lines? (*c*) With this same flux what is the loss at 1,000 r.p.m.?

364. If the hysteresis loss in a generator is 600 watts with a speed of 800 r.p.m., what is it when the speed is increased to 1,000 r.p.m.? To 1,200 r.p.m.? The flux is the same in each case.

365. A shunt generator delivers 100 amp. at 115 volts. The total losses in the machine are 1,200 watts. What is its efficiency?

366. A shunt motor takes 35 amp. at 220 volts. Its field resistance is 180 ohms, its armature resistance is 0.3 ohm and its stray power loss is 400 watts. What is its output in horsepower and what is its efficiency at this load?

367. Assume that the stray power loss of problem 366 is in error by 10 per cent. What error does this introduce into the efficiency?

368. A shunt generator delivers 250 amp. at 220 volts. The shunt field current is 5 amp., the armature resistance 0.035 ohm and the stray power loss is 1,800 watts. (*a*) What is the horsepower input to the generator? (*b*) What is its efficiency? (*c*) If the generator speed is 400 r.p.m. what is the torque applied to its shaft?

369. A shunt generator is running light as a motor. Its armature takes 15 amp. from 220-volt mains. The armature resistance is 0.08 ohm. What is the stray power loss of the machine under these conditions?

370. A motor running light takes 25 amp. from 110-volt mains. Its shunt field resistance is 12 ohms and its armature resistance is 0.016 ohm. What is the stray power loss in this motor?

371. It is desired to measure the stray power loss in the generator of problem 368 by running it light as a motor. To what values should the terminal volts and speed be adjusted? Make a diagram of connections showing the instruments and methods of adjustment.

372. Two similar 10-kw. 220-volt generators are connected as shown in Fig. 330 for the purpose of having their losses measured. When the motor is taking its rated current of 48 amp. the line current I is found to be 7.5 amp. The generator field current is 2.4 amp. and the motor field current is 1.8 amp. Each machine has an armature resistance of 0.2 ohm. What is the stray power of each machine at this load? What is the efficiency of each machine at this load?

373. When the current in the generator armature of problem 372 is 24 amp. (half load) the line current I is 5 amp. The generator field current is now 2.2 amp. and the motor field current is 1.8 amp. Determine the stray power and the efficiency of each machine at this load. Why are the field currents different in the two machines?

374. The armature and field resistances of a 550-volt shunt generator are measured after the machine had been standing idle for some time in an engine

room whose temperature is 30° C. The voltage across the field winding exclusive of the rheostat is found to be 420 volts and the field current 4.8 amp. The armature resistance between two marked commutator segments is found to be 0.21 ohm. After the machine had been running under load for 2 hours these same measurements were repeated. The field voltage is now 450 volts and the field current 4.8 amp. The armature resistance is now 0.225 ohm. What is the temperature rise of each? Are these maximum temperatures safe for untreated cotton insulation?

375. Two 50-kw. 220-volt generators are operating in parallel. They both are adjusted to 230 volts at no load and are then paralleled. In generator No. 1 the voltage drops 8 volts from no load to full load and in No. 2 it drops 12 volts. When the aggregate load on the system is 360 amp., what current does each generator deliver? What kilowatt load does each deliver? Assume that the voltage drops in a straight line in each case.

376. Repeat problem 375 for an aggregate load of 400 amp.

377. It is desired to operate a 100-kw. 220-volt generator and a 60-kw. 220-volt generator in parallel. The voltage of the first drops 8 volts from a no-load voltage of 230 when its rated load is applied. What should be the voltage drop of the second generator in order that each may take its proportionate share of the load at all times? Assume that the voltage drops follow a straight line in each case. How much current does each deliver when the system demand is 700 amp.?

378. Two compound generators are operating in parallel. One has a rating of 100 kw. and the other a rating of 75 kw. The resistance of the series field of the first is 0.002 ohm. What should be the resistance of the series field of the second machine for proper division of load?

QUESTIONS ON CHAPTER XIV

1. Why cannot the ordinary direct current voltages be used for transmitting considerable amounts of power over long distances? Where is direct current most commonly utilized? What are its advantages under these conditions?

2. What is the general scheme of getting large amounts of power from a remotely situated power station to the consumers' premises? What are the ranges of transmission voltages? Of distribution voltages? What part does the substation play in the system?

3. How does the weight of conductor vary with the transmission voltage? If the transmission voltage were doubled how would the weight of copper be affected the other factors remaining unchanged?

4. What five conditions in general determine the size of conductor to be used? For what conditions does the question of heating particularly apply? How may the economics of the problem determine the size of conductor? What is the disadvantage of having too large a conductor? Too small a conductor?

5. Why is 110 volts most convenient for incandescent lighting? Why is a higher voltage undesirable? What are the advantages and disadvantages of a lower voltage for this purpose?

6. What are the common trolley voltages? Why are these voltages so chosen?

7. What is meant by distributed loads? Where do such loads occur? Where are conductors of uniform cross-section throughout most commonly used?

8. Theoretically, what type of conductor is most economical for uniformly distributed loads? What is the practical condition that most nearly approaches this theoretical condition?

9. Why is the "return loop" system of distribution used? What is its one disadvantage?

10. What system overcomes the disadvantage of the return loop system? Make a sketch and show how this system may be further modified to form a still more efficient system.

11. What advantage is gained by connecting 110-volt loads in series groups of two and utilizing 220-volt supply? What are the disadvantages of so grouping the loads?

12. How are the objections to the series-parallel system overcome? What are the relations existing among the voltages of the Edison 3-wire system?

13. If the neutral wire be of the same size as the two outers, what are the relative weights of copper in the 3-wire system with 220 volts across outers and in the simple 110-volt system?

14. What is meant by balanced loads? Under this condition how much current flows through the neutral?

15. In what direction does the neutral current flow if the positive load is the greater? The negative load? What relation does the neutral current bear to the current in the outer wires? What type of ammeter should be used in the neutral? What is the commercial limit of unbalancing?

16. State briefly the effect of opening the neutral with (*a*) balanced loads and (*b*) unbalanced loads. Why is the neutral usually grounded?

17. What in general is the effect of putting too heavy a load on one side of a 3-wire system upon the voltage on that side of the system? Upon the voltage on the other side of the system?

18. Sketch a method of obtaining a neutral by the use of two shunt generators? What is the principal disadvantage of this method?

19. How may a storage battery be used for obtaining a neutral? In general how does the current in the neutral wire divide when it reaches the center of the battery?

20. Upon what principle does the balancer set operate? What determines which machine shall operate as a motor? As a generator? What two methods are used to accentuate the motor and the generator actions?

21. Upon what principle does the 3-wire generator operate? Where does the alternating current flow? The returning direct current from the neutral? How is the direct current able to pass so readily back into the armature?

22. How in general is power supplied to congested direct current loads? What is the function of the feeders? The mains? The junction boxes?

Where are the house services connected? How are the voltages at feeding points generally determined?

23. What type of generator is most commonly used for railway purposes? How is it connected to the system?

24. Under what conditions does a single trolley suffice for getting the power to the car? If a single trolley of the ordinary size is of insufficient cross-section, what means can be taken to assist it in supplying the required power? Why is the size of trolley not increased?

25. Under what conditions are multiple feeders employed? What is the disadvantage of their use? How may this disadvantage be overcome?

26. Why does the return current from a trolley car leave the track? What determines the paths which it follows? What damage, if any, occurs at the point where the current enters a pipe? Where it leaves the pipe?

27. Name two methods by which electrolysis may be reduced. What measurements give a good idea of stray currents between pipes and track?

28. Sketch a typical central station load curve. Show how the habits of a community determine the general shape of such a curve. Why is such a load curve far more undesirable than a uniform load curve having the same total kilowatt-hours?

29. What is meant by load factor? Is a high or a low load factor desirable? Why?

30. How may a storage battery smooth out a station load curve? When should the battery be charged? Discharged? Why are storage batteries not more generally used for this purpose?

31. Where can storage batteries be used efficiently to carry the load in off-peak times? For what purposes are they now commonly used by central stations? Where should they be located? Under what conditions is a battery very useful to a central station?

32. What difficulty is met when an attempt is made to operate storage batteries in conjunction with a power plant? What simple method may be used to control the battery load? What is the objection to this method?

33. Upon what simple principle do the counter-electromotive force cells operate? What is the chief advantage of this method of control over the resistance method?

34. What is meant by end cell control? How is such a battery charged? In what manner is the connection changed from one cell to the next without opening the circuit or dead-short circuiting the batteries?

35. What is meant by a "floating" battery? What is the purpose of such a battery? Why is it often necessary to install auxiliary means for accentuating the battery charge and discharge with change of load? Sketch the connections of one simple method for accomplishing this purpose.

36. Why will a battery placed at the end of a long feeder tend to equalize the station load without auxiliary apparatus for charging and discharging? Under what conditions does such a battery "float?"

37. What is the essential difference between the series system and the parallel system of distribution? In the series system what is the effect of

attempting to remove a load by opening the circuit? How is a load cut out in a series system?

38. By what devices is a series system supplied? What are the advantages of the series system? Where does its field of application lie? Sketch the layout of two different systems of series arc distribution. Name the advantages of each.

PROBLEMS ON CHAPTER XIV

379. 140 kw. are transmitted a distance of 1,000 ft. over a cable of such size that there is a potential difference of 215 volts at the load with 225 volts at the bus-bars. (*a*) What size of feeder is used, assuming that a mil-foot of copper has a resistance of 10 ohms? (*b*) What is the weight of copper if a cubic inch weighs 0.32 lb.?

380. Repeat problem 379 with the same power, the same loss, the same distance and the same percentage line drop, but with 550 volts at the load. How do the weights of copper compare with the respective voltages in the two cases?

381. A 10-hp. motor is fed from a switchboard the bus-bars of which are maintained at 115 volts. The motor is located at a distance of 500 ft. from the switchboard and it is desired to have a voltage of 110 at the motor terminals when the motor is carrying its full load of 10 hp. What must be the diameter (mils)?

(*a*) Of the copper wire used to connect the motor to the switchboard? Assume a temperature of 50° C. 1 hp. = 746 watts. The efficiency of the motor is 86 per cent.

(*b*) If copper weighs 0.32 lb. per cu. in., what will be the weight of the wire in (*a*)?

(*c*) Repeat (*a*) and (*b*) for a switchboard voltage of 230 and the same per cent. drop to the motor.

(*d*) Repeat (*a*) and (*b*) for a switchboard voltage of 550 and the same per cent. drop to the motor.

382. A certain street is 2,000 ft. long. It is illuminated by eleven 200-watt multiple-connected lamps placed 200 ft. apart. No. 4 A.W.G. conductors are used to supply this system. The voltage at the feeding end of the street is 120 volts. What is the voltage drop between each two adjacent lamps? What is the voltage at the last lamp? Assume that each lamp takes 2.0 amp.

383. If the lamps of problem 382 are fed by the anti-parallel system (see Fig. 343*a* page 385), No. 4 wire still being used, determine the voltage the lamps on the two ends of the street. Compare their absolute volta and their difference of voltage with the results of problem 382.

384. A load of 100 amps. is situated 800 ft. from 600-volt bus-bars. 1,000 ft. farther on a second load of 65 amps. is located. A 4/0 annealed copper feeder runs from the bus-bars to the first load. No. 1 wires run from the 100 amp. load to the 65 amp. load. (*a*) What is the voltage at each load? (*b*) What is the weight of copper used?

385. (*a*) Determine the size of a uniform feeder which will have the same weight as the two feeders of problem 384. (*b*) Determine the voltage at each load with this uniform feeder. (*c*) Under which condition is the copper most effectively utilized?

386. It is desired to operate 40 75-watt lamps on one circuit. Compare the sizes of wire necessary to feed these lamps when all are connected in parallel across 110 volts and when the lamps are connected in series groups of two across 220 volts. (Use Table Appendix D, page 410.)

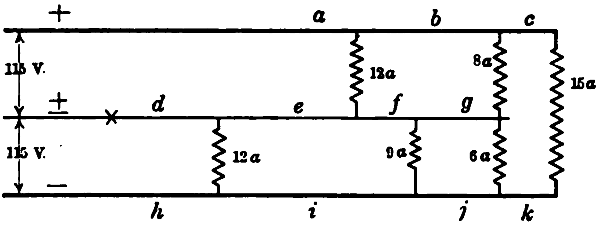


FIG. 388*a*.

387. Repeat problem 386 for an Edison 3-wire system, assuming that the neutral is the same size as the outer wires.

388. Fig. 388*a* shows an Edison 3-wire system with various loads. Indicate the current and its direction at each of the points *a*–*k* inclusive.

389. If the neutral is cut at point *X*, Fig. 388*a*, find the voltages across the two sides of the system, assuming that the load resistances do not change. Neglect the drop in the mains themselves.

390. Find the voltages across loads *A* and *B*, Fig. 390*a*, if loads *A* and *B* are each 40 amp.

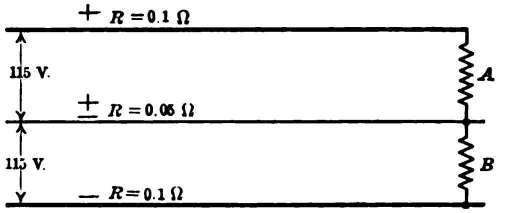


FIG. 390*a*.

391. Repeat problem 390 when load *A* is 60 amp. and load *B* is 20 amp.

392. Find the voltage across each load, problem 391, which would occur if the neutral were opened.

393. Determine the voltage across each of the loads *AB* and *BC*, Fig. 393*a*, and also the voltage across the motor.

394. Repeat problem 393 with the motor connected between the neutral and the negative conductor. Owing to the fact that the voltage is halved the motor must now take approximately 200 amp. to develop its former power.

395. Find the current in each machine of the balancer set of Fig. 395*a* and indicate which machine is the motor and which is the generator. There is 110 volts across each machine and the efficiency of each machine is 80 per cent. How much current does the main generator deliver?

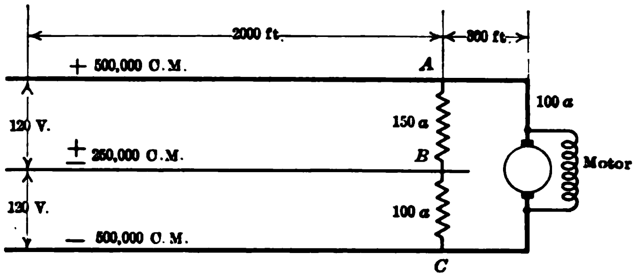


Fig. 393*a*.

396. Solve problem 395 when there is a total load of 100 amp. connected between the neutral and negative main and there is no load on the positive side.

397. A 4/0 hard-drawn copper trolley wire runs from 600-volt bus-bars to a station 6 miles out. For 4 miles it is paralleled by a 350,000 C.M. feeder which feeds it every quarter mile. (See Fig. 357*b*, page 397.) The 4/0 wire has a resistance of 0.26 ohm per mile and the 350,000 C.M. feeder has a resistance of 0.163 ohm per mile. The resistance of the track and ground return is 0.05 ohm per mile. Find the voltage at a car 5 miles out and taking 60 amp. What is the voltage at the end of the line?

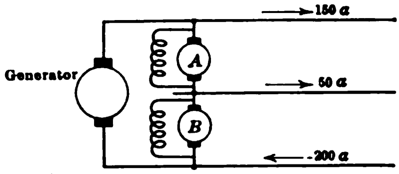


Fig. 395*a*.

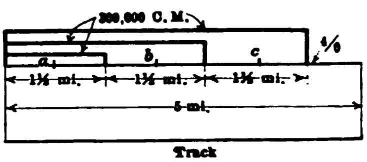


Fig. 399*a*.

398. (*a*) Find the voltage at the car in problem 397, when the car is 4 miles from the power station. (*b*) When the car is 2 miles from the station.

399. Fig. 399*a* shows a 5-mile length of hard-drawn 4/0 copper trolley wire. This is fed by three 300,000 C.M. multiple feeders each feeding at points 1½ miles apart. Find the equivalent resistance of the trolley and feeders to the end of the line.

400. Find the voltage at a car, Fig. 399*a*, when it is at the end of the line and taking 100 amp. The station voltage is 600 volts and the ground and track resistance is 0.04 ohm per mile.

401. Find the voltage at the car of problem 400 when the car is 3 miles from the station and taking 100 amp.

402. Repeat problems 400 and 401 for a sectionalized trolley, the insulated sections being at *a*, *b*, *c*, Fig. 399*a*. Which system of feeding is the more economical of copper?

403. A central station has a peak load of 6,200 kw. It delivers 52,000 kw.-hr. over a 24-hour period. What is its daily load factor?

404. A storage battery helps the station of problem 403 by taking 1,500 kw. off the peak, it being necessary to use this battery for an hour and a half. If the battery efficiency is 85 per cent. and it is charged off peak, what is the new load factor of the station?

405. A storage battery consists of 64 cells connected in series, each cell having an electromotive force of 2.1 volts and a resistance of 0.004 ohm. It is desired to allow this battery to discharge at a 40-amp. rate into 115-volt bus-bars. (*a*) What series resistance is necessary? (*b*) How much power is developed within the battery? (*c*) How much is lost in the battery resistance? (*d*) How much is lost in the series resistance?

406. If counter-electromotive force cells, each having an electromotive force of 2.08 volts, were used in problem 405, how many would be necessary? Neglect the internal resistance of the counter-electromotive force cells.

407. If end cell control were used in problem 405 how many cells would be cut off the end of the battery?

408. A 4/0 hard-drawn copper trolley, having a resistance of 0.26 ohm per mile, extends 5 miles from 600-volt bus-bars. The track return has a resistance of 0.05 ohm per mile. A storage battery consisting of 260 cells floats at the end of the line. Each cell has an electromotive force of 2.08 volts and a resistance of 0.002 ohm. For what value of current at the battery terminals will the battery "float?"

409. What current must a car 3 miles from the station of problem 408 take in order that the battery may "float?"

410. At what rate does the battery charge when there is no car at all on the system?

411. When a car at the battery takes 120 amp., how much current does the station supply and how much does the battery supply? How much power is supplied by each?

412. Repeat problem 411 for a car taking 120 amp., 3 miles from the station.

413. A series arc generator supplies 60 500-watt 6.6-amp. magnetite arcs over a No. 6 cable, which has a resistance of 0.395 ohm per 1,000 ft. The length of the arc circuit is 10 miles. What is the terminal voltage of the generator and what is the efficiency of transmission?

414. Repeat problem 413 for an 80-lamp circuit in which the length of the circuit is 15 miles.

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